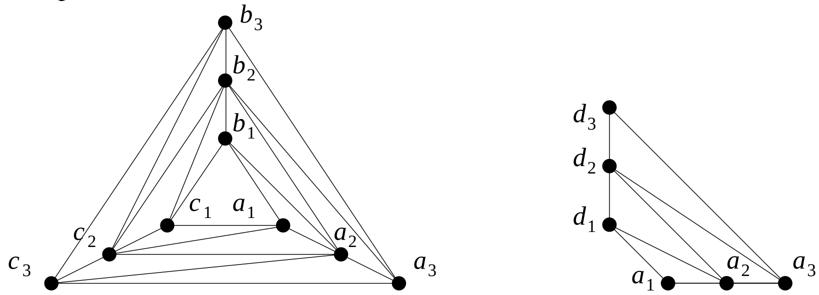
Problem Sheet 4 — Non-regular triangulations

1. Draw (again) the graph of triangulations of the mother of all examples. See the back of this page.

2. Count the number of flips in the 2-dimensional triangulation of the (left) figure below. You have $3k$ points forming k concentric triangles. The figure shows the case $k = 3$.



3. Starting with that 2-dimensional configuration, we are now going to build a 3-dimensional one.

   - Place another k points $d_1, \ldots, d_k$ vertically above your planar configuration.
   - Cone the ith concentric layer of the 2D triangulation to the ith point in the vertical line (note: the first layer is just the central triangle. The other layers consist of 6 triangles ecah.
   - In order to "fill-in" the space between different layers, insert the tetrahedra $\{a_i, b_i, d_i, d_{i+1}\}$, $\{a_i, c_i, d_i, d_{i+1}\}$ and $\{b_i, c_i, d_i, d_{i+1}\}$, for $i = 1, \ldots, k-1$. That is, make the **link** of the edge $\{d_i, d_{i+1}\}$ be the three edges of the triangle $\{a_i, b_i, c_i\}$. The figure in the right shows a vertical cut of what you get in the $a's$ and $d's$.

   Count the number of flips in this 3D triangulation.

4. Give a direct proof that no heights exist triangulating the boundary of the cube-octahedron in the "skew" way of page 7.

5. Show that there is a set of 8 points in general position in $\mathbb{R}^3$ and a triangulation of it with only three flips (i.e., prove the remark at the very bottom of page 8).

6. Let P be any 3-polytope with more vertices than faces. Prove that the point set consisting of the vertices of P together with any single interior point has a non-regular triangulation.
   (OOOUUPS, I did this one; sorry about that!)