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COMPUTER-AIDED GEOMETRIC DESIGN AND COMPUTER GRAPHICS:

LINE DRAWING ALGORITHMS

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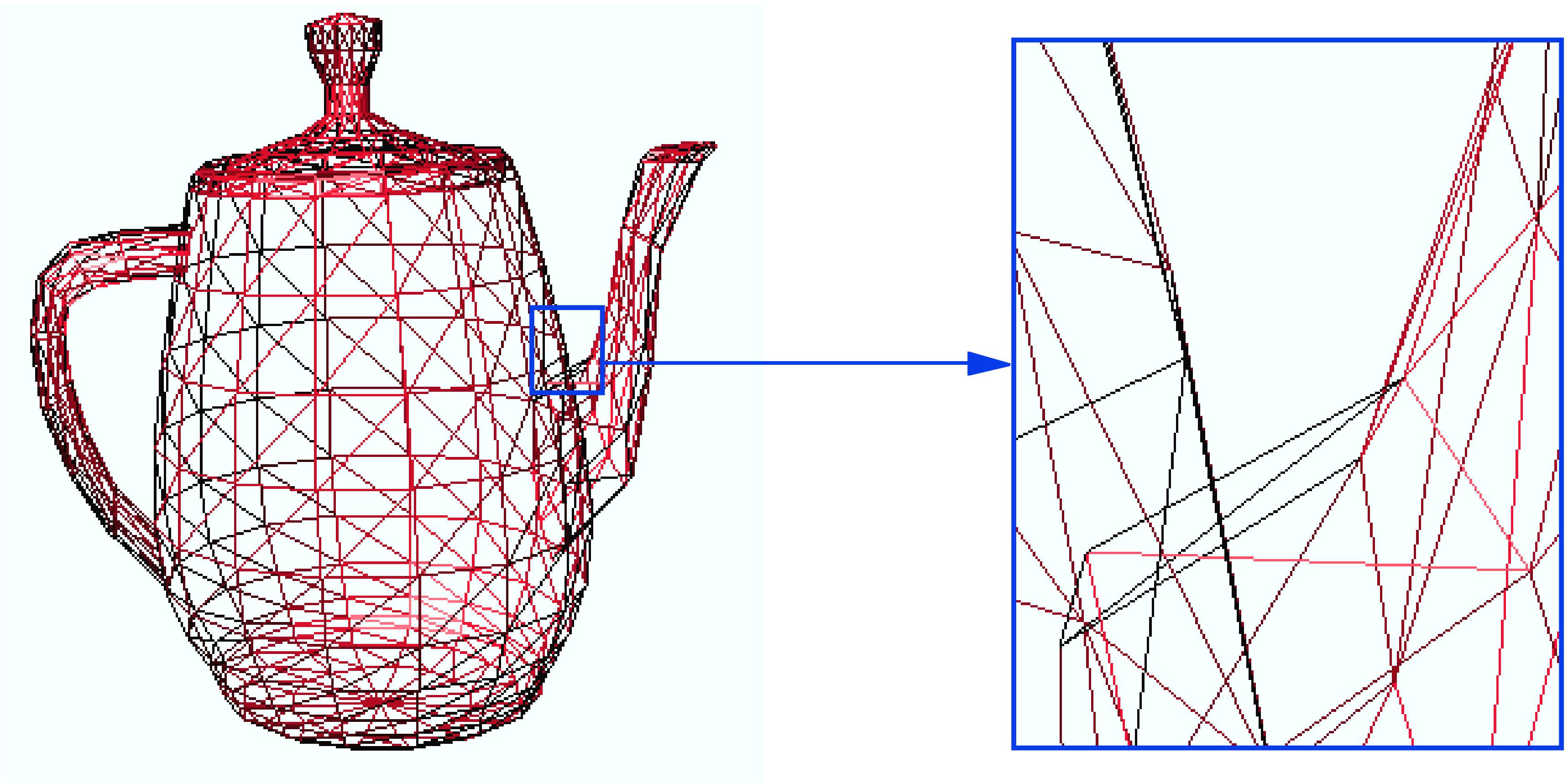
Line Drawing Algorithms



The lines of this object appear ***continuous***

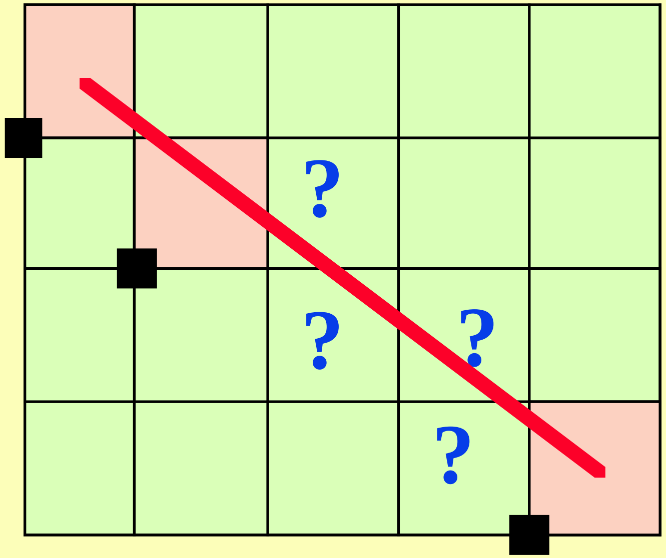
However, they are ***made of pixels***

Line Drawing Algorithms

We are going to analyze how this process is achieved.

Some useful definitions

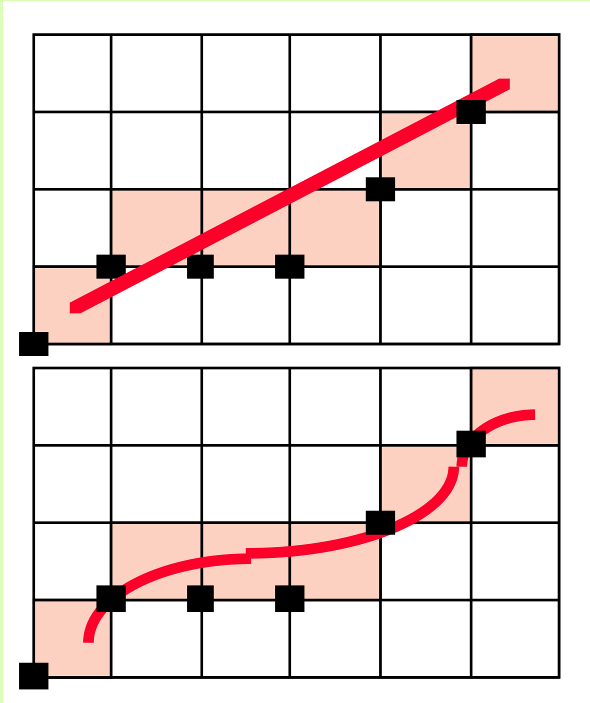
Rasterization: Process of determining which pixels provide the best approximation to a desired line on the screen.



Scan Conversion: Combination of rasterization and generating the picture in scan line order.

General requirements

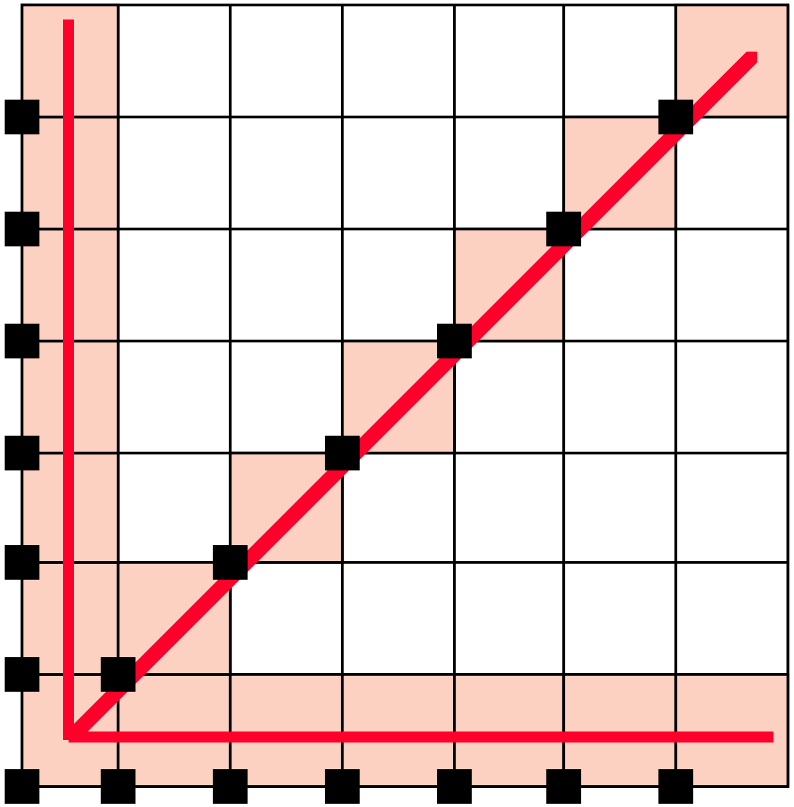
- Straight lines must appear as straight lines.



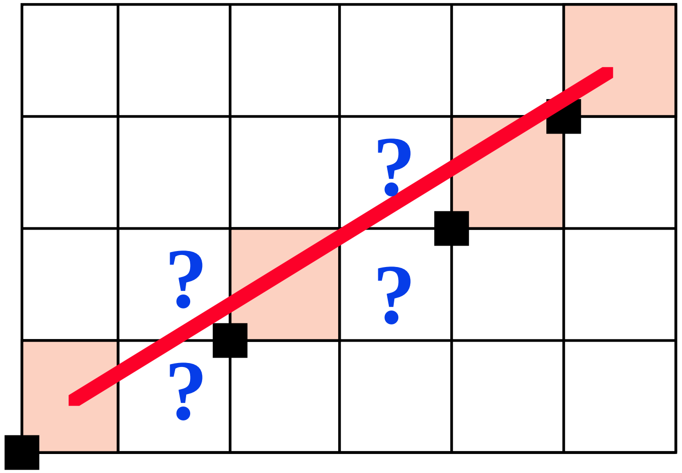
- They must start and end accurately
- Lines should have constant brightness along their length
- Lines should drawn rapidly

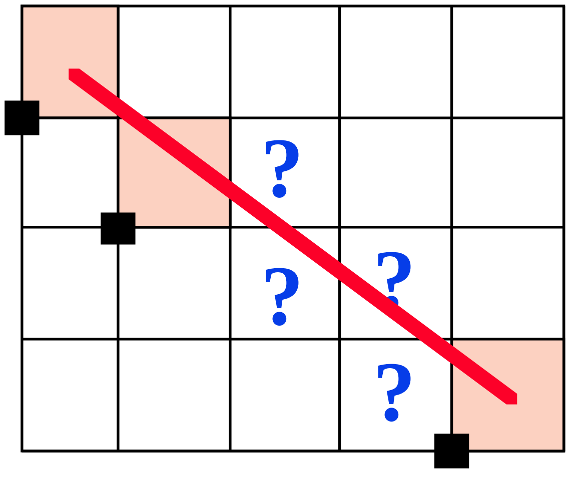
Line Drawing Algorithms

For horizontal, vertical and 45º lines, the choice of raster elements is obvious. This lines exhibit constant brightness along the length:



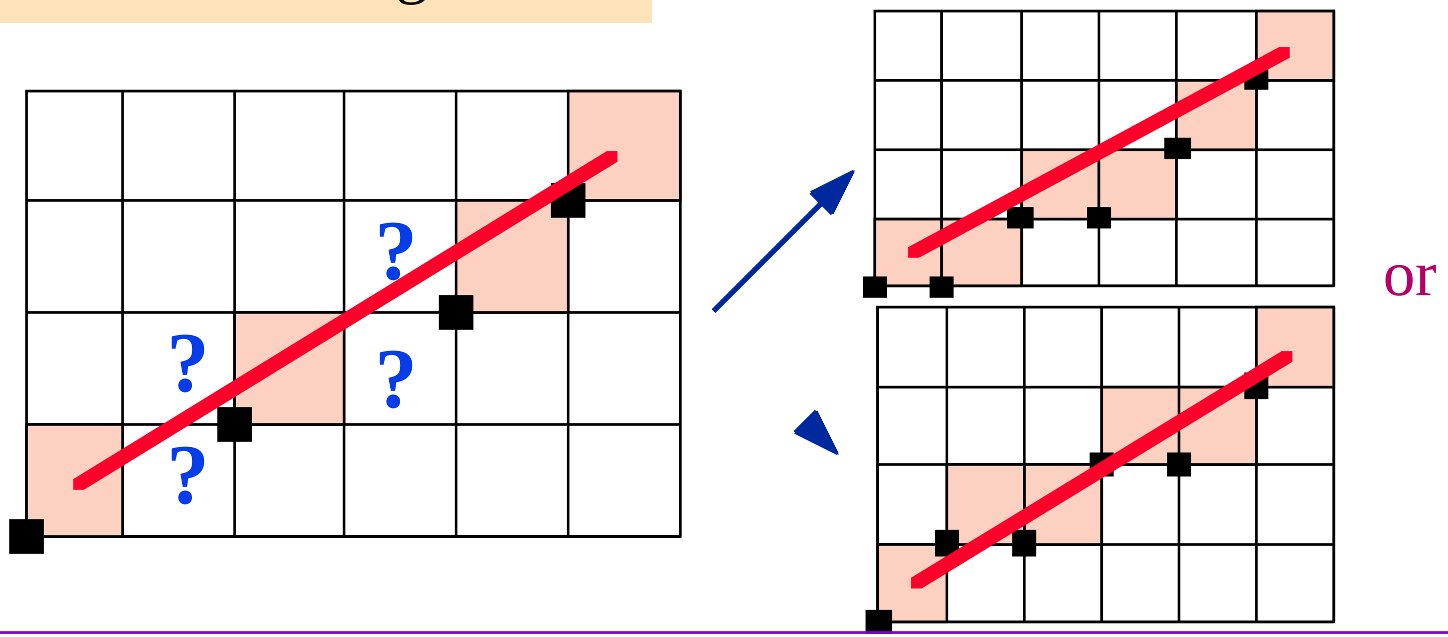
For any other orientation the choice is more difficult:





Line Drawing Algorithms

Rasterization of straight lines.



Rasterization yields uneven brightness: Horizontal and vertical lines appear brighter than the 45º lines.

For fixing so, we would need:

1. Calculation of square roots (increasing CPU time)
2. Multiple brigthness levels

=>

Compromise:

1. Calculate only an approximate line
2. Use integer arithmetic
3. Use incremental methods

Line Drawing Algorithms

The equation of a straight line is given by: $y = m.x + b$

Algorithm 1: Direct Scan Conversion

1. Start at the pixel for the left-hand endpoint x1
2. Step along the pixels horizontally until we reach the right-hand end of the line, xr
3. For each pixel compute the corresponding y value
4. round this value to the nearest integer to select the nearest pixel

```
x = xl;

while (x <= xr){

    ytrue = m*x + b;
    y = Round (ytrue);
    PlotPixel (x, y);
      /* Set the pixel at (x,y) on */
    x = x + 1;

}
```

The algorithm performs a floating-point multiplication for every step in *x*. This method therefore requires an enormous number of floating-point multiplications, and is therefore expensive.

Line Drawing Algorithms

Algorithm 2: Digital Differential Analyzer (DDA)

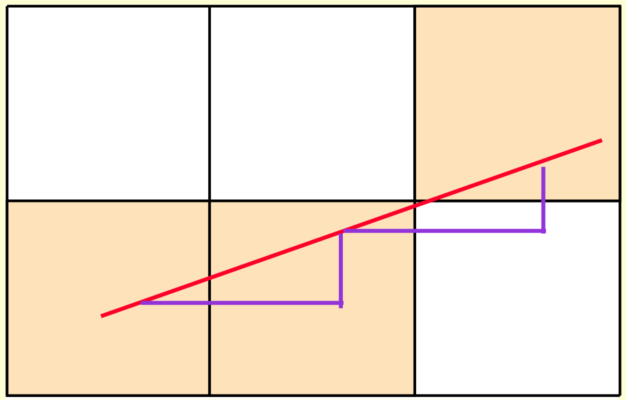
The differential equation of a straight line is given by:

$$\frac{dy}{dx} = constant \quad \text{or} \quad \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The solution of the finite difference approximation is:

$$x_{i+1} = x_i + \Delta x$$

$$y_{i+1} = y_i + \frac{y_2 - y_1}{x_2 - x_1} \Delta y$$



DDA uses repeated addition

We need only compute *m* once, as the start of the scan-conversion.

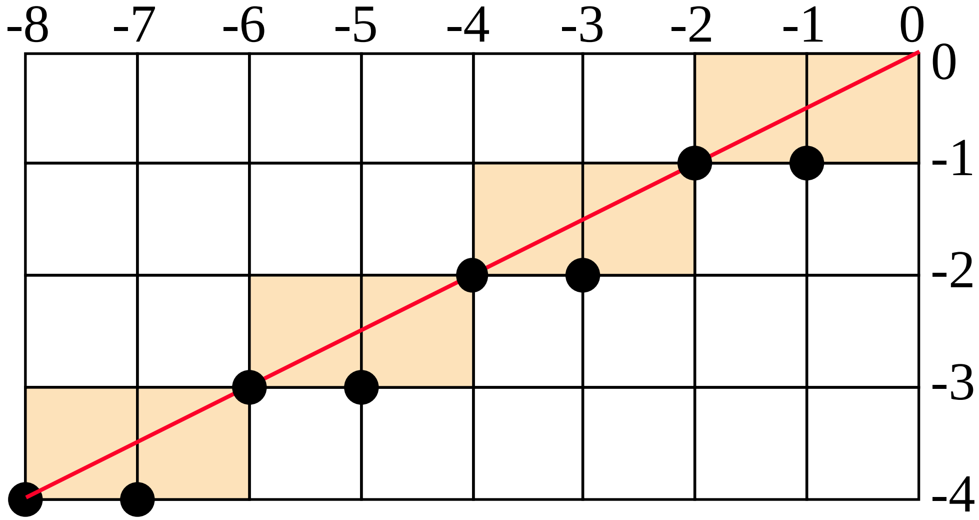
The DDA algorithm runs rather slowly because it requires real arithmetic (floating-point operations).

Line Drawing Algorithms

DDA algorithm for lines with $-1 < m < 1$

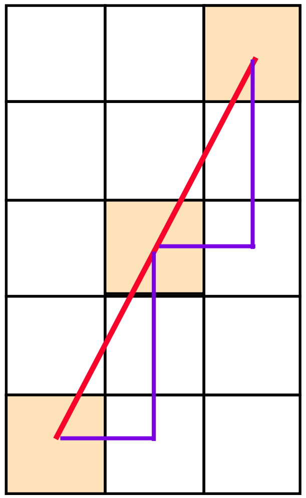
```
x = xl;
ytrue = yl;
while (x <= xr){
      ytrue = ytrue + m;
      y = Round (ytrue);
      PlotPixel (x, y);
      x = x + 1;
}
```

Example: Third quadrant

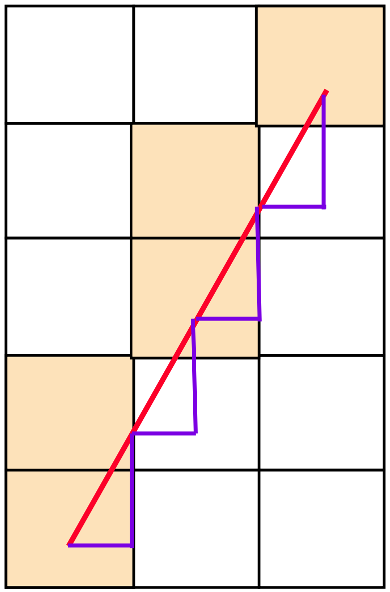


Switching the roles of *x* and *y* when $m>1$

Gaps occur when $m > 1$



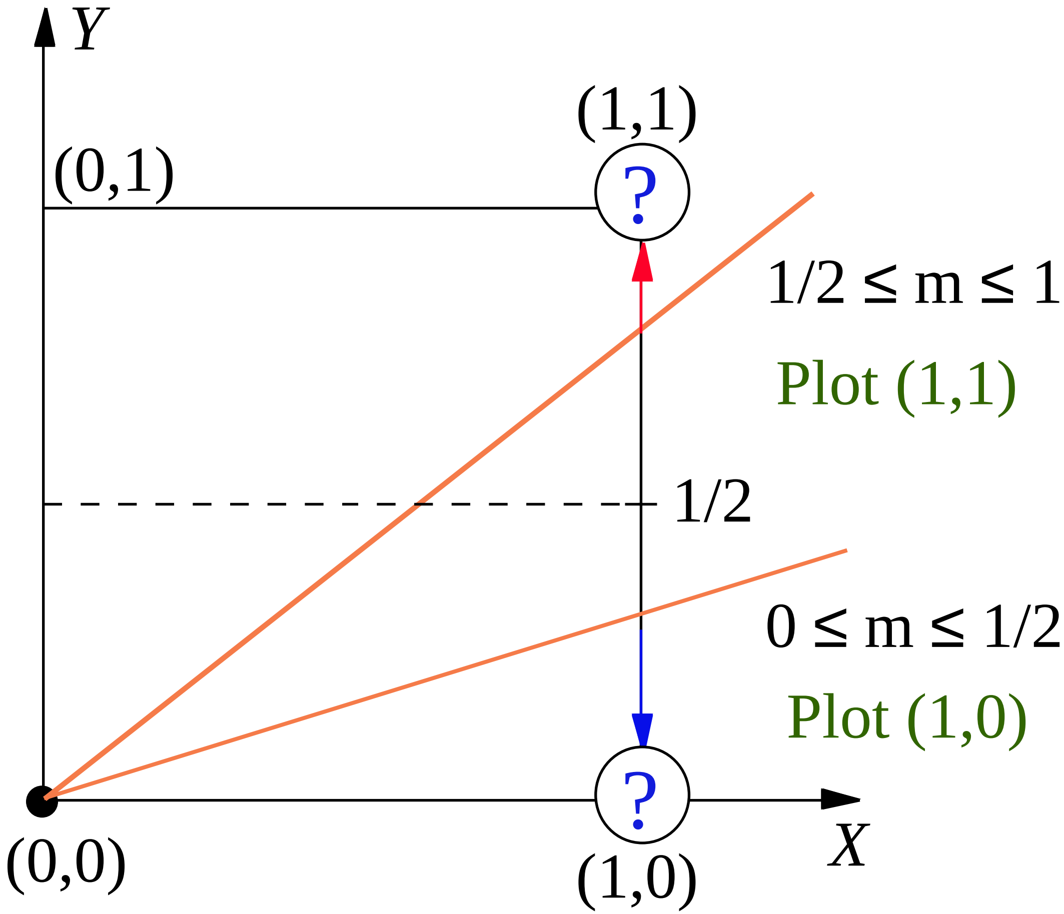
Reverse the roles of *x* and *y* using a unit step in *y*, and $1/m$ for *x*.



Algorithm 3: Bresenham's algorithm (1965)

Bresenham, J.E. *Algorithm for computer control of a digital plotter*, IBM Systems Journal, January 1965, pp. 25-30.

This algorithm uses only integer arithmetic, and runs significantly faster.



Key idea: distance between the actual line and the nearest grid locations (error).

Initialize error:

$e=-1/2$

Error is given by:

$e=e+m$

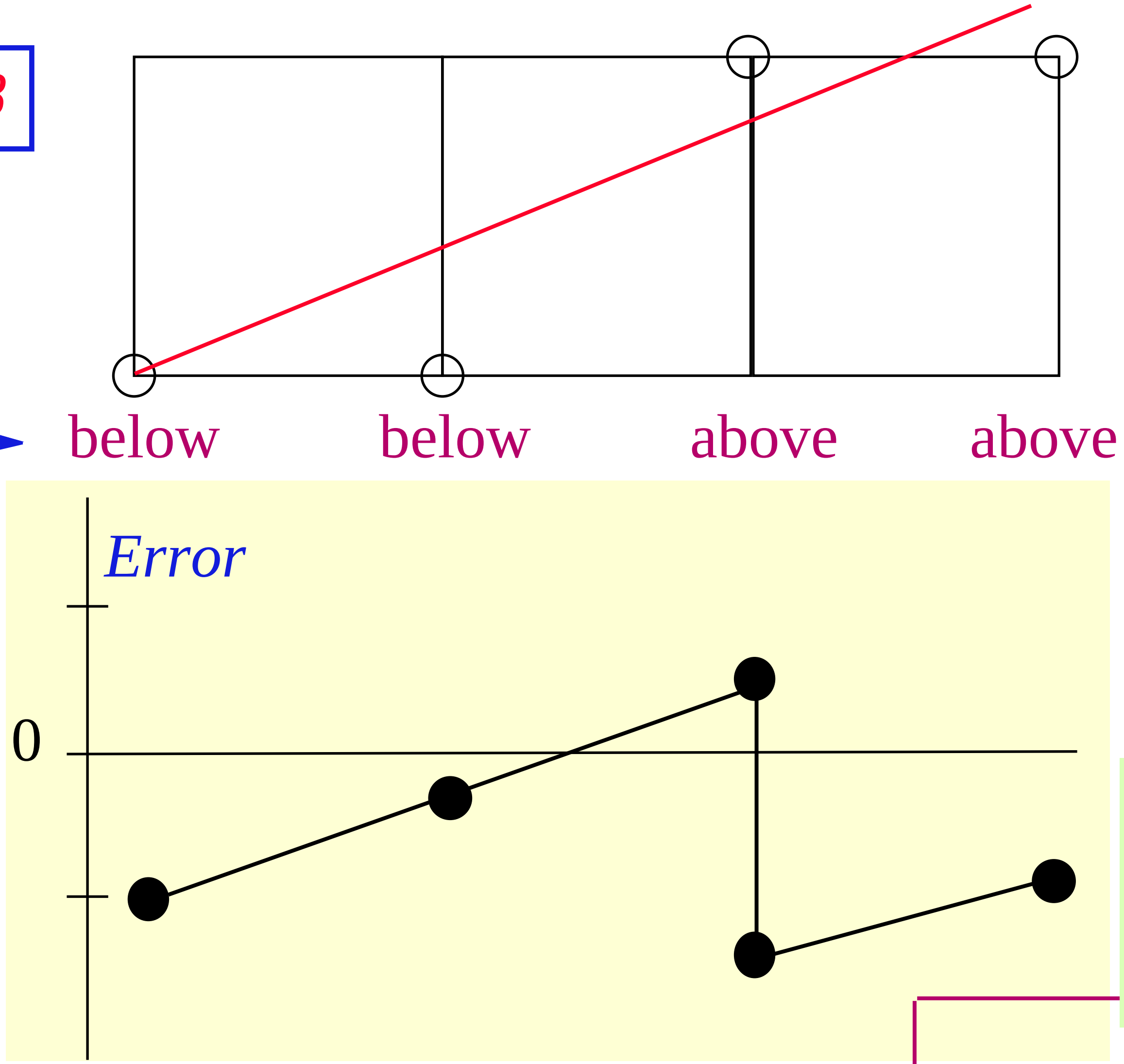
Reinitialize error:

when $e>0$

Line Drawing Algorithms

Example: *m=3/8*

If e<0 below
else above



Reinitialize error:
e = 1/4 -1
= -3/4

Error: *e=e+m*

Initial value: $e = -1/2$	$e = -1/2+3/8$ $=-1/8$	$e = -1/8+3/8$ $= 1/4$	$e = -3/4+3/8$ $= -3/8$

Line Drawing Algorithms

However, this algorithm does not lead to integer arithmetic. Scaling by: *2* dx*

```
void Bresenham (int xl, int yl, int xr, int yr)
    {
      int x,y;                       /* coordinates of pixel being drawn  */
      int dy, dx;
      int ne;                        /* integer scaled error term      */
      x = xl; y = yl;                /* start at left endpoint          */
      ie = 2 * dy - dx;              /* initialize the error term */
       while (x <= xr){              /* pixel-drawing loop */
          PlotPixel (x,y);           /* draw the pixel   */
          if (ie > 0) {
             y = y + 1;
             ne = ne - 2 * dx;       /* replaces e = e - 1  */
          }
          x = x + 1;
         ne = ne + 2 * dy;           /* replaces e = e + m  */
       }
    }
```

Line Drawing Algorithms

Bresenham's algorithm also applies for circles.

Bresenham, J.E. ***A linear algorithm for incremental digital display of circular arcs*** **Communications of the ACM, Vol. 20, pp. 100-106, 1977.**

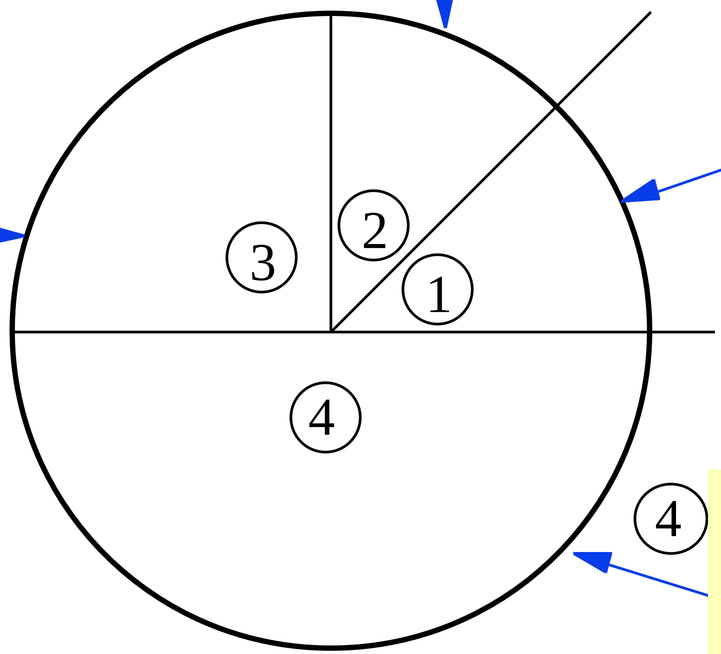
Key idea: compute the initial octant only

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

② Reflect first octant about $y=x$

③ Reflect first quadrant about $x=0$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$



① Generate first octant

④ Reflect upper semicircle about $y=0$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Line Drawing Algorithms

Bresenham's incremental circle algorithm.

Example:

circle of radius 8

Bright pixels:

initial pixel → (0,8)
(1,8)
(2,8)
(3,7)
(4,7)
(5,6)
(6,5)
(7,4)
(7,3)
(8,2)
(8,1)
end pixel → (8,0)