Diamonds, Kazhdan-Lusztig Polynomials, and Hecke Algebra Representations

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In their seminal paper [Representations of Coxeter groups and Hecke algebras, Invent. Math., 53 (1979), 165-184] Kazhdan and Lusztig defined, for every Coxeter group W, a family of polynomials, indexed by pairs of elements of W, which have become known as the Kazhdan-Lusztig polynomials of W. These polynomials play a role in several areas of mathematics, including the algebraic geometry and topology of Schubert varieties, and representation theory (see, e.g., [J. E. Humphreys, Reflection Groups and Coxeter Groups, Cambridge Studies in Advanced Mathematics, no.29, Cambridge Univ. Press, Cambridge, 1990, Chap. 7] and [A. Björner and F. Brenti, Combinatorics of Coxeter Groups, Graduate Texts in Mathematics, 231, Springer-Verlag, New York, 2005, Chap. 5], and the references cited there).

Our purpose in this talk is to show that Kazhdan and Lusztig's construction can be carried out in a much more general (and entirely combinatorial) context. More precisely, we introduce a new class of partially ordered sets, which we call diamonds, which have a very rich combinatorial and algebraic structure and which include, in a very precise sense, Coxeter groups. In the talk we will define diamonds and study their basic combinatorial properties (in particular, that they are always Eulerian posets), and show how one can define a family of polynomials, indexed by pairs of elements in the diamond, which reduce to the Kazhdan-Lusztig polynomials in the case that the diamond is a Coxeter group. We will then show that every diamond contains, in a natural way, a Coxeter group, and hence a Hecke algebra. Finally, we will show that this Coxeter group, and the corresponding Hecke algebra, act naturally on the diamond, and that the resulting representations include those constructed by Kazhdan and Lusztig, but contain several new ones. We will conclude with a list of open problems, mainly of a combinatorial nature, concerning diamonds. This is joint work with F. Caselli and M. Marietti.