

Gröbner Bases and yet another Generalization of Associahedra

Volkmar Welker (University of Marburg)
welker@mathematik.uni-marburg.de

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In this talk we consider Gröbner bases of the following ideals:

- The ideal $I_{n,r}$ generated by the $r \times r$ minors of a general $n \times n$ matrix.
- The ideal $P_{n,r}$ generated by the Pfaffians of degree r of a general $n \times n$ skew-symmetric matrix.
- The defining ideal of the Grassmannian $G_{n,r}$ of r -planes in complex n -space.

Even though there are Gröbner basis known in each case, our study reveals new or conjectured new Gröbner basis that have the additional property that their initial ideals are Stanley-Reisner ideals of polytopes or simplicial spheres. The polytopes/spheres occurring are generalizations of Associahedra for $P_{n,r}$ and the Grassmannian and Cyclohedra for $I_{n,r}$.

These Gröbner basis are minimal in a certain sense and their existence has strong implications on the Hilbert-series of the corresponding rings.

The proof methods include methods from geometric combinatorics, commutative algebra and enumerative combinatorics.