f-Vectors and Flag Vectors of 4-Dimensional Polytopes

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The face number f_i of a polytope or complex is the number of i-dimensional faces. In 1906, Steinitz gave a complete characterization of the f-vectors (f_0, f_1, f_2) of the 3-dimensional convex polytopes: They are the integer points in a certain 2-dimensional convex polyhedral cone. His result is remarkably simple; it is easily extended to the larger generality of strongly regular cell decompositions of the 2-dimensional sphere, and of Eulerian lattices of length 4.

In this lecture, I will survey what 99 years after Steinitz' paper we know "one dimension higher", about the f-vectors (f_0, f_1, f_2, f_3) and the flag vectors $(f_0, f_1, f_2, f_3; f_{03})$ of 4-dimensional convex polytopes, and of 3-dimensional regular CW spheres, etc.

Thus we will discuss the crucial parameters of *fatness* and *complexity*, and survey various classes of examples (old and new) that are "extremal" and thus interesting in terms of their f- and flag vectors: This includes stacked and truncation polytopes, neighborly polytopes, neighborly cubical polytopes, projected deformed products, as well as lots of 2-simple 2-simplicial polytopes.