Can Social Security be welfare improving when there is demographic uncertainty?

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Abstract

This paper studies the welfare implications of a PAYG pension system in an overlapping generations (OLG) model with demographic uncertainty and sequentially incomplete markets. In the absence of public pensions, small cohorts tend to be favoured by the changes in relative prices implied by demographic shocks. As described in Bohn (2001), PAYG Defined-Benefit systems can help to share the financial risks created by this type of demographic uncertainty across generations. The overall welfare impact depends on the balance between this insurance effect and the well-known crowding-out effect stemming from the unfunded character of the system and reductions in precautionary savings. Therefore, the question of the total welfare impact of PAYG pensions is intrinsically quantitative. In this paper we use a four-periods OLG model calibrated to the US economy to provide a quantitative assessment of the relative size of the different effects involved. The findings are not favorable for PAYG pension systems: **the gains from risk sharing offsets a very small fraction of the impact of crowding-out, making the introduction of public pensions a welfare decreasing process (even in *ex ante* terms)**. In particular, with a marginal PAYG pension scheme (providing a 2% replacement rate of the average wage) small cohorts lose the equivalent to a 1.32% of their life cycle income as young workers, while the loss for larger cohorts is 1.06%. The overall welfare reduction is 1.2%. This quantitative result is very robust to reasonable changes in the model parameters.

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1 Introduction

In the last two decades, developed countries have witnessed growing concerns regarding the financial sustainability of their Pay As You Go (PAYG) pension systems. It is widely agreed that the ability to extend current pension benefits to future generations of retirees is severely compromised.[1] This gloomy prospect has triggered a very lively public debate, and moved the study of the properties of alternative intergenerational arrangements to the top of the academic research agenda. This paper is a contribution to that fast-growing literature.

Two different demographic processes are commonly held responsible for the problems ahead: a universal trend towards higher longevity and the short-term problems posed by the retirement of the Baby-Boomers (the large cohorts born during the fifties and the sixties). In this paper we concentrate on the latter process: the consequences of large swings in the size of cohorts. We take a long view on changes in the age-composition of the population, and envisage the large cohorts of Baby-Boomers as one particular instance of a cyclical pattern governing the size of the workforce. We provide empirical evidence supporting the existence of such a "demographic cycle" in the case of the US economy and explore whether it may lead to a justification for PAYG pensions entirely based on efficiency grounds.

Even after several decades of strong research efforts, the existence of such a justification is still an open question. Classical Overlapping Generations (OLG) models formulated in deterministic, dynamically efficient settings leave no room for a welfare improving PAYG pension system (Diamond (1965) and Auerbach and Kotlikoff (1987))[2]. This is mainly due to the crowding-out of private savings. In stochastic economies, in contrast, social security can enhance welfare by substituting some missing or imperfect private insurance markets (Diamond (1977)). Once equipped with truly quantitative general equilibrium models, economist have explored this possibility intensively. Early efforts focused on idiosyncratic risks and found that the overall impact of social security was negative[3]. Attention then shifted to the intergenerational risk sharing of aggregate risks in the context of incomplete financial markets. Krueger and Kubler (2003b) find that in economies with stochastic production where returns to labor and capital are imperfectly correlated the positive diversification effect of PAYG pensions does not overcome the welfare costs of the crowding out effect.

The consideration of the uncertainties involved in the demographic process itself has progressed more slowly, hindered by the computational difficulties involved. Initial research efforts have taken place in the only tractable setting: the two-period OLG model pioneered in Bohn (2001). He explores the optimal dynamic response to a demographic shock by analytically solving a log-linear approximation of the economy's equilibrium conditions. He finds that defined-benefit social security systems are ex ante more efficient than defined-contribution or privatized systems. This is a consequence of the favourable movements in wages and interest rates enjoyed by small cohorts of workers. As individuals do not know ex ante if they are going to belong to a large or small cohort, and they cannot insure against this risk in private markets, there is room for public transfer schemes to improve upon a purely private solution. In particular, Defined-Benefit (DB) pension schemes may achieve a more balanced distribution of the burden implied by demographic shocks by mitigating the effect of changes in the relative prices (small cohorts pay larger contributions, while retirees also participate in wage increases via higher pensions).

Our target in this paper is to quantify the welfare impact of the insurance role of PAYG-DB pension systems, and compare it to that of the crowding-out of private savings. We add to the literature by exploring whether previous theoretical results extend to models with larger

[1]See Casey et al (2003) for an updated evaluation of the fiscal pressures lying ahead for OECD countries. A comprehensive analysis for the US economy can be found in Kotlikoff and Burns (2004).

[2]The possibility of dynamic inefficiencies has been consistently rejected in empirical studies (see, for example, Abel et al (1989)). It is easier to find a positive role for Social Security in dynastic economies or in economies populated with altruistic agents (see eg Fuster (1999) and Fuster et al (2003)).

[3]See, for example, Hubbard and Judd (1987), Imrohoroglu et al (1995), Storesletten et al (1998),Huang et al (1997) and Miles and Sefton (2002).

time-disaggregation. In these models big and small cohorts overlap in the labour force, with the result of smaller fluctuations in the labour input. Besides, individuals are simultaneously wage earners and capital earners during large parts of their life-cycle, smoothing the effects of changes in relative prices. Finally, shorter periods allow individuals to adapt their behaviour to the state of the economy more frequently. All these elements are important for a truly quantitative evaluation of the welfare impact of public pensions with demographic shocks. We address them in this paper by proceeding in the following way. We start by presenting evidence on demographic shocks and their empirical link with factor prices in section 2. In section 3 we illustrate the insurance role of DB pensions in a two-period exchange economy with a financial asset. We then move on to our basic model: an OLG economy with a standard neoclassical production function. The model is described in section 4, calibrated to the US economy in section 5, and solved in section 6, where we present our basic simulation findings. The robustness of the results to the particular parameters employed is the topic of section 7. The paper concludes with some final comments and directions for future research in section 8.

Our main results can be summarized as follows. We show first that, in the absence of crowding out, PAYG-DB pension systems can be welfare improving in an ex ante way (our 2-period exchange economy is an example). Secondly, we show how the insurance role of public pensions extends to a 4-period, carefully calibrated production model. In this case, however, the positive effect is not strong enough to compensate for the welfare consequences of the crowding out of private capital (with the exception of the cohorts of advanced age at the time the system is introduced). In the long run, small cohorts lose the equivalent of 1.321% of their life cycle income with a pension paying 2% of average wages. Large cohorts fare better, but still suffer a 1.060% loss in life cycle income. These general conclusions are robust to changes in the calibrated values of preferences, technology and demographic shocks. Of course, one should be cautious in interpreting the numbers in our experiment as an overall rejection of real-world public pension systems [4]. Finally, these findings only apply to the US economy, but the qualitative message may be of broader interest as demographic cycles seem to be fairly common in countries with mature populations, and international trade and capital flows may extend their effects to younger and faster growing economies (see Kenc and Sayan (2001)). Evidence supporting the stochastic treatment of demographic patterns and the linkage of the population's age structure and factor prices is presented in the next section.

2 Demographic shocks and factor prices: empirical evidence

Almost 100 million people of less than 25 years of age were living in America in 2000 (figure 1). At the beginning of the century, this age group numbered barely over 40 million, implying that the size of the incoming generations has increased at a considerable 0.9% annual cumulative growth rate. Actual annual growth rates, however, have varied a great deal, following a pattern of persistent cyclical fluctuations around a long term average. Over the period 1900-1920 the mean annualized growth rate was around 1.2 %, well above the global average. In contrast, between 1920 and 1940 the annualized rate was 0.4 %, falling well short of the long-run tendency. After the Second World War the population growth caught up with the long run trend and eventually passed it by, resulting in the post-war Baby Boom. Overall, the growth rate in the interval 1940-1960 was 2.0 %, but this process reversed again in the following years, with a mere 0.29% rate in the last two decades of the century. It makes perfect sense, then, to talk of a "demographic cycle" that reveals itself in generational frequencies of around 20 years. Similar demographic fluctuations have been described

[4] *Steady state* figures do not take into account the welfare gains provided to the first cohorts of pensioners. This inter-generational redistribution should be counted along with the pure efficiency impact. Besides, computational limitations force a separate, one-by-one confrontation of the crowding-out effect with some specific insurance effect. There is still some time to come before a truly *overall* assessment may be undertaken of the various insurance roles played by public pensions, not to mention additional impacts on labour supply, fertility, etc.

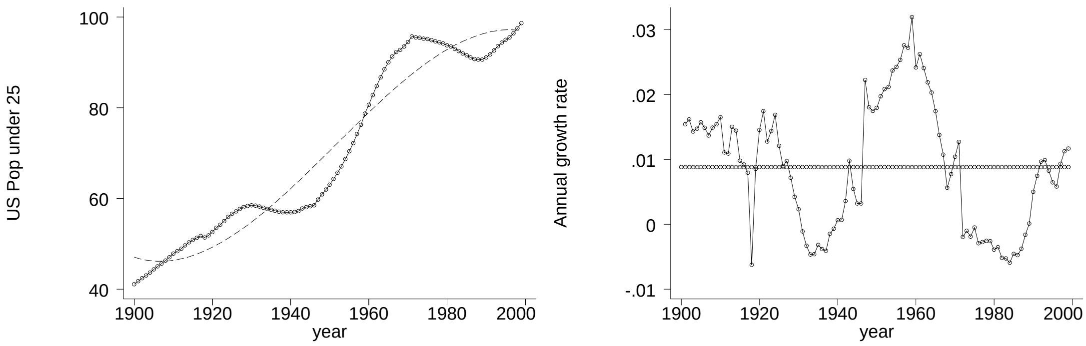

Figure 1: US population under the age of 25. Left panel: levels (in millions) (∘) and adjusted polynomial tendency (- -); Right panel: annual growth rates (∘) and long run average. Source: historical data from the US Census Bureau.

for other economies in advanced stages of the demographic transition (see Boldrin et al (2005) for evidence in 9 European countries).

The cycle itself is the product of two different processes: changes in fertility behaviour and modifications to immigration patterns. Note, in particular, that the low-frequency fluctuations described so far are perfectly compatible with the existence of a long term trend towards lower fertility. In this first approach to the topic, however, we do not look further into the ultimate causes influencing the size of the population. We simply take the stochastic process which generates the size of the incoming cohorts as one of the fundamentals of the economy[5]. This is a reasonable first step, given our target of highlighting the interactions between fluctuations in the size of the workforce and the design of the pension system.

The empirical link between demographic fluctuations and factor prices has been explored in the literature, but without obtaining very conclusive results. The basic difficulty is the low frequency variation in the population age structure, implying that each generational observation requires 20 years of data. This results in an insufficient number of degrees of freedom for a direct application of the standard statistic tools (Bohn (2001) and Brooks (2002)). Regarding wages, the evidence is moderately favorable, albeit somewhat indirect. There is a considerable amount of literature reporting changes in relative wages induced by the arrival of the large cohorts of Baby-Boomers (eg. Welch (1979), Murphy and Welch (1992) and Katz and Murphy (1992)). These analyses document drops in the new entrants' earnings (relative to those of more experienced workers) coinciding with the arrival of the peak-sized baby boom cohorts. The empirical elasticities are roughly compatible with those normally implemented in the calibration of OLG models (Bohn (2001)). Additional evidence of a negative relation between population growth and wages is found in cross-country comparisons of per capita income (of the type frequently used in growth literature

[5]There are, admittedly, strong links between the pension system, the immigration process and fertility behaviour. See, for example, Storesletten (2000) for an analysis of the impact of immigration policy on the future balance of the pension system. In Boldrin et al (2005) the impact of pensions on fertility is explored. These connections highlight the potential of this topic for future research.

(eg Barro (1996)). Poterba (2001) is the most relevant reference for the empirical links between asset prices and demographics. He does not find a clear pattern of correlations, but this may reflect that the size of the effects is small in relation to other lower frequency, asset market specific shocks or, perhaps, a reduced elasticity associated with low depreciation rates (Bohn (2001)).[6] All in all, our assessment of the empirical findings is that they are not inconsistent with the results obtained in OLG models.

3 The insurance role of PAYG pensions: a minimum model

This section illustrates the insurance that PAYG-DB systems provide against demographic shocks in a stylized setting where crowding out effects are ruled out by construction: a two-period, exchange economy. We consider a one-sector model of a closed economy in discrete time, populated by individuals of two possible ages ("workers" and "retirees"). Every period t, an endowment of the consumption good, w, is granted to the workers. They can transfer some consumption to the second stage of their lives by holding a certain amount of a purely financial asset, which we assume has a fixed supply of one unit per period. Therefore, workers consume part of their endowment (c_1^t) and save the rest by purchasing a certain amount of the asset (s^t), at the price p^t, from the cohort born in the immediately preceding period. The proceeds from that sale in turn finance the consumption of the current retirees (c_2^t). We follow the convention of denoting the age as a subscript and calendar time as a superscript. The only uncertainty at period t concerns the size of incoming generations, $N_1^l \quad l > t$, which we assume follows a 2-state Markov chain ($N_i^t \in \{\mathbf{N}^L, \mathbf{N}^S\} \quad i = \{1, 2\}$ ie, the size of any cohort can be either large, $\mathbf{N}^L$, or small, $\mathbf{N}^S$), with transition matrix Π:

$$\Pi = \begin{bmatrix} \pi_L & 1-\pi_L \\ 1-\pi_S & \pi_S \end{bmatrix} \qquad \pi_j = Prob\,[N_1^{t+1} = \mathbf{N}^j |\, N_1^t = \mathbf{N}^j\,] \quad j = \{L, S\}$$

Note in particular that there is no risk of early death. The model includes a system of intergenerational transfers providing the retirees with a fixed pension b, financed from contributions levied on the worker's endowment. For concreteness, we make both b and the annual contribution a fixed proportion (θ and ς^t respectively) of the period endowment w. The system is self-balanced on a year-by-year basis by adjusting the contribution rate:

$$N_1^t \varsigma^t\, w = N_2^t\, b \tag{1}$$

Individuals' behaviour is represented with a standard life-cycle optimization problem:

$$\begin{array}{cc} \max\limits_{s^t} & u(c_1^t) + \beta E_t[\,u(c_2^{t+1}))\,|N_1^t\,] \\ & c_1^t + p^t s^t = w\,(1-\varsigma^t) \\ & c_2^{t+1} = p^{t+1} s^t + b \end{array} \tag{2}$$

where β is a discount factor, u stands for a monotone increasing and strictly concave period utility function, and the expectation E_t is conditional on the information available in t.

The equilibrium of this economy is a set of time series for consumption, contributions and prices such that workers solve the problem above, the pension system is balanced and the asset market clears:

$$N_1^t\, s^t = 1 \tag{3}$$

Owing to the assumption that the asset is in fixed offer, the optimal saving behaviour is completely determined by the asset market equilibrium condition. This makes it possible to infer the

[6] Some strong correlations do emerge, specially between Treasury bills and Long-term government bond returns and the share of the population in the "prime saving years" (40 to 64). This author also rejects the *asset prices meltdown* conjecture, although his conclusions (obtained by using current life-cycle asset profiles to project future prices) are seriously challenged by Abel (2001).

equilibrium asset prices from (3), the budget constraints in (2) and the first order condition of the worker's problem:

$$p^t\, u_c(c_1^t) = \beta\, E_t[\, p^{t+1}\, u_c(c_2^{t+1})\,] \tag{4}$$

with $u_c = d\,u/d\,c$. A recursive definition of the equilibrium is, however, more fruitful, as there is no closed-form solution (with the exception of the logarithm utility) and we have to use numerical methods to calculate it. In this simple model, a *recursive* equilibrium is simply the 3-tuple of the asset's price and consumption allocation, in every possible combination of current and 1-period lagged shocks (represented by N_1 and N_2 respectively; note that we drop the t when referring to the components of the recursive version), with the same properties mentioned above:

$$\{p, c_1, c_2\}(N_1, N_2) \qquad N_1, N_2 \in (\mathbf{N}^L, \mathbf{N}^S)$$

The system of equations defining the recursive equilibrium is presented in appendix A.

Equilibrium in the purely private economy

We start by considering the solution in the absence of public pensions. Certain general observations immediately stand out. First, it is clear that a demographic shock (the arrival of a small cohort ($N^t = \mathbf{N}^S$) after a normal-size one ($N^{t-1} = \mathbf{N}^L$)) implies an unavoidable loss in consumption for at least one of the contemporaneous cohorts. This is apparent by exploring the shift in the *consumption feasibility frontier* of the economy[7]. No system of intergenerational transfers is capable of avoiding these losses (an aspect that is often overlooked in the arguments about reforming or privatizing current PAYG systems), though it is true that different institutional arrangements lead to different distributions of the welfare costs implied. The second direct observation from the model is that purely private systems place all the burden from the shock on the large cohort of retirees. This result is driven by falls in the asset price, stemming from the interaction of a small number of young buyers with a normal-size number of old sellers. An illustrative example can be found in appendix A.1. Such an extreme distribution of the welfare losses does not seem efficient, an intuition that can be confirmed by solving the planner's problem for this economy (see appendix A). An efficient outcome demands an immediate adjustment in the consumption of all living cohorts at the time the shock is revealed.

Equilibrium with PAYG transfers

Market incompleteness makes the private solution inefficient. In such a context a system of intergenerational transfers financed on a PAYG basis may result in welfare gains, a possibility that can only be explored through numerical simulation.[8] When a small cohort arrives in the private economy each young worker must increase her holdings of the asset in the private case from $1/\mathbf{N}^L$ to $1/\mathbf{N}^S$ in order to clear the assets market. This drives asset prices down, reducing retiree consumption while pushing worker consumption up. Things change in several ways in the presence of PAYG pensions. The system keeps its balance by charging a higher payroll rate ς to the smaller cohorts, in accordance with equation (1). This, in turn, reduces the demand for private savings and, consequently, the equilibrium price of the asset. Therefore, the impact on pensioners is double: they receive a new and safe source of income (the public pension), but the return on their private

[7] The consumption feasibility frontier is the maximum amount of old-age consumption compatible with any amount of young-age consumption. Its explicit equation at any point in time is simply $N_1 c_1 + N_2 c_2 = N_1\, w \Leftrightarrow c_2 = (N_1/N_2)(w - c_1)$. None of the combinations (c_1, c_2) which are feasible when a normal-size cohort is followed by another normal-size one remains so with the arrival of a reduced number of young workers (with the exception of the corner solution where the young consume everything). Of course, the absence of productivity growth is essential for this result.

[8] Table 6 in appendix A.1 illustrates the results of such a simulation by comparing a private economy with an economy including a marginal PAYG system, providing retirees with a pension equivalent to 2% of the workers' endowment.

savings is reduced. Accordingly, their total disposable income may move in either direction. If the change in the asset price is "mild" then c_2 is larger and c_1 is smaller than in the private case, making the distribution of consumption closer to the efficient benchmark. The PAYG system acts, therefore, as an insurance mechanism that improves the condition of the *ex post* poorer cohorts (the large ones) at the expense of the *ex post* better off cohorts (the small ones). It is not difficult to find standard economies where the inclusion of a marginal PAYG scheme is welfare improving in an *ex ante* sense: the expected utility computed before knowing the size of one's own cohort is higher with social security. We present in appendix A.1 an example of such a possibility.[9]

4 Demographic uncertainty in a production economy

Our basic model extends the standard one-sector, neoclassical, OLG model to incorporate demographic uncertainty in the form of shocks to the size of the workforce. In contrast with our exchange economy, both the insurance effect of PAYG pensions and the crowding out of private savings are in operation in this setting.

4.1 The model economy

Time is discrete and the economy is populated by overlapping generations of otherwise identical individuals. There is no life-time uncertainty and each individual's life spans over I periods: first as children (periods 1 to $I_W - 1$), then as workers (periods I_W to $I_R - 1$) and finally as retirees (from I_R to I). The calendar time is represented by t and individual age by i. Individual variables are indexed by the age (as a subscript) and by the calendar time (as a superscript). Aggregate variables are only indexed by calendar time. Each period a new generation of individuals of size N_1^t appears. The growth rate of this initial cohort $n^t = N_1^t/N_1^{t-1}$ is assumed to follow a two-state Markov chain, featuring "high" and "low" growth states: $n^t \in \mathcal{N} \equiv \{\mathbf{n}_h, \mathbf{n}_l\}$. The one-period-ahead probability of state-j, conditional on today's growth rate is represented by $\Gamma_j(n^t)$. The population dynamics are driven by stochastic changes in fertility, immigration and mortality patterns, although for the reasons discussed in section 2, we do not take these differences into account in this paper. Shocks in this model are accordingly best interpreted as fertility shocks in a closed economy.[10] Individuals cannot write contracts contingent upon the realization of the shock, ie. they cannot insure against the risk of being born in a large generation or being followed by a small cohort.

4.1.1 Institutional arrangements

The public sector in the model is made up of two institutions. First, there is a PAYG-DB Social Security system that taxes current workers at rate ς_t to pay a pension b_t to retirees. For simplicity, pension benefits are a fixed proportion, θ, of the current average per-worker earnings, and ς_t is adjusted period by period to ensure the balance of the system. Second, the public sector taxes personal income (asset returns and net labour earnings) to finance some public consumption

[9]The result in appendix A.1 serves our purpose of showing an illustrative example of the role of PAYG pensions as insurance against demographic uncertainty, but is not a general result. It is possible to find regular economies where asset prices "overreact" leading to a more extreme distribution of consumption than that prevailing in a purely private world. The analytical condition for this overreaction is:

$$\theta < (p^t - p_s^t)/N_1^{t-1} \Leftrightarrow \varsigma^t < (p^t - p_s^t)N_1^t$$

where p^t and p_s^t represent the asset prices in the private and PAYG economies respectively. The elasticity of the asset price is therefore crucial for PAYG systems to have a welfare improving role.

[10]A formal treatment of mortality changes is one of our immediate research targets. Consideration of immigration is also possible, though its quantitative importance is smaller. The lack of overseas capital flows is an (admittedly quite extreme) representation of current imperfections in global capital markets. They reveal themselves in eg. high correlations between national saving and investment rates (both across countries and within countries over time), massive home bias in equity ownership and very limited consumption smoothing with respect to country-specific output fluctuations.

expenditures, P_t. The size of these outlays is modelled as a constant fraction ν of total aggregate output. The fiscal system is balanced every period by adjusting the (proportional) income tax rate τ_t.

4.1.2 Technology and firms

The time-invariant production technology of the economy is given by a constant returns to scale, Cobb-Douglas function that uses capital, K, and efficient units of labour, H, to produce the single good in the model: $Y = K^{\alpha} H^{1-\alpha}$, where Y is aggregate output and $\alpha \in (0,1)$ stands for the share of capital in aggregate income. Period t aggregate labour input is $H_t = \sum_{i=I_W}^{I_R-1} N_i^t \varepsilon_i^t$, where ε_i^t is the age-i endowment of efficient labour units in period t. The time-dependency of the productivity schedule reflects the existence of exogenous, labour-augmenting, technological growth (with constant growth rate λ). Aggregate capital depreciates at a constant rate $\delta \in (0,1)$. The technology is operated by a large number of identical, profit maximizing firms. These firms interact with the economy's households through competitive spot markets for labour and for the consumption/investment good.

4.1.3 Household's decisions

The economy is populated by utility maximizing individuals who form rational expectations about the uncertain variables in the economy (ie, we assume they know the stochastic process governing population growth rates). Generation-g individuals (namely, those born in calendar year g) start making decisions after joining the labour force at the age of I_W. From this age onwards, they choose how much to consume c_i^{g+i-1} and how much to hold as assets a_i^{g+i-1} during the rest of their lives ($i \in \{I_W, \ldots, I\}$), both to smooth consumption over the life-cycle and for precautionary reasons. The vehicle for saving is the acquirement of some of the capital stock of the economy. Individuals are not restricted to have a positive amount of assets at every age.[11] There is no labour supply decision, as workers are assumed to provide their endowment of efficient labour units, ε_i^{g+i-1}, inelastically.

Individual preferences are represented with an Epstein and Zin (1989), recursive utility function, which allows for a separate treatment of risk aversion and preferences for inter-temporal substitution. This separation is important for the analysis of public programs that simultaneously alter the risk properties of the economy and the timing of life-cycle income. The optimal consumption sequence for a cohort-g individual is therefore obtained by recursively maximizing:

$$U_i(c_i^t|n^t) = \begin{cases} \left[(c_i^t)^{\rho} + \beta \left(E_t[U_{i+1}(c_{i+1}^{t+1}|n^{t+1})^{\sigma}] \right)^{\rho/\sigma} \right]^{1/\rho} & i = \{I_W, \ldots, I-1\} \\ c_i^t & i = I \end{cases} \tag{5}$$

where $t = g + i - 1$ (ie, t is the calendar time when the cohort-g individual is i years old). The expectation above takes the form:

$$E_t[U_{i+1}(c_{i+1}^{t+1}|n^{t+1})^{\sigma}] = \sum_{j=\{h,l\}} \Gamma_j(n^t) \left(U_{i+1}(c_{i+1}^{t+1}|n^{t+1} = \mathbf{n}_j) \right)^{\sigma} \tag{6}$$

The parameters ρ and σ control the constant intertemporal elasticity of substitution (IES$=1/1-\rho$) and the degree of relative risk aversion (CRRA$= 1-\sigma$). Note that when $\rho = \sigma$ the Epstein and Zin's utility becomes the standard isoelastic utility function.

The optimization program is subject to a standard sequence of budget constraints:

$$c_i^t + a_{i+1}^{t+1} \leq \chi_i^t + a_i^t \left(1 + r_t(1-\tau_t)\right) \qquad I_W \leq i \leq I \tag{7}$$

[11] This is partly for computational reasons (to make full use of the optimality properties of our solution algorithm) and partly for internal coherence (there is no idiosyncratic income uncertainty in the model). This assumption has no consequences in the computed equilibria.

where r_t stands for the gross return to capital and χ_i^t is net labour/pension earnings:

$$\chi_i^t = \begin{cases} w_t(1-\varsigma_t)(1-\tau_t)\,\varepsilon_i^t & I_W \le i < I_R \\ (1-\tau_t)\,\theta\,\bar{w}_t & i \le I_R \end{cases} \tag{8}$$

with w_t denoting the unique wage per efficient unit of labour.[12] $\bar{w}_t$ is the average wage in period-t: $\bar{w}_t = w_t \sum_{I_W}^{I_R-1} \mu_i^t\,\varepsilon_i^t$, where μ_i^t is the weight of age i workers in the total labour force ($\mu_i^t = N_i^t / \sum_{i=I_W}^{I_R-1} N_i^t$). Note that we abstract from the costs associated with raising children.

4.1.4 Recursive Competitive Equilibrium

At any point in time, the *state* of the economy is thoroughly determined by the age structure of the population and the asset holdings by age. For computational convenience we derive the former from the history of (discrete) population growth shocks:

$$\bar{n} = (n_0, n_{-1}, \ldots, n_{-(I-2)}) \in \bar{\mathcal{N}} \equiv \overbrace{\mathcal{N} \times \cdots \times \mathcal{N}}^{I-1}$$

where n_{-j} stands for the j-periods lagged growth rate, and the latter from the per-worker capital stock, k, and its distribution by age:

$$\bar{s} = (s_{I_W}, \ldots, s_{I-2}) \in \bar{\mathcal{S}} \equiv \overbrace{[0,1] \times \cdots \times [0,1]}^{I-I_W-1}$$

Our state vector $x = (\bar{n}, k, \bar{s}) \in \mathcal{X} \equiv \bar{\mathcal{N}} \times R_+ \times \bar{\mathcal{S}}$ brings all these variables together.[13] As usual, all variables are corrected for the exogenous growth of labor productivity and aggregate variables are also expressed in *per worker* terms.

A Recursive Competitive Equilibrium in our model economy is a set of individuals' policy functions $\{c_i(x), a_{i+1}(x)\}_{i=I_W}^{I}$, public policies $(\tau(x), \varsigma(x))$ and prices for capital and labour ($r(x)$, $w(x)$) such that the following properties apply:

(i) At every age $I_W \le i \le I$ and for every $x \in \mathcal{X}$, the policy rules $c_i(x)$ and $a_{i+1}(x)$ recursively solve the individual problem given the public policies, price functions and the optimal future policy rules $\{c_j(x), a_{j+1}(x)\}_{j=i+1}^{I}$:

$$(c_i,\, a_{i+1}) = Arg\,Max\; U_i(c_i) = \left[c_i^\rho + \beta(1+\lambda)^\rho \left(\sum_{x' \in \mathcal{X}} \Gamma(x'|x)\ \{U_{i+1}(c_{i+1}(x'))\}^\sigma \right)^{\rho/\sigma} \right]^{1/\rho} \tag{9}$$

with x' standing for the next period state vector and subject to a recursive version of the budget constraints (7) and (8) above:

[12] There is a certain amount of evidence in labor economics literature (reviewed in section 2) pointing to a more complex specification in which workers of different ages are imperfect substitutes in the production function, leading to age-specific wages. This specification is relevant for our problem because it reinforces the positive insurance role of PAYG pensions. It is, in our view, quite unlikely that such a change may reverse the results in this paper, but it is an extension well worth considering in the future.

[13] The state vector includes $I-1$ demographic variables plus $I-I_W$ economic variables (including k), while the number of age groups is I and the number of economically active years is $I-I_W+1$. The two missing variables originate as follows. In the computable model, aggregate variables are expressed in per-worker terms and corrected of productivity growth. This means that the total population does not have to be part of the state vector. This accounts for the missing variable in $\bar{n}$. As for the missing asset holding, it is clear that optimal behavior leads to $s_I = 0$ in the absence of bequest motives. Furthermore, as $\sum_{i=I_W}^{I-1} s_i = 1$ and we keep track of the per-worker capital stock, we only need to store the asset shares from age I_W to $I-2$.

$$c_i + (1+\lambda)\, a_{i+1} \leq \chi_i(x) + s_i\, k\, (\, 1 + r(x)(1-\tau(x))\,)$$
$$\chi_i(x) = \left\{ \begin{array}{cc} w(x)\,(1-\varsigma(x))\,(1-\tau(x))\,\varepsilon_i & i < I_R \\ (1-\tau(x))\,\theta w(x) h(x) & I_R \leq i \leq I \end{array} \right.$$

where ε_i is the invariant age profile of efficiency labour units, current asset holdings are given by $s_i\, k$ and labour supply, $h(x)$, is the weighted sum of the workers' endowment of efficiency labour units:

$$h(x) = \sum_{i=I_W}^{I_R-1} \mu_i(\bar{n})\,\varepsilon_i$$

The analytical expression of the functions $\mu_i\,(\bar{n})$ relating the age-population shares with the demographic state $\bar{n}$ is presented in appendix B.

The various elements of the state vector x' are updated as follows:

- The demographic component is straightforward: simply drop the oldest growth rate $n_{-(I-2)}$ from the current growth rate $\bar{n}$, shift all components one space and substitute with the new growth rate:

$$\bar{n}' = (n', n_0, n_{-1}, \ldots, n_{-(I-2)+1}) \quad n' \in \mathcal{N}$$

- Next period capital and asset shares are then obtained from the new population shares and the optimal saving decisions:

$$k' = \sum_{i=I_W}^{I-1} \mu_i\,(\bar{n}')\, a_{i+1}(x) \qquad s_i' = \frac{\mu_i\,(\bar{n}')\, a_{i+1}(x)}{k'} \qquad I_W \leq i \leq I-1$$

- Conditional distribution of future states, $\Gamma(x'|x)$: the one-period-ahead state can take one of two possible values, determined by the support of the stochastic population growth rates:

$$x' = (\, n', n_0, \,\ldots\,, n_{-(I-2)+1}, k'(\bar{n}'), \bar{s}(\bar{n}')\,) \qquad n' \in \{\mathbf{n}_h, \mathbf{n}_l\}$$

  with probabilities given by $\Gamma_j(n_0)$ $j = \{h, l\}$. Therefore, the conditional distribution of future states, $\Gamma(x'|x)$, is fully determined by the discrete probabilities $\Gamma_j(n_0)$, the optimal decision rules and the law of motion of the state vector detailed above.

(ii) Pre-tax factor prices equal marginal productivities (ie, firms behave competitively in the factor markets):

$$r(x) = f_k(k(x), h(x)) - \delta \tag{10}$$
$$w(x) = f_h(k(x), h(x)) \tag{11}$$

where $f_i() = \frac{\partial f}{\partial i}()$; $i = \{k, h\}$ and $f = k^\alpha h^{1-\alpha}$

(iii) Both public programmes have balanced budgets

- Contributions match pension expenditures in the Social Security system:

$$\varsigma(x)\, w(x)\, h(x) = b(x) \sum_{i=I_R}^{I} \mu_i(x) \qquad \text{with pension } b(x) = \theta\, w(x)\, h(x) \tag{12}$$

- Public consumption outlays match fiscal revenues:

$$p(x) = \left[\{h(x)w(x)(1 - \varsigma(x)) + k(x)r(x)\} + b(x) \sum_{i=I_R}^{I} \mu_i \right] \tau(x) \tag{13}$$

(iv) Individual and aggregate behaviour are consistent with each other:

$$y(x) + (1 - \delta)k(x) = \sum_{i=I_W}^{I} \mu_i(\bar{n})c_i(x) + p(x) + k'(x)(1 + \lambda)\,\psi(x)$$

where $\psi(x)$ is the growth rate of the total number of workers, k' is next period capital stock and $y = f(x)$ as in point (ii) above.

4.2 Numerical solution

Solving the recursive equilibrium is a rather daunting task, due to the presence of infinite-dimensional objects in the state vector. The literature has dealt with this problem in two ways: approximating the mathematical objects involved with smooth low-dimensional parametric functions (typically orthogonal polynomial, as in Krueger and Kubler (2003)) and approximating the state vector itself (following the general method pioneered by Krusell and Smith (1998)). In this paper we have opted for the first approach, implementing the Smolyak algorithm to obtain multivariate approximations of the saving functions.[14] Solving the model for any given set of parameters accordingly implies two steps: obtaining the savings functions numerically (in our case, by minimizing the quadratic norm of the vector of errors in the Euler equations of problem (9)). And, secondly, simulating the resulting model to generate long time series of the endogenous variables. It is by analysing the behaviour of the simulated series that the properties of the particular economy solved emerge.

5 Calibration

We calibrate our model to reproduce empirically observed aspects of risk and life-cycle savings behavior in the US economy[15]. Table 1 summarizes parametrization of our model.

Since our variables do not exactly correspond to the National Income and Product Accounts (NIPA) definitions, we manipulate NIPA series to generate a consistent set of measurements for the aggregate capital stock, investment, output and income flows in the model as briefly described in appendix C.

Parameter values

Demography The demographic cycle we consider is well captured by considering periods of 20 years of length. Given the life expectancy of 77 years in 2000, it seems reasonable to consider a

[14] The Smolyak algorithm is a high dimensional interpolation method, characterized by the use of (1) low order (orthogonal) polynomials as interpolating functions and (2) a sparse grid (extrema of Chebyshev polynomials) in which to perform the function evaluations. With this grid choice, it is possible to increase the problem's dimension (for a fixed degree of the approximating polynomials) with a small increase in computational cost. Furthermore, the grids obtained with polynomials of higher degree (for a fixed dimension) are nested. All in all, this method provides almost optimal error bounds using a much lower number of points than with tensor products of univariate polynomials. See Bathelmann et al (2000) and Krueger and Kubler (2003) for detailed descriptions of the method.

[15] A good reproduction of the risk/insurance behaviour can be achieved by carefully modelling the degree of risk aversion and the stochastic properties of the shocks. For life-cycle savings we pay special attention to three elements: the preferences for intertemporal consumption (controlled by the discount factor and the intertemporal elasticity of substitution), the life-cycle profile of income (including labour productivity, taxes and the details of the pension system) and the determinants of the returns to private capital (controlled by the capital parameter in the production function, the income tax and the depreciation rate).

Category	Parameters	Target properties of the model's solution
Demography	I_W, I_R, I	Relative length of working life versus retirement
	$\{\Gamma_j\}$ $\{\mathbf{n}_j\}$ $i = \{h, l\}$	Mean, variance and persistence of population growth.
Institutions	$\theta,\ \nu$	Pension replacement rate, Public Expenditure/GDP
Productivity	$\lambda, \{\varepsilon_{I_W}, \ldots, \varepsilon_{I_{R-1}}\}$	Exogenous growth rate; Individual life-cycle income profile
Technology	$\alpha,\ \delta$	Capital income share and depreciation rate
Preferences	$\sigma;\ \ \beta, \rho$	Risk Aversion; Life-cycle saving behaviour

Table 1: Calibration process: matching of the model parameters and target properties.

lifespan of 4 periods: an initial childhood period, two periods of active labour force participation and a final retirement period. In terms of our general notation, we set I_W to 2, I_R to 3 and I to 4. The parameters of the Markov chain governing the population process are chosen to mimic the empirical mean and variance of the growth rate of the US population under 25 years of age (0.194 and 0.164 respectively, over 20-year periods). We set the population shocks to alternate between a high growth state, $\mathbf{n}_h$, featuring a 1.5% annual rate, and a low growth state, $\mathbf{n}_l$, with a 0.15% annual rate. Since assessing the persistence of the process is very difficult with the small number of generational observations available (Brooks (2002)), we take the uncorrelated case ($\Gamma_j = 0.5$) as our benchmark, and explore the robustness of our results in section 7.

Public Institutions The ratio of public consumption expenditures to GDP, ν, is set to the average empirical value in our consistent measurements from the NIPA (18.5%). This is financed with a proportional income tax of 19.5%. For the pension system, we target the current value of the pay-roll tax rate (15%) by fixing the pension replacement rate, θ, at 39% (consistent with the empirical estimations in eg. Brooks (2002)).

Productivity profiles We calibrate the life-cycle profile of efficiency labour units $\{\varepsilon_1, \varepsilon_2\}$ to reproduce the empirical age-profile of labour earnings. Specifically, we reproduce the relative earnings of males in the age groups 45-64 and 25-44 in the Current Population Survey 1970-2000. We find that senior workers earn 17% more than their younger workmates. These calculations account for an annual productivity growth rate λ of 2.1% (set to the average growth rate of per capita product).

Technology Both the capital parameter in the Cobb-Douglas production function, α, and the depreciation rate, δ, are obtained from our consistent set of measurements. The former is set to the average capital income share in 1970-2000 (0.36) and the latter is set to reproduce the observed capital stock dynamics (4.1% per annum). The obtained values are standard in macro-calibration literature.

Preferences We characterize attitudes towards risk with a benchmark CRRA $(1 - \sigma)$ of 2, a common value in macroeconomic applications. To reproduce the saving behaviour we target the gross rate of return on aggregate capital. Our estimated annual empirical average in the interval 1970-2000 is 7.56%, a figure well within the dynamically efficient range (meaning that PAYG pensions can only improve welfare through risk sharing). There is a continuum of pairs of values $(1/(1 - \rho), \beta)$ for the intertemporal elasticity of substitution and the discount factor that can reproduce the targeted interest rate. In the absence of clear guidance from econometric estimations, we adopted as our benchmark a elasticity value of 0.7. This figure is slightly larger than that consistent with Expected Utility (0.5, given our CRRA choice), but it lies well within the range of available estimations (eg. Attanasio and Weber (1999) or Blundell et al (1994)). More importantly, it ensures that our benchmark case is a fair environment to judge the merits of PAYG

n_t	w_1^{t+1}	w_2^{t+2}	r_2^{t+2}	r_3^{t+3}	EU
Low growth state (0.15%)	0.205	0.207	2.119	2.245	0.0184
High growth state (1.5%)	0.196	0.194	2.449	2.310	0.0178

Table 2: Simulation results in the private economy: average life-cycle prices and expected utility by cohort size.

pensions[16]. Finally, we set the discount factor to the value that generates the targeted interest rate (an annual 0.2 % rate or $\beta = 0.97$). Section 7 explores the robustness of our findings to these particular values.

6 Simulation results

We start by exploring the effects of demographic shocks in a private economy, a version of our benchmark economy with no PAYG transfers. To describe the economic forces at work, we consider first the impact of an isolated negative shock: the arrival of a cohort whose size is only slightly larger than that of its predecessor, implying a growth rate well below the average. All immediately preceding and ensuing cohorts expand at the larger rate. Appendix D presents all the figures and more detailed comments.

A reduction in the population growth rate in t has a 1-period delayed impact over the *per worker* labour, h, and capital, k. Both go up one year after the shock, but the increase in k is relatively larger driving k/h down in $t+1$. This leads to an increase in salaries and a reduction in the return to savings. The capital to labour ratio reaches a maximum in $t+2$ and progressively returns to its initial value afterwards, with k serving as the propagation mechanism of the impact of the shock. The induced price changes occur at different points in the life-cycle of the several cohorts involved, causing unbalanced welfare changes. Cohorts born in $t-2$ suffer from the drop in saving returns at $t+1$. Cohorts born in $t-1$ and t experience both gains and drawbacks from the change in prices. The overall impact is negative in the former case (as the gains from larger wages in t+1 do not compensate for the severe drops in interest rates in $t+2$), and positive in the latter (t-cohort's members enjoy the wage rises fully, while interest rate drops affect them only marginally). Cohorts born in $t+1$ enjoy higher wages in $t+2$ (the last direct effects stemming from the shock) and respond by increasing their savings. This, in turn, spreads part of the benefits to the ensuing cohorts in the form of a larger capital stock. In summary: relatively small cohorts benefit from the price changes induced by their small size, save more and transfer some of the benefits to the ensuing cohorts. In contrast, previous (relatively large) cohorts are hit by the low returns precisely when they are more dependent upon the performance of their assets. The only differences with the exchange economy are (1) a general weakening of the strength of the effects (as cohorts experience opposite forces at different points of their life-cycles), and (2) the extension of the effects to the following cohorts via changes in the capital stock.

General simulation results

Cohorts are normally hit by shocks of opposite sign at different stages of their life-cycle. Average values in the entire simulation show how demographic shocks generate a significant degree of volatility in factor prices, especially regarding the interest rate (the coefficient of variation of the

[16] PAYG pensions tend to be more favourable in economies populated by more elastic individuals. This is so because inelastic individuals prefer flatter life-cycle consumption profiles. Therefore, economies populated by these types of individuals exhibit smaller aggregate capital stocks and larger interest rates, exacerbating the differences in the returns to public and private savings. Furthermore, borrowing constraints become more frequently binding in this type of economy.

n_t	ς_1^{t+1}	ς_2^{t+2}	$b^{t+3}{\cdot}100$	$\bar{\varsigma}^t$	$a_{2\,SS}^{t+2}/a_2^{t+2}$	EV(%)
Unconditional	0.793	0.794	0.429	0.991	97.441	-1.201
Low growth state (0.15%)	0.849	0.895	0.431	1.130	97.320	-1.321
High growth state (1.5%)	0.733	0.682	0.427	0.873	97.576	-1.060

Table 3: Simulation results in the economy with a 2% PAYG-DB pension system: life-cycle contribution rates, pension, effective contribution, ratio of savings with and without the pension system, and Equivalent Variation by cohort size.

simulated interest rates is around 10%, while that of wages is half that figure)[17]. Table 7 and 8 in Appendix D.2 provides the full record of prices and utilities by demographic state. We simplify the presentation of this information here by conditioning the results on the size of the own cohort. Although this simplification overlooks the impact of the variability induced by the size of the preceding and ensuing cohorts, the expositional gains more than compensate for the implied loss of detail. Accordingly, table 2 shows the model predictions conditional on the relative size of the cohort at birth. Relatively small cohorts enjoy higher salaries throughout their working lives. They also face relatively lower returns on savings, but this is mainly concentrated at the beginning of their working lives, when their dependency on asset income is low. Overall, their utility is clearly larger than that of relatively large cohorts, which suffer low wages and only slightly higher returns on their retirement assets. Accordingly, the general conclusion in the absence of PAYG transfers is that relatively small cohorts are better off. This is similar to the exchange economy, though the gains and losses are more evenly distributed in the four period model, reducing the quantitative importance of the welfare differences.

6.1 The impact of a marginal PAYG-DB pension system

In this section we undertake the basic experiment of the paper: the introduction of a *marginal* PAYG pension scheme (providing a 2% replacement rate of the average wage) to test the potential welfare improving role of social security. We start by reviewing the impact of an isolated shock in period t (see the tables in Appendix D). As the pension system is self-balanced on a period by period basis, the privileged cohort in the private case (the relatively small cohort born in t) is now hit by higher contribution rates throughout its working career and by relatively lower pensions. In contrast, cohorts badly battered by the shocks in the private world can now participate, through higher pensions, in the wage increases induced. Finally, the cohorts born after the shock, which also benefit from it in the private case, are now struck either by higher contributions (cohort $t+1$) or indirectly via larger reductions in the capital available per worker.[18] In this way the cost of demographic shocks is effectively extended to the ensuing cohorts.

PAYG-DB pensions, then, redistribute the benefits and costs from shocks in a more balanced way than the market does, although the effect is less clean than in the exchange economy as effects of opposite sign occur at different stages of the life-cycle. For instance, cohort $t-1$ members are relatively better treated under a PAYG system, but also suffer from high contribution rates at the

[17] An important prediction of the OLG model is that quite strong fluctuations in the size of the incoming generations (the coefficient of variation of the empirical net growth rate is close to 1) only generate a mild degree of volatility in factor prices. When we add the ability to smooth these fluctuations by means of savings (and a one-period advance notice of the arrival of a shock), our model predicts a CV of consumption ranging from 2.1 to 5.5% depending on the age. This smoothing is significantly larger in the four-period model than in the 2-period one, for the reasons discussed in section 1.

[18] This can be appreciated by checking the ratio of retirement savings with and without social security. As PAYG pensions crowd-out private savings, this ratio is always less than one. But the scope of this effect changes with the cohort in a systematic way: cohorts t and $t+1$ protect themselves from the system's discriminatory treatment by saving relatively less (thereby partly offsetting the larger contributions placed on them). Consequently, they leave smaller capital stock behind them.

n_t	EV[t-3]	EV[t-2]	EV[t-1]	EV[t]	EV[t+1]
Unconditional	1.732	0.716	-0.186	-0.568	-0.714
Low growth state (0.15%)	1.727	0.807	-0.081	-0.667	-1.035
High growth state (1.5%)	1.738	0.628	-0.287	-0.472	-0.405

Table 4: Short run welfare impact of the introduction of a marginal PAYG-DB pension system in period t. Average percentage Equivalent Variation by cohort (conditional on the size of the cohort born in t and unconditional, overall effect).

age when their time endowment is most productive. All in all, risk averse individuals who do not know which cohort they are going to be born into, should regard this insurance mechanism highly. Their overall evaluation, however, also depends on the crowding-out of their life-cycle wealth implied by compulsory transfers. Table 3 provides the average results in the entire simulation. A PAYG-DB system forces relatively small cohorts to pay higher contributions and share in this way the cost of demographic shocks. The impact on pensions tends to be blurred by the mixing of the shocks, but the overall effect on life-cycle wealth is clear: small cohorts pay larger *effective* contribution rates ($\bar{\varsigma}^t$)[19].

Welfare impact of PAYG pensions

The introduction of the pension system favours contemporary cohorts of retired and advanced-age workers. The former receive pensions without having made any contributions and without suffering any unfavourable movement in factor prices. The latter enjoy their pensions after having contributed only as senior workers, but suffering wage reductions from the crowding-out at that stage. Younger cohorts at the system's inception date and all future cohorts enjoy the insurance provided by the PAYG pension but also inherit the debt implied by the gift granted to older cohorts (materialized as lower levels of capital stock). The reduction in precautionary savings results in additional reductions in the aggregate capital stock. We evaluate the welfare changes induced by computing the Equivalent Variation (EV) associated with the introduction of the pension scheme[20]. The quantitative importance of these transitory effects is presented in table 4. As expected, they are monotone decreasing ranging from a 1.7% gain (in terms of life cycle income) for the first cohort of pensioners to significant losses for the cohorts arriving a few periods later. This pattern depends on the state of the economy when the pension system is introduced, but the differences are small.

The effects of PAYG pensions in the *long run* are summarized in the last column of table 3. The insurance effect results in sizable reductions in the coefficient of variation of consumption at all ages (ranging from 0.5 to 3.5% depending on age), while the crowding out manifests itself in an average capital per worker 2.4% lower than in the private case. This second effect is eventually dominant, leading to an average welfare change equivalent to a 1.2% *reduction* in life cycle income. The specific figures conditional upon the (relative) cohort size are a 1.06% *reduction* for large

[19]The effective contribution rate is the change in life-cycle wealth (expressed as a proportion of the net labour income of young workers) induced by the introduction of the pension system. Its analytical expression is (with cot_i^{t+i} representing the contributions made at age i by a member of cohort t and defining χ_1^t as in (8)) :

$$\bar{\varsigma}^t = [\ cot_1^{t+1} + cot_2^{t+2}/(1+r^{t+2}) - b^{t+3}/(1+r^{t+2})(1+r^{t+3})\ ]/\chi_1^{t+1}$$

[20]We compute the Equivalent Variation as the proportional change in life cycle income (materialized in the first period of life) required for the private economy to display the same utility level as that observed in the presence of social security. Note that we let consumers adjust their optimal behaviour in response to the wealth changes during the experiment. We have also computed an *adjusted* Equivalent Variation, where the percentage change is measured with respect to the corresponding steady state figure, but this refinement does not alter the results and is therefore not reported.

Parameter	EV	EV($\mathbf{n}_l$)	EV ($\mathbf{n}_h$)	Parameter	EV	EV ($\mathbf{n}_l$)	EV ($\mathbf{n}_h$)
IES ($1/(1-\rho)$)				CRRA ($1-\sigma$)			
0.3	-1.201	-1.436	-0.913	2*	-1.201	-1.321	-1.060
0.5	-1.245	-1.430	-1.026	4	-1.194	-1.317	-1.056
0.7*	-1.201	-1.321	-1.060	8	-1.184	-1.307	-1.046
0.9	-1.127	-1.205	-1.041	12	-1.174	-1.293	-1.036
1.1	-1.063	-1.125	-1.032	20	-1.154	-1.273	-1.015
Persistence (Γ_j)				Volatility (CV)			
0.1	-1.030	-0.995	-1.070	0.45	-1.060	-1.124	-1.000
0.5*	-1.201	-1.321	-1.060	0.85*	-1.201	-1.321	-1.060
0.9	-1.200	-1.588	-0.820	1.25	-1.365	-1.555	-1.142

Table 5: Sensitivity Analysis: Equivalent Variation (EV) associated with the introduction of a marginal PAYG-DB pension system in a range of economies. All EV are expressed as percentages, with benchmark parameters marked as *. Conditional values depend on ones' cohort size representing either a high, $\mathbf{n}_h$, or low, $\mathbf{n}_l$, growth rate of the workforce.

cohorts and a 1.32% *reduction* for small cohorts, revealing the insurance effect of PAYG pensions. As it is shown in appendix D.2, this is mostly *ex ante* insurance (ie. insurance against uncertainty about the state of nature at birth), as welfare fluctuations conditional on birth are relatively minor. **The overall significance of this insurances is, in any case, very small when compared to the Crowding Out effect. We estimate that only around a 2 % of the impact of the Crowding Out is offset by the risk sharing provided, leading to a very clear welfare reducing role for public pensions in the long run.**[21]

7 Robustness of the results

The relevance of the quantitative findings in our benchmark economy is supported by its rigorous link to an explicit set of empirical properties in section 5. However, as different combinations of reasonable values for σ, ρ and β can reproduce our targeted return on capital, it is important to check that the impact of PAYG pensions in these alternative economies is unaltered. This is explored here by repeating our basic experiment in a wide range of economies differing in the preferences of the population. We simultaneously adjust one of the remaining preference parameters to guarantee that the measured return on capital is still reproduced.[22] We also explore the robustness of our findings to changes in the variance and persistence of the demographic shocks, as they are quite poorly measured in the data. The findings obtained are presented in table 5 and summarized as follows:

- **Degree of risk aversion (CRRA)** Our simulations confirm the intuition that higher degrees of risk aversion should imply a higher appreciation of the insurance role provided by PAYG pensions (top-right panel of table 5), though the quantitative importance of this

[21] There is not an obvious and unique way of decomposing the effects involved. The 2% figure is obtained by isolating the insurance role in the following way. We construct an instrumental economy with a marginal PAYG pension system, but where prices and aggregate quantities are those observed in the private case. This eliminates the effects of the crowding out that manifests through changes in prices. Besides, individuals are compensated for the reduction in life-time resources created by the PAYG financing of the system (due to the "free" transfers made to the initial cohorts at the system first inception date). The compensation takes the form of an extra income equivalent to the average effective contribution to the PAYG system. We gauge the strength of the insurance effect by computing the equivalent variation in life cycle income needed to make people indifferent between this setting (where all crowding-out effects are eliminated) and the private economy.

[22] As a complement, we present in appendix E "non-adjusted" calculations of the robustness of our basic finding to changes in the preference and technological parameters.

effect is remarkably small. This is due to the mild degree of volatility in factor prices and consumption found in our simulations (consumption volatility ranges from 2.1 to 5.5% depending on age) and the very modest reductions that the PAYG-DB pensions achieves in those figures (ranging from 0.5 to 3.5%).

- **Intertemporal elasticity of substitution (IES)** Individuals with higher IES are less reluctant to have steeper consumption profiles, which in equilibrium leads to larger savings and capital stocks. This drives the return differential between private savings and contributions down, creating a more favourable environment for PAYG pensions. The welfare impact of public pensions is, however, not significantly altered in these new contexts (top-left panel of table 5).

- **Properties of demographic shocks** More persistent shocks result in a more clear-cut pattern of factor prices. Relatively large cohorts are rewarded with consistently lower returns on capital than in the benchmark, while the opposite occurs for smaller cohorts. The insurance role of PAYG pensions is therefore reinforced, leading to more extreme welfare changes in either case (bottom left panel in table 5). With negatively correlated shocks, in contrast, people experience price movements of opposite sign at different points in their life-cycles, resulting in a general dampening of the impact of the shocks and leaving less scope for the pension insurance role. Similar remarks can be made about changes in the volatility of the shocks (bottom right panel in table 5).

All in all, the negative welfare impact of PAYG pensions emerges as a robust characteristic of the general class of empirically plausible economies.

8 Conclusions

In a world where the size of the incoming cohorts fluctuates along a long run trend, factor prices can experience significant cyclical variation. In a purely private economy these price movements systematically favour smaller-size cohorts at the expense of larger-size cohorts. Defined-Benefit, Pay As You Go pension systems counteract these swings in factor prices by charging higher effective contributions on smaller cohorts. PAYG-DB pensions have, then, an insurance effect against aggregate demographic risk that may lead to welfare gains. In this paper we quantitatively explore the trade off between this insurance effect and the classical crowding out of private savings. Our results are not positive for public pensions: we find significant welfare benefits deriving from risk sharing, but they are not enough to compensate for the reductions in per capita income generated by the crowding out. This should not be interpreted as an overall rejection of *real world* pension systems, as we are focusing on a very specific insurance effect and abstracting from a range of other aspects already discussed in the literature. Our contribution is a precise quantitative evaluation of this highly specific trade-off.

The present analysis may be improved in several ways. First, it may be important to include the costs associated with raising children, as in eg. Brooks (2002). This could either dampen or reinforce the welfare impact of changes in factor prices depending on the sign of its auto-correlation, a quite difficult empirical matter. Second, it may be interesting to consider the uncertainty surrounding human longevity. This is important because public pensions help to cope with lifespan risk at the individual level and because there is substantial uncertainty about the speed and scope of longevity increases at the aggregate level, a type of risk that private markets find very difficult to insurance against. One last modelling aspect that may be reconsidered is the use of a one-sector model where installed capital can be consumed without costs. When capital is only partially reversible retirees may incur large capital losses when trying to sell their assets to the younger cohorts. This reinforces the damaging effect of demographic contractions, increasing the possibility of a positive role for PAYG pensions.

References

Abel, A. Mankiw, G., Summers, L. and Zeckhauser, R., 1989. Assessing dynamic efficiency: theory and evidence. The Review of Economic Studies 56, 1-19.

Abel A., 2001. Will bequest attenuate the predicted meltdown in stock prices when baby boomers retire? The Review of Economic and Statistics 83, 589-595.

Abel A., 2003. The effect of the baby boom on stock prices and capital accumulation in the presence of social security. Econometrica 71, 551-578.

Attanasio, O. and Weber, G., 1995. Is consumption growth consistent with intertemporal optimization? Evidence for the consumer expenditure survey. Journal of Political Economy 103, 1121-57.

Auerbach A. J. and Kotlikoff, L J., 1987. Dynamic fiscal policy. Cambridge University Press, Cambridge.

Barthelmann V., Novak, E. and Ritter, K., 2000. High dimensional polynomial interpolation on sparse grids. Advances in Computational Mathematics 12, 273-288.

Blundell R., M. Browning and C Meghir, 1994. Consumer Demand and the Life Cycle Allocation of Household Expenditures. The Review of Economic Studies vol 61, No.1, 57-80.

Barro, R. J., 1996. Determinants of economic growth: a cross-country empirical study. NBER Working Paper 5698.

Bohn H., 1999. Social security and demographic uncertainty: the risk sharing properties of alternative policies. NBER Working Paper 7030.

Bohn H., 1999. Should social security trust tund hold equities? An intergenerational welfare analyisis. Review of Economic Dynamics 2, 666-697.

Bohn H., 2001 Social security and demographic uncertainty: the risk sharing properties of alternative policies, in: Campbell, J. and Feldstein, M. (Eds), Chapter 6. Risk aspects of investment based social security reform. The University of Chicago Press.

Boldrin M., De Nardi , M. and Jones, L. E., 2005. Fertility and Social Security. NBER Working Papers 11146.

Brooks, R., 2002. Asset-market effects of the baby boom and social security reform. American Economic Review 92, 403-406.

Casey, B., Oxley, H. Whitehouse, E. Antolin, P., Duval, R. and Leibfritz, W., 2003. Policies for an ageing society: recent measures and areas for further reform. OECD Economics Department Working Paper 369.

Cooley, T. and Prescott, E., 1995. Economic growth and business cycles, in: Frontiers of business cycle Research Cooley, T. (Ed), Chapter 1. Princeton University Press, pp. 1-38.

Diamond, P., 1965 National debt in a neoclassical growth model. American Economic Review 59,1127-55.

Diamond, P., 1977. A framework for social security analysis. Journal of Public Economics 83, 275-298.

Epstein, L. and Zin, S., 1989. Substitution, risk aversion and the temporal behaviour of asset returns: a theoretical framework. Econometrica 57, 937-69.

Fuster L., 1999. Is altruism important for understanding the long-run effects of Social Security? Review of Economic Dynamics 2, 616-37.

Fuster, L., Imrohoroglu, A. and Imrohoroglu, S., 2003. A welfare analysis of social security in a dynastic framework. International Economic Review 44, 1247-74.

Huang, H., Imrohoroglu, S. and Sargent. T.J., 1997. Two Computations to Fund Social Security. Macroeconomic Dynamics 1, 7-44.

Hubbard, G. and Judd, K L., 1987. Social security and individual welfare: precautionary saving, borrowing constraints, and the payroll tax. The American Economic Review 77, 630-646.

Imrohoroglu A., Imrohoroglu, S. and Joines, D. H., 1995. A life cycle analysis of social security. Economic Theory 6, 83-114.

Katz, L. and Murphy, K., 1992. Changes in relative wages, 1963-1987: supply and demand factors. Quarterly Journal of Economics 107, 35-78.

Kenc T. and Sayan.S., 2001. Demographic shock transmision from large to small countries. An overlapping genertions CGE analysis. Journal of Policy Modelling 23, 677-702.

Kotlikoff L., 2003 Fixing social security and medicare for good, in: Improving social insurance programs conference held at University of Maryland.

Kotlikoff, L. and Burns, S., 2004. .The coming generational storm. The MIT Press.

Krueger D. and Kubler, F., 2004. Computing equilibrium in OLG models with stochastic production. Journal of Economic Dynamics and Control 28, 1411-36.

Krueger D. and Kubler, F., 2003. Pareto improving social security reform when financial markets are incomplete. NBER Working Paper 9410.

Krusell, P. and Smith, A., 1998. Income and wealth heterogeneity in the macroeconomy. Journal of Political Economy 106, 867-96.

Miles D. and Sefton, J., 2002. Optimal Social Security Design. CEPR Discussion Paper 3290.

Murphy, K. and Welch, F., 1992. The structure of wages. Quarterly Journal of Economics 107, 285-326

Poterba, J., 2001. Demographic structure and assets returns. The Review of Economic and Statistics 83, 565-84.

Ríos-Rull, J.V., 2001. Population changes and capital accumulation: the aging of the baby boom. Berkeley Economic Press, Advances in Macroeconomics 1.

Storesletten K, Telmer, C.I. and Yaron, A., 1999. The risk-sharing implications of alternative social security arrangements. Carnegie-Rochester Conference Series on Public Policy.

Storesletten K., 2000. Sustaining fiscal policy through immigration. Journal of Political Economy 108, 300-23

Welch, F., 1979. Effects of cohort size on earnings: the baby boom babies's financial bust. Journal of Political Economy 85, 65-97.

A The insurance role of PAYG pensions: a minimum model

The solution of the exchange economy of section 3 (expressed in recursive form) is implicitly given by the following system of 4 non-linear equations (in the 4 contingent prices $p(N_1, N_2)$, $N_i = \{\mathbf{N}^L, \mathbf{N}^S\}$ $i = \{1, 2\}$):

$$p(N_1,N_2)\, u_c\left(\underbrace{-p(N_1,N_2)\frac{1}{N_1} + w(1-\varsigma(N_1,N_2))}_{c1(N_1,N_2)}\right) = \beta\, E\left[\, p(N_1',N_1)\, u_c(\underbrace{p(N_1',N_1)\frac{1}{N_1} + \theta\, w}_{c2(N_1',N_1)}) \mid N_1 \right]$$

All notation is as in section 3, with N_1' standing for the unknown size of the next future cohort. The equation is obtained when the asset market clearing condition (equation (3)) and the budget constraints of the individual problem (2) are particularized in the first order condition (4). The contingent prices are the only variables left once contributions take the value that guarantees the balance of the pension system $\varsigma(N_1, N_2) = \theta N_2/N_1$ (equation (1)).

A.1 An illustrative example

As an example of the arguments in section 3, we present here the equilibrium values obtained in a particular economy. We explore a world where the incoming cohorts can be either *normal-sized* ($\mathbf{N}^L = 1$) or *small* ($\mathbf{N}^S = 0.8$), cohort sizes are identically and independently distributed, the endowment w is normalized to 1 and individuals maximize an isoelastic utility function $u(c_i) = c_i^\sigma/\sigma$ with $\sigma = -1$ and β equivalent to a 1% annual discount. This is not an explicitly calibrated example, but it is enough to highlight the essential qualitative properties of the solution. Results are presented in table 6.

Economy	(N_1, N_2)	c_1	c_2	p	ς %	Eu_1
Private	(1,1)	0.529	0.471	0.471	0	-3.569
	(0.8,1)	0.567	0.346	0.346	0	-3.106
	(1,0.8)	0.529	0.588	0.471	0	-3.569
	(0.8,0.8)	0.567	0.433	0.346	0	-3.106
						Ex ante: -3.338
PAYG	(1,1)	0.531	0.469	0.449	2.0	-3.558
θ=2%	(0.8,1)	0.564	0.349	0.329	2.5	-3.118
	(1,0.8)	0.533	0.584	0.451	1.6	-3.553
	(0.8,0.8)	0.566	0.434	0.331	2.0	-3.112
						Ex ante: -3.335
Planner	(1,1)	0.550	0.450	-	-	-3.439
	(0.8,1)	0.522	0.383	-	-	-3.294
	(1,0.8)	0.577	0.528	-	-	-3.353
	(0.8,0.8)	0.550	0.450	-	-	-3.197
						Ex ante: -3.332

Table 6: Recursive equilibrium (with an without a PAYG pension system) and Planner solution of the numerical example in section A.1. Distribution of consumption among the living generations, price, contribution rate and young-cohort expected utility (Eu_1), according to the current and one-period-lagged population sizes (N_1, N_2). The *ex ante* utility is computed by taking expectations before the state of nature is revealed.

A.2 Planner solution

The efficient benchmark in the exchange economy of section 3 can be obtained by maximizing an aggregate welfare function W, given the number of senior citizens at the initial period, N_2^1, and specific weights Ω_t for all current and future cohorts:

$$W_1(N_2^1) = E_1\left[\Omega_0 \beta u(c_2^1) + \sum_{t=1}^{\infty} \Omega_t U_t\right] \qquad U_t = u(c_1^t) + \beta u(c_2^{t+1})$$

The maximization is made subject to the aggregate resource constraint and the Markov process for the demographic shocks. Again, all notation is as in section 3. The contingent plans that solve this problem provide the allocations that would be chosen by a planner willing to spread risk across generations.[23] We are particularly interested in the way this planner allocates consumption among coexisting generations. This is controlled by the following first order condition:

$$\frac{u'(c_1^t)}{(1+R)\,\beta u'(c_2^t)} = \frac{N_1^t}{N_2^t} \Leftrightarrow \frac{c_1^t}{c_2^t} = \frac{1}{\tilde{\beta}^{1/(1-\sigma)}}\left(\frac{N_2^t}{N_1^t}\right)^{1/(1-\sigma)}$$

where we assume that the weights applied to different cohorts are defined by a constant discount rate $(1+R) = \Omega_{t-1}/\Omega_t$, that the planner use the same isoelastic utility function (with constant relative risk aversion $1-\sigma$) as in the example in section A.1 and where $\tilde{\beta}$ stands for $(1+R)\beta$. This equation implies a proportional relation between the efficient consumption levels of coexisting cohorts, though one that depends on the state of nature. Going one step further, it is easy to prove that although the ratio c_1^t/c_2^t should increase when a small cohort arrives, both cohorts should share the costs of the shocks.[24] This means that although senior citizens may have to pay a proportionally larger share of the costs, the young should always contribute to this effort. In these circumstances, it is clear that the extreme distribution of costs in the private solution is inefficient. Table 6 displays the efficient solution in the example economy of section A.1 (assuming R=0).

B Demographic uncertainty in a production economy

The recursive equilibrium for the production economy of section 4.1.4 is completed here with the formal expression linking the population weights $\mu_i = N_i / \sum_{i=I_W}^{I_R-1} N_i$ with $\bar{n}$, the demographic component of the state vector (ie, the history of previous growth rates of the incoming cohort):

$$\mu_j(\bar{n}) = \frac{N_j}{\sum_{j=I_W}^{I_R-1} N_j} = \frac{N_j/N_{I_R-1}}{\sum_{j=I_W}^{I_R-1} N_j/N_{I_R-1}} = \begin{cases} \frac{\prod_{i=-j+1}^{-I_R+3} n_i}{1+\sum_{j=I_W}^{I_R-2}\prod_{i=-j+1}^{-I_R+3} n_i} & 0 \le j \le I_R - 2 \\ \frac{1}{1+\sum_{j=I_W}^{I_R-2}\prod_{i=-j+1}^{-I_R+3} n_i} & j = I_R - 1 \\ \frac{\prod_{i=-I_R+2}^{-j+2} 1/n_i}{1+\sum_{j=I_W}^{I_R-2}\prod_{i=-j+1}^{-I_R+3} n_i} & I_R \le j \le I - 1 \end{cases}$$

Accordingly, the law of motion of the population weights is simply $\mu_i' = \mu_i(\bar{n}')$.

C Consistent measurements in the calibration process.

For the reasons discussed in section 5, the original NIPA series (we use tables 1.1.5; 1.7.5; 1.12; and 5.7.5B, and Fixed Assets Table 1.1) have to be modified to obtain a proper match with the variables in our stylized model. The following are the main steps involved:

[23] A planner who distinguishes between generations born in different states of the world would rely on the (much weaker) *interim* criterion. Instead, here we are using the *ex ante* welfare criterion.

[24] In Bohn (2001) the ratio c_1^t/c_2^t is constant in all possible states. This result is a special feature of the welfare function employed by this author, and it does not hold in the more general form used here.

1. Construct a consistent series of income on *fixed private capital*. We make the same assumptions as in Cooley and Prescott (1995) to solve the ambiguities related to the imputation of proprietors' income and the difference between national product and national income in the NIPA series.

2. Estimate the depreciation of capital stocks (including those not explicitly present in the model as eg. consumer durables). This is achieved by using a steady-state version of the law of motion of any capital stock: $(1+n)(1+\lambda)k = x + (1-\delta)k$. n is the average population growth rate and λ the productivity growth rate. Take the NIPA series for the investments (x) and the stock value.

3. Determine the return on *fixed capital*, i, from the general equation:

$$IC^j = (i + \delta^j)\, K^j \tag{14}$$

   where IC^j stands for income from stock j and δ^j is the stock depreciation rate. We use the income series constructed in point 1, the depreciation series obtained in point 2 and the corresponding NIPA series for the stock.

4. Equipped with the depreciation, stock series, and return on fixed capital i, use equation (14) to compute the income from any other capital stock.

5. Capital in the model includes household capital (consumer durables and residential structures) and inventories, over and above the stock of fixed private capital. Consequently, the income from capital in the model must account for the flow of services from durable goods computed above. Output series must also include an imputation of the flow of services from consumer durables. With these corrections we compute a consistent measure for the capital's share in income, α.

6. The crucial return on the model's capital, r, is estimated from equation (14), computing δ as in point 2 above and using the capital stock and capital income estimated in point 5, ie:

$$r = \text{Capital Income}/K - \delta$$

D Simulation results in the general model

D.1 Simulation of isolated shocks

In section 6 we analyse the effects of demographic shocks in two steps: first, by studying an isolated shock, and second by exploring the average performance in long simulated series. In this section we provide the details of the examples involving an isolated shock.

Example 1. An isolated shock in the private economy

In a world without persistence, isolated shocks are rare. Only by running large simulations is it possible to find a relatively large sequence of positive shocks, interrupted half-way through by a negative one. This is how we have generated the examples shown in the tables below and discussed in the main text.

- **Simulated time series**.

Calendar year	n_t	k	h	k/h	r	w
t-2	1.35	0.037	1.072	.0341	2.568	0.190
t-1	1.35	0.036	1.072	.0336	2.600	0.189
t	1.03	0.035	1.072	.0328	2.645	0.187
t+1	1.35	0.040	1.084	.0370	2.411	0.195
t+2	1.35	0.042	1.072	.0391	2.308	0.199
t+3	1.35	0.036	1.072	.0339	2.582	0.189

- **Simulated prices, decisions and welfare by cohort.**

Year of birth	n_t	w_1^{t+1}	w_2^{t+2}	r_2^{t+2}	r_3^{t+3}	a_1^{t+1}	a_2^{t+2}	EU
t-3	1.35	0.190	0.189	2.600	2.645	0.0288	0.0727	0.0177
t-2	1.35	0.189	0.187	2.645	2.411	0.0288	0.0735	0.0176
t-1	1.35	0.187	0.195	2.411	2.308	0.0269	0.0728	0.0172
t	1.03	0.195	0.199	2.308	2.582	0.0278	0.0729	0.0180
t+1	1.35	0.199	0.189	2.582	2.606	0.0314	0.0750	0.0182
t+2	1.35	0.189	0.188	2.606	2.404	0.0288	0.0735	0.0176
t+3	1.35	0.188	0.195	2.404	2.085	0.0272	0.0739	0.0173

Detailed description of the effects

A reduction in the population growth rate in t has a 1-period delayed impact on the relative scarcity of capital and labour, having effects from date $t+1$ onwards. One year after the shock, both *per worker* labour, h, and capital, k, increase. The former effect reflects that older workers, endowed with a larger amount of efficient labour units, become a bigger share of the labour force. The latter results from the simultaneous change in population composition and the first endogenous changes in individual's savings. The overall effect of these opposite forces is, in our simulation, a reduction in k/h in $t+1$, as labour becomes relatively more scarce than capital. This leads to an increase in salaries and a reduction in the return to saving. These changes go further away in $t+2$, as k reaches a maximum while h goes back to its initial value. From that point on, k progressively returns to its initial levels, driving wages and interest rates back to their starting values. Note that all changes after $t+2$ stem from changes in the saving decisions triggered by the shock, as changes in k serve as propagation mechanism of the demographic shock. These price changes occur at different points in the life-cycle of the several cohorts involved, causing uneven welfare effects:

- **Cohorts preceding the shock:** Cohort (born in) $t-2$ suffers from the drop in savings returns at $t+1$. This is in spite of the advanced notice of the arrival of the shock, which allows older workers in t to protect themselves by saving more. Cohort $t-1$ also ends up suffering badly, as they bear the brunt of the fall in interest rates (r hits a minimum in $t+2$, precisely when this cohort goes into retirement). Note that this cohort benefits from the large wages prevailing in $t+1$, but this is not enough to compensate for the drops in interest rates.

- **Cohort t**: The relatively small cohort born in t, as with cohort $t-1$, experiences both gains and drawbacks from the change in prices. The overall welfare impact is, however, very different in each case: t-cohort members enjoy a substantial gain in life-cycle utility. This is because this cohort fully benefits from the wage rises, while interest rate drops affect them only marginally (when they go into retirement the *direct* demographic effect is over, and r is quite close to the initial level).

- **Cohorts born after the shock**: Cohort $t+1$ is the last to experience direct effects from the shock, in the form of high wages in $t+2$. All other effects along this cohort's life-cycle are induced changes, with the recovery of the interest rates being particularly important. The behavioral reply to this new scenario takes the form of higher savings, which guarantee a progressive convergence of k to its pre-shock values, spreading in the process the welfare gains from the shock to the ensuing cohorts.

Positive shocks (the occurrence of a large population growth amid slowly growing cohorts) have entirely symmetric effects to those describe above.

Example 2: An isolated shock with a marginal PAYG scheme

We reproduce here the results corresponding to the simulation of an isolated shock discussed in section 6.1. They include the contribution rates experienced at different ages by the cohorts affected by the shock, the pensions they receive, the ratio of their retirement savings with and without the PAYG system and the equivalent variation in welfare associated with the introduction of the system.

Year of birth	n_t	ς_1^{t+1}	ς_2^{t+2}	b^{t+3}	$a_{2\ SS}^{t+2}/a_2^{t+2}$	EV%
t-3	1.35	.0063	.0063	.0040	.9772	-0.6727
t-2	1.35	.0063	.0063	.0042	.9779	-0.9342
t-1	1.35	.0063	.0073	.0043	.9774	-0.4064
t	1.03	.0073	.0083	.0040	.9760	-0.7768
t+1	1.35	.0083	.0063	.0040	.9763	-1.0498
t+2	1.35	.0063	.0063	.0042	.9773	-0.9648

D.2 Welfare analysis in the main simulation

Tables 7 and 8 present detailed results of the simulation of our benchmark economy, conditional on the demographic state ($\bar{n}$), ie. conditional on the size of the demographic shocks corresponding to the own, previous and immediately following cohorts. Table 7 corresponds to the private economy while table 8 reflects the impact of the introduction of PAYG pensions.

n_t	n_{t-1}	n_{t+1}	w_1^{t+1}	w_2^{t+2}	r_2^{t+2}	r_3^{t+3}	EU
1.03	1.03	1.03	0.211	0.213	1.990	2.071	0.0186
1.03	1.03	1.35	0.211	0.203	2.202	2.391	0.0188
1.03	1.35	1.03	0.198	0.210	2.052	2.126	0.0179
1.03	1.35	1.35	0.198	0.209	2.265	2.433	0.0182
1.35	1.03	1.03	0.202	0.200	2.299	2.120	0.0180
1.35	1.03	1.35	0.202	0.191	2.533	2.455	0.0183
1.35	1.35	1.03	0.190	0.197	2.374	2.167	0.0173
1.35	1.35	1.35	0.189	0.188	2.621	2.535	0.0176

Table 7: Simulation results in the private economy: average life-cycle prices and expected utility by demographic state of nature (ie, size of the demographic shocks corresponding to the own, previous and following cohorts).

n_t	n_{t-1}	n_{t+1}	ς_1^{t+1}	ς_2^{t+2}	$b^{t+3}\cdot 100$	$\bar{\varsigma}^t$	$a_{2\ SS}^{t+2}/a_2^{t+2}$	EU	EV(%)
1.03	1.03	1.03	0.957	0.957	0.447	1.270	0.972	0.0184	-1.5658
1.03	1.03	1.35	0.957	0.826	0.419	1.193	0.973	0.0187	-1.5699
1.03	1.35	1.03	0.730	0.957	0.442	1.057	0.974	0.0178	-0.9865
1.03	1.35	1.35	0.730	0.826	0.416	0.976	0.975	0.0180	-1.0811
1.35	1.03	1.03	0.826	0.730	0.443	0.983	0.974	0.0179	-1.2830
1.35	1.03	1.35	0.826	0.630	0.415	0.943	0.975	0.0182	-1.3521
1.35	1.35	1.03	0.630	0.730	0.439	0.794	0.977	0.0172	-0.7174
1.35	1.35	1.35	0.630	0.630	0.408	0.755	0.978	0.0175	-0.8020

Table 8: Simulation results in the economy with a 2% PAYG-DB pension system: life-cycle contribution rates, pension, effective contribution, ratio of savings with and without the pension system, expected utility and Equivalent variation by demographic state of nature (ie, size of the demographic shocks corresponding to the own, previous and immediately following cohorts).

E Sensitivity analysis

The results in this section complete the sensitivity analysis in section 7. We repeat our basic experiment of introducing a marginal PAYG pension system in economies differing in terms of the preference parameters but reproducing the empirical rate of return on capital. This was achieved by simultaneously changing the discount factor whenever the resulting economy produced a different return on capital. In this section we relax the "calibration restriction" and simply explore the results of our basic experiment in economies differing by just one parameter at a time. These results are easier to interpret but less relevant for our problem, as the resulting economies tend to diverge from the targeted US economy in a fundamental way. This can be appreciated by computing the predicted interest rate (with a realistically calibrated pension system), and comparing with the empirical value. The results are presented in table 9 and arranged as follows. We explore the sensitivity to changes in the IES first. In the top left panel we show the results when all other parameters are fixed at their benchmark values. In the top right panel we consider the same elasticities, but when the discount factor is cero. All other parameters are as in the benchmark. Finally in the middle-left panel the discount factor is 1%. The results of changing the discount factor itself (with all the other parameters fixed at the benchmark values) are reported in the middle-right panel. The effect of changes in the technological parameters (again with all other parameters as in the benchmark) is displayed in the bottom panels. As in section 7, we can only conclude that our main finding is very robust to changes in the parameter values, with the well known exception of economies close to dynamic inefficiency (which manifests in implausibly low interest rates, as in the simulation with α =0.16).

Parameter	EV	EV($\mathbf{n}_l$)	EV($\mathbf{n}_h$)	r	Parameter	EV	EV($\mathbf{n}_l$)	EV($\mathbf{n}_h$)	r
IES					IES†				
0.5	-1.596	-1.570	-1.433	9.6	0.5	-1.578	-1.720	-1.418	9.5
0.7	-1.201	-1.321	-1.061	7.6	0.7	-1.179	-1.303	-1.035	7.4
0.9	-0.725	-0.829	-0.599	6.4	0.9	-0.684	-0.796	-0.555	6.2
1.1	-0.766	-0.857	-0.660	5.7	1.1	-0.736	-0.834	-0.621	5.5
IES‡					% Discount				
0.5	-1.660	-1.781	-1.520	10.4	0	-1.176	-1.303	-1.035	7.4
0.7	-1.302	-1.407	-1.182	8.3	0.2	-1.201	-1.321	-1.060	7.6
0.9	-0.906	-0.997	-0.806	7.0	0.5	-1.247	-1.364	-1.122	7.9
1.1	-0.919	-0.973	-0.827	6.3	1	-1.303	-1.407	-1.182	8.3
					3	-1.439	-1.517	-1.352	10.2
α					δ				
0.16	+506	+0.517	+0.501	4.6	1	-1.293	-1.495	-1.140	7.8
0.26	-0.619	-0.679	-0.533	6.2	4	-1.201	-1.321	-1.060	7.6
0.36	-1.201	-1.321	-1.060	7.6	7	-1.136	-1.246	-1.010	7.4
0.46	-1.570	-1.730	-1.405	9.0					

Table 9: "Non-adjusted" sensitivity analysis: Equivalent Variation associated with a 2% PAYG-DB pension system (both unconditional and conditional on the size of the own cohort), and interest rate in the economy with a calibrated pension system. All implicit parameters are fixed at their benchmark values with the exception of $\beta = 1$ in † and $\beta = 0.82$ (1% annual discount factor) in ‡. IES= Intertemporal elasticity of substitution, α= capital's share in output, δ= % annual depreciation rate.