IMSE2014: The 13th International Conference on Integral Methods in Science and Engineering

Karlsruhe Institute of Technology (KIT) 21 -25 July 2014

Minisymposium

“Asymptotic analysis: homogenization and thin structures”

Tuesday 22 July -Wednesday 23 July

Speakers / schedule / abstracts of the talks

Andrey A. Amosov, Moscow Power Engineering Institute, Moscow, Russia

Tuesday 22, 17:17.30

Title: Semidiscrete and asymptotic approximations for the nonstationary radiative-conductive heat transfer problem in a periodic system of grey heat shields

Abstract:

We consider semidiscrete and asymptotic approximations to the nonstationary nonlinear initial-boundary-value problem governing the radiative-conductive heat transfer in a periodic system consisting of n grey parallel plate heat shields of width $\varepsilon = 1/n$, separated by vacuum interlayers. We study properties of special semidiscrete and homogenized problems whose solutions approximate the solution to the problem under consideration. We establish the unique solvability of the problem and deduce a priori estimates for the solutions. We obtain error estimates of order $O(\sqrt{\varepsilon})$ and $O(\varepsilon)$ for semidiscrete approximations and error estimates of order $O(\sqrt{\varepsilon})$ and $O(\varepsilon^{3/4})$ for asymptotic approximations.

Jose M. Arrieta, Universidad Complutense de Madrid, Madrid, Spain

Tuesday 22, 16:16.30

Title: Locally periodic thin domains with varying period

Joint work with Manuel Villanueva-Pesqueira

Abstract:

We analyze the behavior of the solutions of the Laplace equation with Neumann boundary conditions in a thin domain with a highly oscillatory boundary. The oscillations are locally periodic in the sense that both the amplitude and the period of the oscillations may not be constant and actually they vary in space. We obtain the asymptotic homogenized limit and provide some correctors. To accomplish this goal, we extend the unfolding operator method to the locally periodic case. The main ideas of this extension may be applied to other cases like perforated domains or reticulated structures, which are locally periodic with not necessarily a constant period.

Loredana Balilescu, U. of Pitesti, Romania, and Federal U. of Santa Catarina, Brasil

Tuesday 22, 16:16.30

Title: Burnett coefficients and laminates

Abstract:

In this talk, we consider a periodic media and we study the dependence of the fourth-order dispersion tensor d or Burnett coefficient in terms of the microstructure. We treat the one dimensional case and also some structures in higher dimension. Considering the one dimensional periodic medium, we completely describe the set in which the dispersion coefficient lies, as the microstructure varies preserving the volume proportion. In higher dimension, the focus of attention is the variation of d on laminated microstructures, where complete bounds are obtained on its quartic form along with the corresponding optimal structures. Differences with corresponding results for the homogenized matrix are pointed out. Using Blossoming Principle, it is shown that d is not negative in the sense of Legendre–Hadamard, even though its quartic form is negative

Alain Brillard, Université de Mulhouse, Mulhouse, France

Tuesday 22, 10:10.30

Title: On the interface boundary conditions between two interacting incompressible viscous fluid flows

Joint work with M. El Jarroudi,

Abstract:

We consider a bounded open subset $\Omega \subset \mathbb{R}^3$ with Lipschitz continuous boundary $\partial\Omega$, such that $\Omega = \Omega^1 \cup \Sigma \cup \Omega^2$, where Ω^1 and Ω^2 are two non-empty open subsets separated by the smooth surface Σ, that we suppose to be contained in the plane $\{x_3 = 0\}$, for simplicity. For every $\varepsilon \in (0,1)$, we denote Σ_ε the thin layer surrounding Σ defined by

$$\Sigma_\varepsilon = \left\{ x \in \mathbb{R}^3 \mid x' = (x_1, x_2) \in \Sigma,\ -\varepsilon h_\varepsilon^1(x') < x_3 < \varepsilon h_\varepsilon^2(x') \right\},$$

where h_ε^1 and h_ε^2 are positive and locally Lipschitz continuous functions satisfying

$$\sup_\varepsilon \|h_\varepsilon^\alpha\|_{L^\infty(\Sigma)} \le 1 \ ; \ \sup_\varepsilon \left(\varepsilon \left\| \frac{\partial h_\varepsilon^\alpha}{\partial x_\alpha} \right\|_{L^\infty(\Sigma)} \right) < +\infty,\ \alpha = 1, 2. \tag{1}$$

We set

$$\begin{array}{lllllllll} \Omega_\varepsilon^1 & = & \Omega^1 \backslash \overline{\Sigma}_\varepsilon, & \Omega_\varepsilon^2 & = & \Omega^2 \backslash \overline{\Sigma}_\varepsilon, & \Gamma_\varepsilon^1 & = & \partial\Omega_\varepsilon^1 \cap \partial\Sigma_\varepsilon, \\ \Gamma_\varepsilon^2 & = & \partial\Omega_\varepsilon^2 \cap \partial\Sigma_\varepsilon, & \Gamma_\varepsilon & = & \Gamma_\varepsilon^1 \cup \Gamma_\varepsilon^2. & & & \end{array} \tag{2}$$

For a given $f \in L^\infty(\Omega; \mathbb{R}^3)$, we consider the stationary fluid flow problem posed in Ω

$$\left\{ \begin{array}{rcll} -\nu^1 \Delta u^\varepsilon + (u^\varepsilon \cdot \nabla) u^\varepsilon + \nabla p^\varepsilon & = & f & \text{in } \Omega_\varepsilon^1, \\ -\nu^2 \Delta u^\varepsilon + (u^\varepsilon \cdot \nabla) u^\varepsilon + \nabla p^\varepsilon & = & f & \text{in } \Omega_\varepsilon^2, \\ -\varepsilon^{r-1} \nu^0 \operatorname{div}\left(|\nabla u^\varepsilon|^{r-2} \nabla u^\varepsilon \right) + (u^\varepsilon \cdot \nabla) u^\varepsilon + \nabla p^\varepsilon & = & f & \text{in } \Sigma_\varepsilon, \\ \operatorname{div}(u^\varepsilon) & = & 0 & \text{in } \Omega, \end{array} \right. \tag{3}$$

where $r \in]1, 2[$ and with the transmission and boundary conditions

$$\left\{ \begin{array}{rcll} [u^\varepsilon]_{\Gamma_\varepsilon} & = & 0 & \text{on } \Gamma_\varepsilon^1 \cup \Gamma_\varepsilon^2, \\ \nu^\alpha \dfrac{\partial u^\varepsilon}{\partial n} - \varepsilon^{r-1} \nu^0 |\nabla u^\varepsilon|^{r-2} \dfrac{\partial u^\varepsilon}{\partial n} & = & 0 & \text{on } \Gamma_\varepsilon^\alpha,\ \alpha = 1, 2, \\ u^\varepsilon & = & 0 & \text{on } \partial\Omega. \end{array} \right.$$

Using Γ-convergence methods, we will prove that the solution $(u^\varepsilon, p^\varepsilon)$ of (3) converges to that of the limit problem $(\alpha, \beta = 1, 2,\ i = 1, 2, 3)$

$$\left\{ \begin{array}{rcll} -\nu^\alpha \Delta u^0 + (u^0 \cdot \nabla) u^0 + \nabla p_\alpha^0 & = & f & \text{in } \Omega^\alpha, \\ \operatorname{div}(u^0) & = & 0 & \text{in } \Omega^\alpha, \\ u^0 & = & 0 & \text{on } \partial\Omega^\alpha \setminus \Sigma, \\ [(u^0)_3]_\Sigma & = & 0 & \text{on } \Sigma, \\ \nu^2 \left(\dfrac{\partial (u^0)_3}{\partial x_3} \right)^+ |_\Sigma - \nu^1 \left(\dfrac{\partial (u^0)_3}{\partial x_3} \right)^- |_\Sigma & = & p_1^0 - p_2^0 & \text{on } \Sigma, \\ \nu^2 \left(\dfrac{\partial (u^0)_\beta}{\partial x_3} \right)^+ |_\Sigma = \nu^1 \left(\dfrac{\partial (u^0)_\beta}{\partial x_3} \right)^- |_\Sigma & = & \nu^0 \left| [u^0]_\Sigma \right|^{r-2} & \\ & & \times [(u^0)_i]_\Sigma\, \mu_{\beta i} & \text{on } \Sigma. \end{array} \right. \tag{4}$$

where $\mu^\bullet = (\mu_{ij})_{i,j=1,2,3}$ is a symmetric matrix of Borel measures which have the same support contained in Σ, which do not charge the polar subsets of Σ and which satisfy $\mu_{ij}(B) \zeta_i \zeta_j \ge 0$, $\forall \zeta \in \mathbb{R}^3$, $\forall B \in \mathcal{B}(\Sigma)$, where $\mathcal{B}(\Sigma)$ denotes the set of Borel subsets of Σ. For the proof, we will first describe the specific functional framework associated to this problem. Then we will build the appropriate test-functions. We will conclude with the description of some special cases. For the details, we refer to the paper [1].

[1] A. Brillard, M. El Jarroudi, On the interface boundary conditions between two interacting incompressible viscous fluid flows. J. Differential Equations 255 (2013) 881–904.

Delfina Gómez, Universidad de Cantabria, Santander, Spain

Tuesday 22, 14.30:15

Title: Spectral problems in banded domains: local effects for eigenfunctions

Abstract:
We consider the asymptotic behavior for the eigenelements of a second order differential operator, with piecewise constants coefficients, in two-dimensional domains composed of a fixed domain surrounded by thin, heavy and/or stiff bands. These models are of interest, for instance, in the study of reinforcement problems. Considering the range of the low, middle and high frequencies, we provide asymptotics for the eigenvalues and the corresponding eigenfunctions. In particular, we highlight the frequencies for which the corresponding eigenfunctions may be localized asymptotically in small neighborhoods of certain points of the boundary. This is a work of collaboration with S. Nazarov and E. Pérez

Andrii Khrabustovskyi, Karlsruher Institut fuer Technologie, Kalsruhe, Germany

Tuesday 22, 14 :14.30

Title: Spectral properties of elliptic operator with double-contrast coefficients near a hyperplane

Joint work with Michael Plum (KIT, Karlsruhe)

Abstract:
We study the spectrum of the elliptic operator governing vibrations of a body occupying a bounded domain and containing many small heavy inclusions surrounded by thin soft layers. The inclusion are located along a plain having nonempty intersection with the body.
Our goal is to describe the asymptotic behavior of the spectrum, when the number of the inclusions tends to infinity, while their radii and the distances between them tend to zero. We prove the Hausdorff convergence of the spectrum and show that in some cases the limit problem may have nonempty essential spectrum.
Also we discuss some applications to spectral theory of periodic differential operators in domains with waveguide geometry.
The results are available online: Andrii Khrabustovskyi, Michael Plum, Spectral properties of elliptic operator with double-contrast coefficients near a hyperplane, arXiv:1404.2555.

Pier Domenico Lamberti, University of Padova, Padova, Italy

Tuesday 22, 15 :15.30

Title: Homogenization of oscillating boundaries of hinged plates via unfolding method

Abstract:
We consider the biharmonic operator subject to suitable intermediate boundary conditions on a bounded domain in the n-dimensional Euclidean space. For n=2 this problem is related to the study of so-called hinged plates. We analyze the limiting behavior of the solutions to the corresponding Poisson problem, as well as of eigenvalues and eigenfunctions, when the boundary of the domain is described by a periodic oscillatory profile depeding on a parameter. We show that there is a critical parameter such that the limiting problem depends on whether we are above, below or just sitting on such critical value. The critical case leads to the study of a somewhat typical homogenization problem and provides a limiting strange term. The analysis of such case is done by means of so-called unfolding method.
This is a joint work with Jose Maria Arrieta

Taras Melnyk, National University of Kyiv, Kyiv, Ukraine

Wednesday 23, 11.30:12

Title: Asymptotic approximations for chemical reactive flows through the exterior thick fractal Junctions

Abstract:

In recent years, materials with complex structure are widely used in engineering devices, biology and other fields of science. It is known that many properties of materials are controlled by their geometrical structure. Therefore, the study of the influence of the material microstructure can improve its useful properties and reduce undesirable effects. Mathematical models for this study are boundary-value problems (BVP's) in domains with complex structures: perforated domains, grid-domains, domains with rapidly oscillating boundaries, thick junctions, etc.

Successful applications of thick-junction constructions in nanotechnologies and microtechnique have stimulated active learning BVP's in thick junctions with more complex structures: thick multi-level junctions, thick cascade junctions (see [1]–[4] and references therein).

In my report I am going to present new results for boundary-value problems in thick junctions that have fractal structure. The main question of my presentation is the asymptotic approximation and homogenization of chemical reactive flows through the exterior of thick fractal junctions. Namely, I will focus my attention on a semi-linear parabolic problem which describes the motion of a reactive fluid having different chemical features on different branching levels of a thick fractal junction.

A model thick fractal junction is the union of some domain, which is called the junction's body, and a lot of joined thin trees situated ε-periodically along some manifold, which is called the joint zone, on the boundary of the junction's body. The thin tree has finite number of branching levels. The small parameter ε characterizes the distance between neighboring thin branches and also their thickness.

The effective behavior of this reactive flow (as $\varepsilon \to 0$) is described by a new nonstandard homogenized parabolic problem containing extra zero-order terms which catch the effect of the chemical reactions on each branch of the thick fractal junction. In addition, the leading terms of the asymptotic expansion for the solution are constructed and the corresponding asymptotic estimate in the Sobolev space $L^2(0,T;H^1(\Omega_\varepsilon))$ is proved.

[1] T.A. Mel'nyk, D.Yu. Sadovyj, Homogenization of boundary value problems in two-level thick junctions consisting of thin disks with rounded or sharp edges. *Journal of Mathematical Sciences*, 191 **(2013)**, no. 2, pp. 254–280.

[2] T. Durante, T.A. Mel'nyk, Homogenization of quasilinear optimal control problems involving a thick multilevel junction of type 3 : 2 : 1. *ESAIM: Control, Optimisation and Calculus of Variations*, 18 **(2012)**, no. 2, pp. 583–610.

[3] G.A. Chechkin, T.A. Mel'nyk, Spatial-skin effect for eigenvibrations of a thick cascade junction with "heavy" concentrated masses *Math. Meth. Appl. Sci.* 37 **(2014)**, pp. 56–74.

[4] D. Blanchard, A. Gaudiello and T.A. Mel'nyk, Boundary homogenization and reduction of dimension in a Kirchhoff-Love plate, *SIAM J. Math. Anal.* 39 **(2008)**, no. 6, pp.1764-1787.

Julia Orlik, Institut für Techno- und Wirtschaftmathematik, Kaiserslautern, Germany

Wednesday 23, 11:11.30

Title: Homogenization of the frictional contact problems on a periodic microstructure.

Joint work with D. Cioranescu, A. Damlamian and V. Shiryaev

Abstract:

We consider the elasticity problem in a heterogeneous domain with an $\varepsilon-$periodic micro-structure, including a multiple micro-contact between the structural components. These components can be a simply connected matrix domain with open cracks or inclusions completely surounded by cracks, which do not connect the boundary. The contact is described by the Signorini and Tresca-friction contact conditions. The Signorini condition is a closed convex cone for the open cracks, while the friction condition is a nonlinear convex functional over the interface jump of the solution on the oscillating interface.
The difficulties appear when the inclusions are completely surounded by cracks and can have rigid displacements. In this case, in order to obtain preliminary estimates for the solution in the ε-domain, the Korn inequality should be modified, first in the fixed context and then for the ε-dependent periodic case. Additionally, for all states of the contact (inclusions can freely move, or are locked at the interface with the matrix, or the frictional traction is achieved on the inclusion-matrix interface and the inclusions can slide in the tangential to the interface direction) we obtain estimates for the solution in the ε-domain, uniform with respect to ε .
An asymptotic analysis (as $\varepsilon \to 0$) for nonlinear functionals over the growing interface is also performed, based on the application of the periodic unfolding method for sequences of jumps of the solution on the oscillating interface.

Grigory Panasenko (Institute Camille Jordan UMR CNRS 5208, PRES University of Lyon/ University of Saint Etienne, France)

Wednesday 23, 10.30:11

Title: Asymptotic expansion of the solution of the Kelvin-Voigt visco-elasticity equation for a thin strip.

The Kelvin-Voigt quasi-steady model for a stratified visco-elastic rod is considered. The complete asymptotic expansion is constructed. The homogenized integro-differential equation of high order is constructed. It confirms the so called memory effect of the stratified media and gives its interpretation in the sense of high order strain gradient theories.

M. Eugenia Pérez, Universidad de Cantabria, Santander, Spain

Tuesday 22, 11:30:12

Title: Spectral problems in porous media: on critical relations for large adsorption parameters in Robin boundary conditions

Abstract: We address asymptotics for spectral problems posed in periodically perforated domains. Ɛ measures the periodicity; it converge towards zero. The operator under consideration is the Laplacian, and the spectral problems are posed in a two or three dimensional domain Ω, outside the cavities. The boundary conditions are of the Dirichlet type on the boundary of Ω and of the Robin type on the boundary of the cavities containing a large Ɛ-dependent parameter (adsorption constant). Thus, several parameters of different orders of magnitude arise in the problem: the periodicity of the structure, the size ofthe cavities (o(Ɛ)) and the adsorption parameters. Depending on the relation between them, different homogenized problems are obtained: both critical sizes for cavities and critical relations for parameters are provided. As a matter of fact, for a certain size of the cavities, a critical relation is obtained in the case where the total area of the cavities multiplied by the adsorption parameter is of order O(1). This is a work of collaboration with D.Gómez, M. Lobo, T. Shaposhnikova, and M.N. Zubova

Andrii V. Popov , National University of Kyiv, Kyiv, Ukraine

Tuesday 22, 11:11:30

Title: Asymptotic analysis of the Steklov spectral problem in thin perforated domains with rapidly varying thickness and different limit dimensions

Abstract:

We consider the following spectral Steklov problem in a thin perforated domain Ω_ε^{n-d} :

$$\begin{cases} L_\varepsilon(u^\varepsilon) = 0 & \text{in } \Omega_\varepsilon^{n-d}, \\ \sigma_\varepsilon(u^\varepsilon) = \lambda(\varepsilon)\,\rho_\varepsilon\, u^\varepsilon & \text{on } G_\varepsilon, \\ \sigma_\varepsilon(u^\varepsilon) = 0 & \text{on } S_\varepsilon^{\pm}, \\ u^\varepsilon = 0 & \text{on } \Gamma_\varepsilon, \end{cases} \tag{1}$$

where L_ε is the second-order symmetric elliptic differential operator with quickly oscillating coefficients; $S_\varepsilon^{\pm}$ are rapidly varying parts of the boundary $\partial\Omega_\varepsilon^{n-d}$; G_ε is the union of the cavity boundaries; $\Gamma_\varepsilon = \partial\Omega_\varepsilon^{n-d} \setminus \left(S_\varepsilon^{\pm} \cup G_\varepsilon\right)$.

The thin perforated domain $\Omega_\varepsilon^{n-d} \subset \mathbb{R}^n$ has the limiting dimension $n-d$, i.e., it degenerates into a domain Ω_0 from $\mathbb{R}^{n-d}$ as $\varepsilon \to 0$ ($d \in \mathbb{N},\ d < n$).

Independently of the limiting dimension of the thin domain (for instance, it can be a thin plate or a rod) with the help of the approach developed in [1, 2], we study the asymptotic behaviour of the eigenvalues and eigenfunctions of the problem (1) as $\varepsilon \to 0$.

Under certain symmetry conditions on the geometry of the thin perforated domain Ω_ε^{n-d} and the coefficients of the equations, we construct full asymptotic expansions for the eigenvalues and eigenfunctions. We also obtain asymptotic estimates for the convergence rate of the eigenvalues and eigenfunctions in that case.

[1] T.A. Mel'nyk, Asymptotic expansions of eigenvalues and eigenfunctions for elliptic boundary-value problems with rapidly oscillating coefficients in a perforated cube, *J. Math. Sci., New York*, **75** (3) (1995), pp. 1646–1671.

[2] T.A. Mel'nyk, A.V. Popov, Asymptotic analysis of the Dirichlet spectral problems in thin perforated domains with rapidly varying thickness and different limit dimensions, *Mathematics and Life Sciences. Edited by Roderick V.N. Melnik and Alexandra V. Antoniouk, Berlin: De Gruyter.* (2013), pp. 89–109.