New Layouts for Midimew Interconnection Networks

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ABSTRACT

Circulant graphs have been deeply studied in technical literature. Midimew networks are a class of distance-related optimal circulant graphs of degree four which have applications in network engineering and coding theory. In a previous work, a new layout for Midimew networks which keeps the maximum link length under $\sqrt{5}$ has been presented. The most interesting sizes have been studied: dense and quasi-dense cases.

This work reviews the layout properties presented in that work and introduces some considerations about the generalization of the folding of Midimew to any number of nodes, and its relationship with the mapping of the nodes around a 3-D Torus.

KEYWORDS: Midimew; Layout; Link length; Folded Torus;

1 Introduction

In current computer systems, performance is being limited by interconnections. Therefore, the use of replicated hardware interconnected by a suitable network is emerging as a design solution for multiple digital systems. Currently, networks are used in the design of systems on chip, cluster microprocessors and on-chip multiprocessors. Nowadays, Torus and Meshes still compete in the design of the latest parallel systems. Circulant graphs are also attractive topologies that have been extensively studied due to their good distance-related properties and their optimal connectivity.

Midimew networks are a family of undirected degree four circulant graphs having optimal distance properties, that is, minimum diameter and average distance [3] for a given number of nodes. Midimew networks can be successfully applied to the design of parallel systems as they exhibit remarkable performance improvements compared to other degree four topologies such as Torus. Peripheral links present implementation problems, which can be solved with adequate node mapping that bound link length. In this work, rectangular mappings and mapping over a 3D-torus, and their relations, are studied.

2 Midimew Networks

A degree four circulant graph with N vertices and jumps $\{a,b\}$ is an undirected graph in which each vertex $n \in \{0,\dots,N-1\}$ is adjacent to the four vertices $n \pm a, n \pm b$, (operations modulo N). Midimew networks [2] are a class of optimal degree four circulants. We will only refer to the dense case here, with $N = 2k^2 + 2k + 1$ nodes and jumps $\{k, k+1\}$. In [3] it was proved that these graphs have minimum average distance, and diameter k. A minimal routing scheme was also proposed. Respect to Torus, diameter is improved by a factor of $\sqrt{2} = 1.41$, and average distance by 1.06.

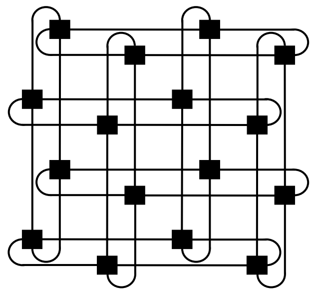

Figure 1: Twisted Torus

Midimew networks, such as 2D-Tori, are mesh-like topologies with wrap-around links, whose lengths grow with the network size. Since internal links are supposed to have unitary length, most applications can be negatively affected by this unbalance. The Folded Torus presented in Figure 1 is a solution to equalize the Torus links by increasing the wire length to 2.

3 Bounded link length Layout for Midimew Networks

Our methodology starts from a base layout having $2k$ rows and $k+1$ columns, plus a node in a separate row, node 0. Every node in the network has an integer label, which identifies the node and determines its adjacencies. Nodes are arranged in label order, and links are laid to connect nodes with jumps k and $k+1$. Figure 2 shows an example for $k = 3$ and $N = 25$. Vertical links represent jumps in $k+1$ and diagonal links represent jumps in k. Apart from the label, we will refer to the position of a certain node by the number of row $\{1,...,2k+1\}$ and column $\{1,...,k+1\}$ where it is located. Given a row with nodes $\{1,2,...,n\}$, we define two shuffle transformations:

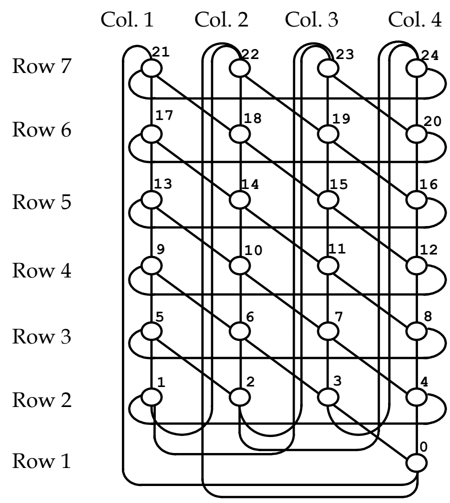

Figure 2: Initial Layout, case N = 25

- **Shuffle A:**

$$x' = 2x - 1 \quad if\ x \le \frac{(n+1)}{2}$$
$$x' = 2n - 2x + 2 \quad if\ x > \frac{(n+1)}{2}$$

- **Shuffle B:**

$$x' = 2x \quad if\ x < \frac{(n+1)}{2}$$
$$x' = 2n - 2x + 1 \quad if\ x \ge \frac{(n+1)}{2}$$

Algorithm: Bounded Layout for Dense Midimews

***Step 1 or Initial layout*:** Arrange the $2k^2 + 2k + 1$ nodes in $2k+1$ rows, beginning with a row containing a single node (node 0, at the end of row 1) and setting the following nodes consecutively as in the initial layout of Figure 2.

Step 2 or Row rotation and shuffle:

- For rows $1 \le i \le k+1$: rotation $m = \left\lfloor \frac{i-1}{2} \right\rfloor$, shuffle A to odd rows and B to even ones.
- For rows $k+2 \le i \le 2k+1$, rotation $\left\lfloor \frac{i}{2} \right\rfloor$, shuffle B to odd rows and A to even ones.

Step 3 or Column shuffle A: Shuffle all columns according to shuffle A.

Mapping example: Previous algorithm [4] has been proved to map any dense midimew network into a grid with bounded link length. As an example, the steps for a $k = 4, N = 41$ nodes network are presented. Figure 3 shows the initial layout for this network, with $k+1=5$ columns and $2k+1=9$ rows. After the second step, links are arranged as in Figure 3 right. Finally, column rotation generates the final layout shown in Figure 4.

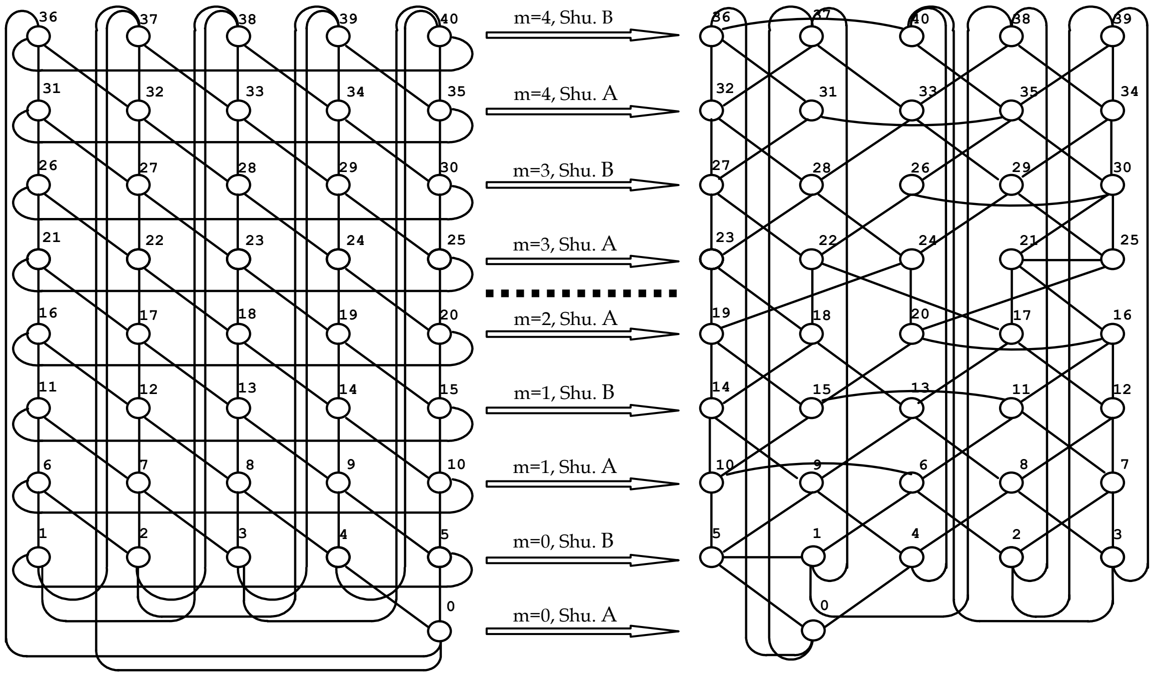

Figure 3: Initial disposition and row shuffle example for case k=4.

4 Midimew Folding and 3-D Torus mapping

Previous works [1] have shown that dense Midimew networks can be embedded into a 3-D Torus with constant link length and without two links crossing each other. Figure 5 shows the mapping of the dense N=61 nodes midimew into a 3D Torus. Links in *k* and *k+1* are plotted with different lines. Each of these two groups conform two helices intersecting each other in the N nodes, this is, two orthogonal Hamiltonian paths.

There is a not obvious relationship between the Torus mapping and the proposed layout for the dense Midimew. The 3-D Torus is composed of two parts, front ($y > 0$) and back ($y < 0$). If the nodes and links in the front were projected over the plane Y = 0, and their position was adjusted to fit a rectangular grid, the result would be approximate to the one presented with solid lines in Figure 6. The dashed lines in that figure represent the projection of the back part of the figure, interleaved with the previous rows.

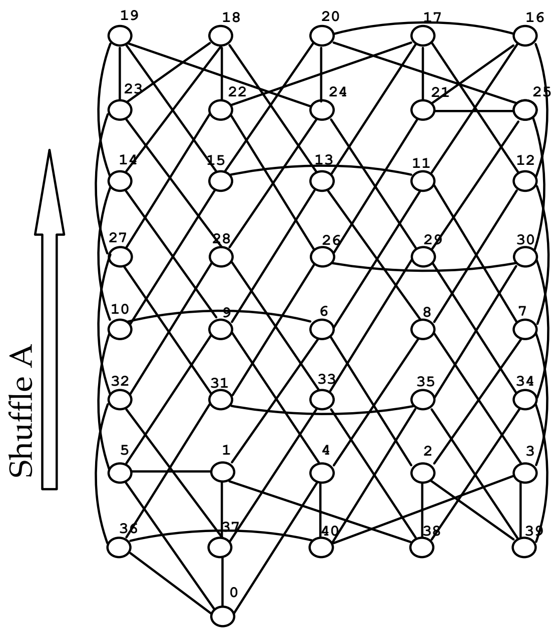

Figure 4: Column shuffle example, k=4

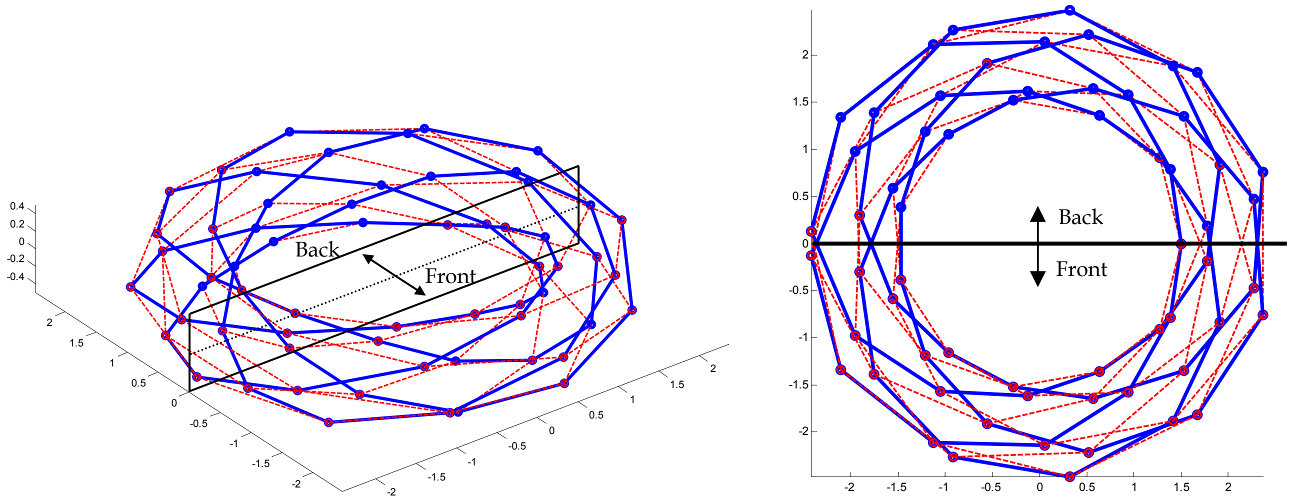

Figure 5: 3D Mapping of a 61-node dense midimiew over the surface of a torus.

Note that, for the sake of simplicity, Figure 6 shows the case $k = 4, N = 41$, and that some links have been removed: those ones that join nodes from the back and front parts, and those ones that join nodes in the same row. Initially, observing the links over the surface of the Torus, there should not be links within the same row; this is, however, an effect of the adjustment of the nodes to the grid.

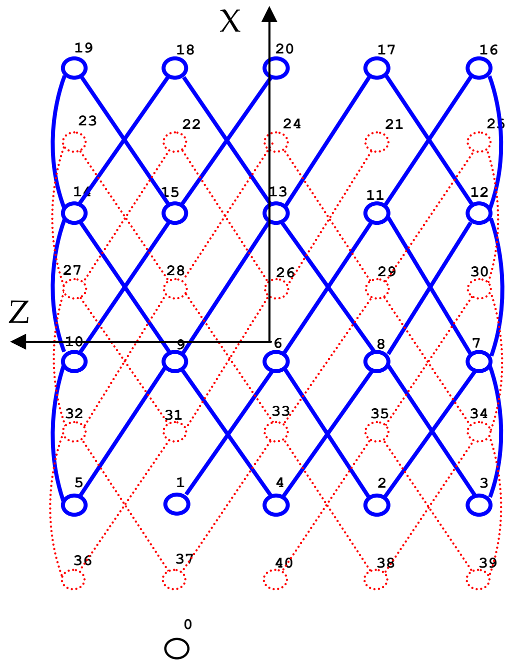

Figure 6: Projection of the front and back

5 Conclusion

The relationship between the 3D mapping of a dense Midimew network and its mapping on a rectangular layout with bounded link lengths is presented. This relationship is proposed as the basis for further study of an algorithm that could map any generic midimew network into a grid with bounded link length.

References

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