On Rayleigh-Taylor instability and mixing*

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Abstract: We develop a new mathematical model for two-fluid interface motion, subjected to the Rayleigh-Taylor (RT) instability.

The problem: The Rayleigh-Taylor (RT) instability of an interface between two fluids of different densities occurs when the high-density fluid is placed over a lower-density fluid under gravity.

RT instability leads to a mixing zone that, after some transient time, satisfies

width of the RT mixing region $\approx 2\delta A g t^2$,

where $A = \frac{\rho^+ - \rho^-}{\rho^+ + \rho^-}$ is the Atwood number, $\rho^\pm$ denote the densities of the fluids and g denotes the gravitational acceleration.

> δ is dimensionless but **non-universal**. The question is: **How to estimate δ?**

The model: The basic mathematical PDE system for RT instability and mixing between two fluids is the Euler equations for irrotational flow.

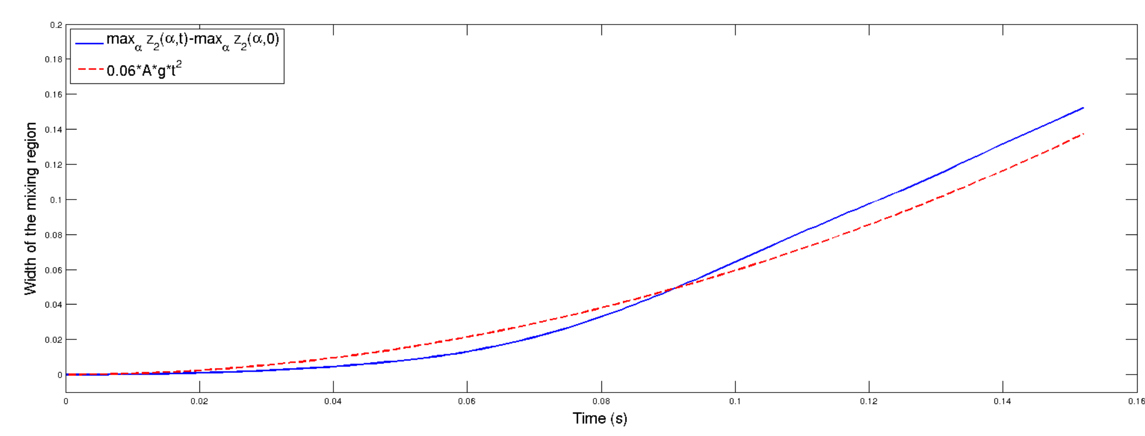
By *restricting the nonlocality of the Euler equations*, we obtain the $z-$**model** [1]

$$z_{tt} = \Lambda\left[\frac{A}{|\partial_\alpha z|^2} H(\gamma H \gamma) + A g z_2\right]\frac{\partial_\alpha^\perp z}{|\partial_\alpha z|^2} + \gamma\left(\frac{\partial_\alpha^\perp z_t}{|\partial_\alpha z|^2} - \frac{\partial_\alpha^\perp z 2(\partial_\alpha z \cdot \partial_\alpha z_t)}{|\partial_\alpha z|^4}\right),$$

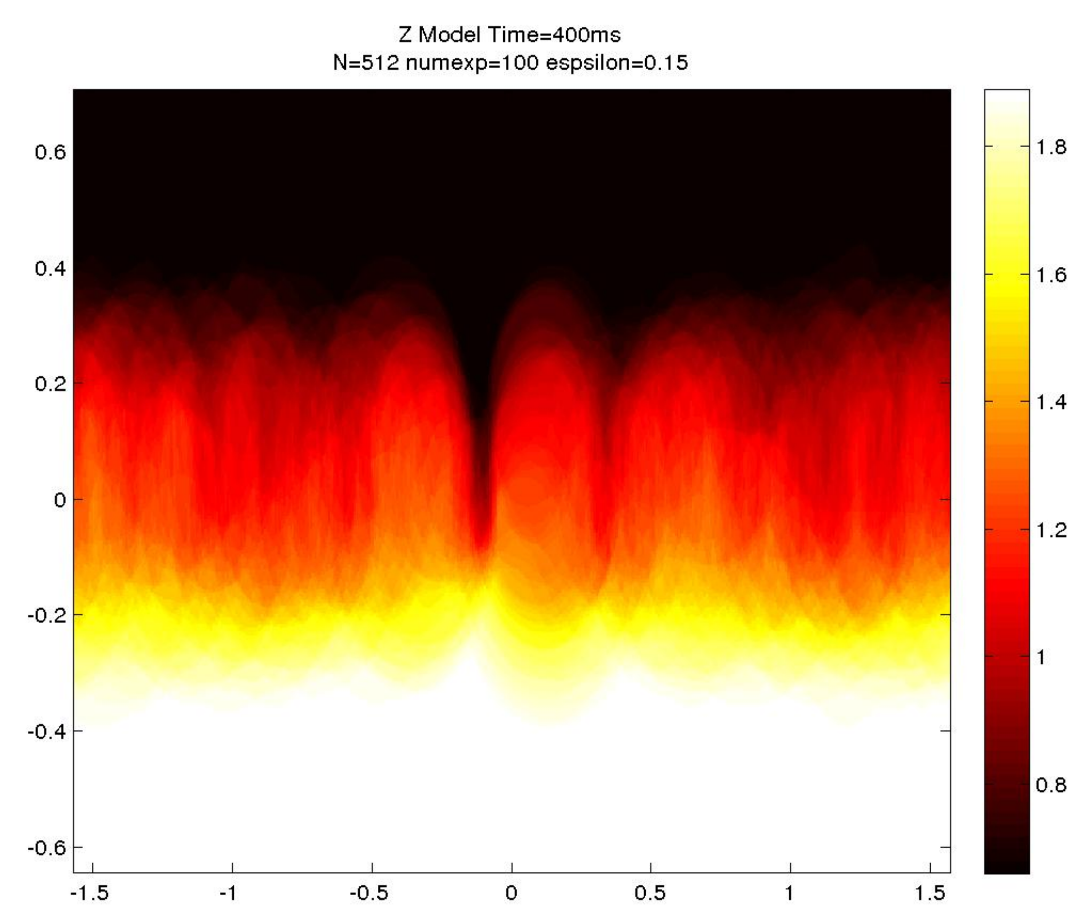
$$\gamma = z_t \cdot \partial_\alpha^\perp z,$$

where $\Lambda = H\partial_\alpha$ and H denotes the Hilbert transform.

The *"rocket rig"* experiment: The *"rocket rig"* experiment of Read and Youngs considers the situation where we have a NaI solution ($\rho^- = 1.89 g/cm^3$) and Hexane ($\rho^+ = 0.66 g/cm^3$) are subjected to a constant acceleration g (acting upwards). The initial interface is given by a *small and random* perturbation of the flat state.



Comparison between the $z-$model (blue) and the empirical value (red) for the width of the mixing region [1]



Mixing region predicted by the $z-$model [2]

References

[1] R. Granero-Belinchón, S. Shkoller, *A model for Rayleigh-Taylor mixing and interface turnover.* To appear in Multiscale Modeling and Simulation, 2017.

[2] R. Granero-Belinchón, S. Shkoller, B. Zhou, *In preparation*

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[1] Assuming that the interface remains a *nearly flat* graph $(\alpha, h(\alpha,t))$, we also obtain a simpler model called the $h-$**model**. For the $h-$**model** we develop a well-posedness theory in [1]