Summary of equations for the best straight line by the method of least squares

n points x_i, y_i

Equal weights

General line $y = mx + c$

$$m = \frac{1}{D}\sum(x_i - \bar{x})y_i \qquad (\Delta m)^2 \approx \frac{1}{D}\frac{\sum d_i^2}{n-2}$$

$$c = \bar{y} - m\bar{x} \qquad (\Delta c)^2 \approx \left(\frac{1}{n} + \frac{\bar{x}^2}{D}\right)\frac{\sum d_i^2}{n-2}$$

$$\bar{x} = \frac{1}{n}\sum x_i \qquad \bar{y} = \frac{1}{n}\sum y_i$$

$$D = \sum(x_i - \bar{x})^2 \qquad d_i = y_i - mx_i - c$$

Line through origin $y = mx$

$$m = \frac{\sum x_i y_i}{\sum x_i^2} \qquad (\Delta m)^2 \approx \frac{1}{\sum x_i^2}\frac{\sum d_i^2}{n-1}$$

$$d_i = y_i - mx_i$$

Unequal weights

General line $y = mx + c$

$$m = \frac{1}{D_w}\sum w_i(x_i - \bar{x})y_i \qquad (\Delta m)^2 \approx \frac{1}{D_w}\frac{\sum w_i d_i^2}{n-2}$$

$$c = \bar{y} - m\bar{x} \qquad (\Delta c)^2 \approx \left(\frac{1}{\sum w_i} + \frac{\bar{x}^2}{D_w}\right)\frac{\sum w_i d_i^2}{n-2}$$

$$\bar{x} = \frac{\sum w_i x_i}{\sum w_i} \qquad \bar{y} = \frac{\sum w_i y_i}{\sum w_i}$$

$$D_w = \sum w_i(x_i - \bar{x})^2 \qquad d_i = y_i - mx_i - c$$

Line through origin $y = mx$

$$m = \frac{\sum w_i x_i y_i}{\sum w_i x_i^2} \qquad (\Delta m)^2 \approx \frac{1}{\sum w_i x_i^2}\frac{\sum w_i d_i^2}{n-1}$$

$$d_i = y_i - mx_i$$

De G.L.Squires, Practical physics, 3ª ed. Cambridge University press, 1976, pg 44