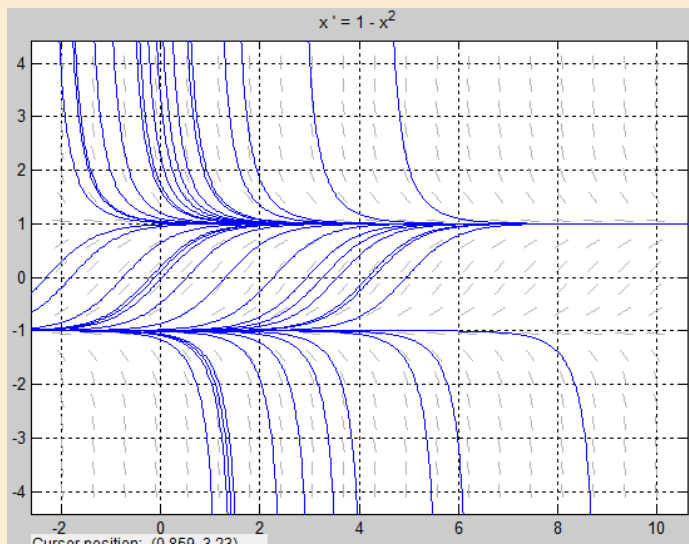


Soluciones a los Ejercicios

Ejercicio 1.

1. Con dfield campo de direcciones con $t \in [-2, 10]$, $y \in [-4, 4]$



2. Soluciones analíticas

$$\blacksquare y(0) = 2 \quad y = -\frac{e^{-(2t+\ln(3))} + 1}{e^{-(2t+\log(3))} - 1}$$

$$\blacksquare y(0) = -0,9 \quad -(\exp(\log(19)-2*t)-1)/(\exp(\log(19)-2*t)+ 1)$$

$$\blacksquare y(0) = 0 \quad -(1/\exp(2*t) - 1)/(1/\exp(2*t) + 1)$$

$$\blacksquare y(0) = -1,1 \quad -(\exp(\log(21) - 2*t)+1)/(\exp(\log(21)-2*t)-1)$$

3. Solución en $[0, 2]$ por RK para los casos $y(0) = -0,9$

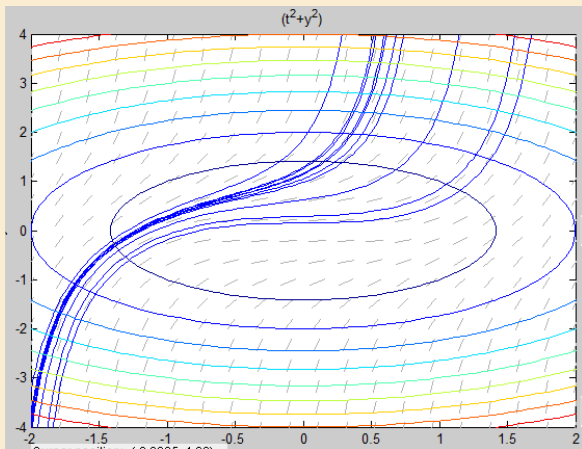
```
f=inline('t.^0-y.^2');yh=[];h=0.05;
[t, y] = rk4(f, [0,2], -0.9, h);yh=[yh; h y(length(t))],h=h/2;
```

Ejercicio 1

Ejercicio 2.

1. Con dfield

```
u=@(t,y)(t.^2+y.^2); ezcontour(u, [-2,2,-4,4]);
```



2. Iteramos hasta que $|y_h(0,9) - y_{h/2}(0,9)| < 0,05$

```
f=inline('t.^2+y.^2');yh=[];h=0.1;  
[t, y] = rk4(f, [0,1], 1, h);yh=[yh; h y(0.9/h+1)],h=h/2;  
diff(yh)
```

$h \approx 0,05$

Ejercicio 2

Ejercicio 3.

$$w_1 = w_0 + h f(x_0, w_0) = 2 + 0,25 \left(1 + \frac{2}{1} \right) = 2,75$$

$$w_2 = w_1 + h f(x_1, w_1) = 2,75 + 0,25 \left(1 + \frac{2,75}{1,25} \right) = 3,55$$

$$w_3 = w_2 + h f(x_2, w_2) = 3,55 + 0,25 \left(1 + \frac{3,55}{1,50} \right) = 4,391667$$

$$w_4 = w_3 + h f(x_3, w_3) = 4,391667 + 0,25 \left(1 + \frac{4,391667}{1,75} \right) = 5,2690476$$

k	x_i	w_i	$y(x_i)$
1	1,25	2,7500000	2,7789294
2	1,50	3,5500000	3,6081977
3	1,75	4,3916667	4,4793276
4	2,00	5,2690476	5,3862944

Ejercicio 4.

$$w_1 = w_0 + h f(x_0, w_0) = 1 + 0,25 (0 \cdot 1) = 1$$

$$w_2 = w_1 + h f(x_1, w_1) = 1 + 0,25 (0,25 \cdot 1) = 1,0625$$

$$w_3 = w_2 + h f(x_2, w_2) = 1,0625 + 0,25 (0,5 \cdot 1,0625) = 1,1953125$$

$$w_4 = w_3 + h f(x_3, w_3) = 1,1953125 + 0,25 (0,75 \cdot 1,1953125) = 1,4194335$$

$$w_5 = w_4 + h f(x_4, w_4) = 1,4194335 + 0,25 (1,0 \cdot 1,4194335) = 1,7742919$$

$$w_6 = w_5 + h f(x_5, w_5) = 1,7742919 + 0,25 (1,25 \cdot 1,7742919) = 2,3287582$$

$$w_7 = w_6 + h f(x_6, w_6) = 2,3287582 + 0,25 (1,5 \cdot 2,3287582) = 3,2020425$$

$$w_8 = w_7 + h f(x_7, w_7) = 3,2020425 + 0,25 (1,75 \cdot 3,2020425) = 4,6029362$$

k	x_i	w_i	$y(x_i)$
0	0,00	1	1
2	0,50	1,0625	1,1331484
4	1	1,4194335	1,6487212
6	1,5	2,3287582	3,0802168
8	2,00	4,6029362	7,3890561

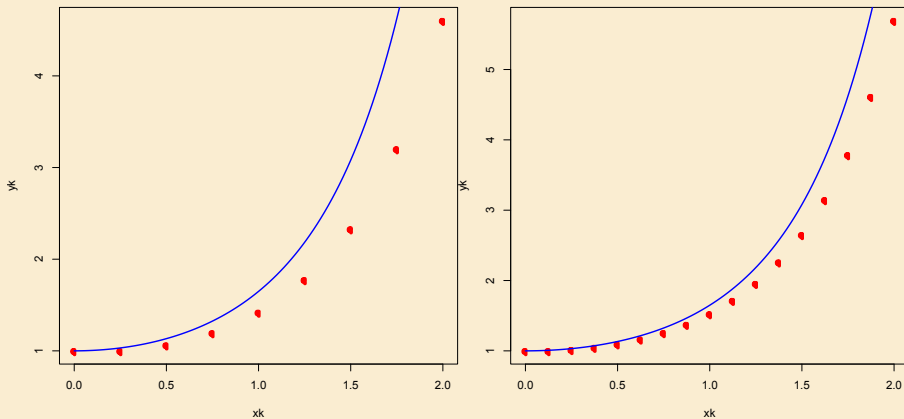


Figura 1: Solución con Euler de $y' = xy$. Izquierda con $h = 0,25$. Derecha con $h = 0,125$. La curva continua es la solución exacta.

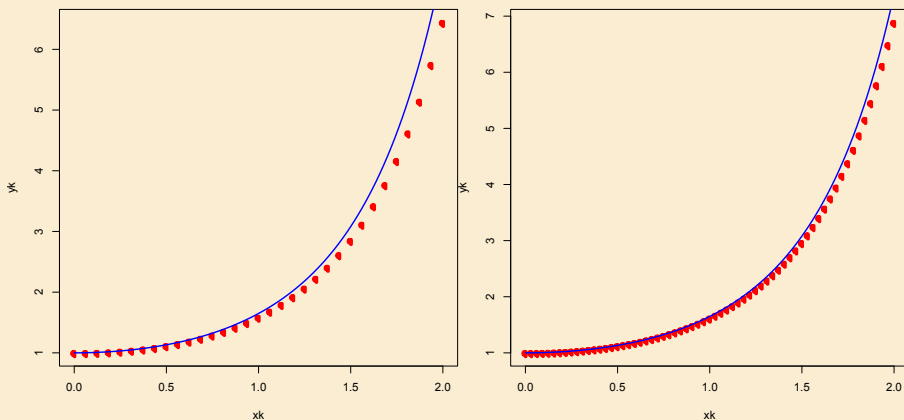


Figura 2: Solución con Euler de $y' = xy$. Izquierda con $h = 0,0625$. Derecha con $h = 0,03125$. La curva continua es la solución exacta.

Ejercicio 5.

$$k_1 = f(x_0, w_0) = 1 + \frac{2}{1} = 3 \quad k_2 = f(x_1, w_0 + h k_1) = 1 + \frac{2 + 0,25 \cdot 3}{1,25} = 3,2$$

$$w_1 = w_0 + \frac{h}{2}(k_1 + k_2) = 2 + \frac{1}{8}(3 + 3,2) = 2,7750000$$

$$k_1 = f(x_1, w_1) = 1 + \frac{2,775}{1,25} = 3,22 \quad k_2 = f(x_1, w_1 + h k_1) = 3,3866667$$

$$w_2 = w_1 + \frac{h}{2}(k_1 + k_2) = 2,775 + \frac{1}{8}(3,22 + 3,3866667) = 3,6008333$$

$$k_1 = f(x_2, w_2) = 3,400555555556 \quad k_2 = f(x_2, w_2 + h k_1) = 3,54341269841$$

$$w_3 = w_2 + \frac{h}{2}(k_1 + k_2) = 4,468829365079365$$

$$k_1 = f(x_3, w_3) = 3,553616780045 \quad k_2 = f(x_3, w_3 + h k_1) = 3,678616780045$$

$$w_4 = w_3 + \frac{h}{2}(k_1 + k_2) = 5,372858560090$$

k	x_i	w_i	$y(x_i)$
1	1,25	2,7750000	2,7789294
2	1,50	3,6008333	3,6081977
3	1,75	4,4688294	4,4793276
4	2,00	5,3728586	5,3862944

Ejercicio 5

Ejercicio 6.

$$k_1 = f(x_0, w_0) = 0 \cdot 1 = 0 \quad k_2 = f(x_1, w_0 + h k_1) = 0,5 \cdot 1 = 0,5$$

$$w_1 = w_0 + \frac{h}{2}(k_1 + k_2) = 1 + \frac{1}{4}(0 + 0,5) = 1,125$$

$$k_1 = f(x_1, w_1) = 0,5 \cdot 1,125 = 0,5625 \quad k_2 = f(x_1, w_1 + h k_1) = 1,40625$$

$$w_2 = w_1 + \frac{h}{2}(k_1 + k_2) = 1,125 + \frac{1}{4}(0,5625 + 1,40625) = 1,6171875$$

$$k_1 = f(x_2, w_2) = 1,0 \cdot 1,6171875 = 1,61719 \quad k_2 = f(x_2, w_2 + h k_1) = 3,63867$$

$$w_3 = w_2 + \frac{h}{2}(k_1 + k_2) = 1,6171875 + \frac{1}{4}(1,61719 + 3,63867) = 2,9311523$$

$$k_1 = f(x_3, w_3) = 1,5 \cdot 2,9311523 = 4,39673 \quad k_2 = f(x_3, w_3 + h k_1) = 10,25903$$

$$w_4 = w_3 + \frac{h}{2}(k_1 + k_2) = 2,9311523 + \frac{1}{4}(4,39673 + 10,25903) = 6,5950928$$

k	x_i	w_i	$y(x_i)$
0	0,00	1	1
1	0,50	1,125	1,1331484
2	1	1,6171875	1,6487212
3	1,5	2,9311523	3,0802168
4	2,00	6,5950928	7,3890561

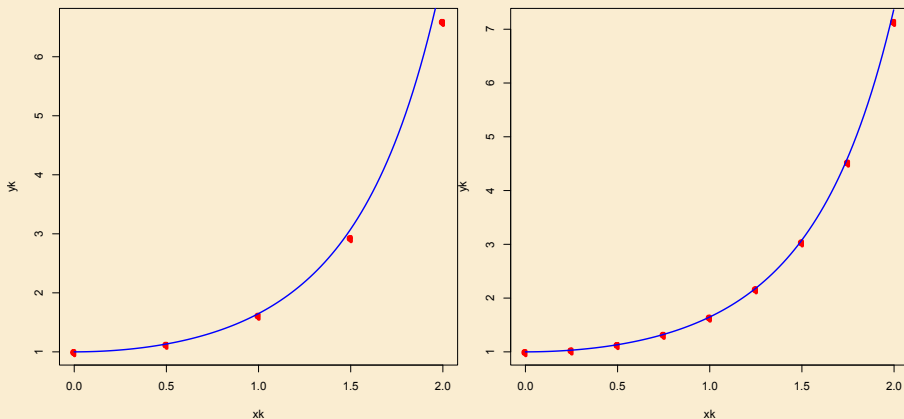


Figura 3: Solución de $y' = xy$. Izquierda con $h = 0,5$. Derecha con $h = 0,25$. Los puntos rojos corresponden a la solución exacta.

Ejercicio 7.

$$k_1 = f(x_0, w_0) = 1 + \frac{2}{1} = 3 \quad k_2 = f(x_0 + h/2, w_0 + 0,5 h k_1) = 3,11111111111111$$

$$w_1 = w_0 + h \cdot k_2 = 2,777777777777778$$

$$k_1 = f(x_1, w_1) = 3,22222 \quad k_2 = f(x_1 + h/2, w_1 + 0,5 h k_1) = 3,31313131313131$$

$$w_2 = w_1 + h \cdot k_2 = 2,775 + \frac{1}{8}(3,22 + 3,3866667) = 3,606060606060$$

$$k_1 = f(x_2, w_2) = 3,40404040404 \quad k_2 = f(x_2 + h/2, w_2 + 0,5 h k_1) = 3,480963$$

$$w_3 = w_2 + h \cdot k_2 = 4,4763014763$$

$$k_1 = f(x_3, w_3) = 3,5578865578 \quad k_2 = f(x_3 + h/2, w_3 + 0,5 h k_1) = 3,6245532$$

$$w_4 = w_3 + h \cdot k_2 = 5,38243978243978$$

k	x_i	w_i	$y(x_i)$
1	1,25	2,77777778	2,7789294
2	1,50	3,60606060	3,6081977
3	1,75	4,47630147	4,4793276
4	2,00	5,38243978	5,3862944

Ejercicio 7