Trends and perspectives in the analysis of PDEs

Roma (Italy) – From 16 to 20 September 2024

Book of abstracts

- **Massimiliano Berti**
  Stable and unstable Stokes waves

  Abstract:
  In the last years much progress has been achieved concerning the problem of determining the stability/instability of Stokes waves, i.e. periodic traveling solutions of the pure gravity water waves equations in an ocean of depth $h > 0$, subject to longitudinal perturbations. In this talk we present some of these results focusing the attention on the existence of unstable spectral bands away from zero for Stokes waves of small amplitude $\epsilon > 0$. We prove that the unstable spectrum is the union of "isolas" of approximately elliptical shape, parameterized by integers $p \geq 2$, with semiaxis of size $|\beta_1^{(p)}(h)|\epsilon^p + O(\epsilon^{p+1})$ where $\beta_1^{(p)}(h)$ is a nonzero analytic function of the depth h that depends on the Taylor coefficients of the Stokes waves up to order p.

- **Roberta Bianchini**
  Hydrostatic modeling in stratified fluids: mathematical perspectives

  Abstract:
  This talk concerns recent mathematical results about the analysis of hydrostatic equations in the context of stably stratified fluids. Beginning with the simpler and better-understood setting of homogeneous fluids, we emphasize the additional mathematical challenges posed by the non-homogeneous framework. We present both positive and negative results, including well-posedness and proof of the hydrostatic limit with suitable regularization, alongside ill-posedness in the fully inviscid setting and the breakdown of the hydrostatic limit in specific scenarios.

- **Jean-Yves Chemin**
  About the asymptotic of fast rotating fluids in a non flat domain

  Abstract:
  We investigate the asymptotic behaviour of fast rotating incompressible fluids with vanishing viscosity, in a three dimensional domain which modelize a ocean with topography including the case of land area. Assuming the initial data is well-prepared, we prove a theorem of strong convergence to the velocity field to a two-dimensional vector field solving a linear, damped ordinary differential equation. The proof is based on a weak-strong uniqueness argument, together with an abstract result implying that the weak convergence of a family of weak solutions to the Navier-Stokes-Coriolis system can be translated into a form of uniform-in-time convergence.

- **Diego Córdoba**
  Finite time blow-up for the hypodissipative Navier Stokes equations

  Abstract:
  In this talk we establish the formation of singularities of classical solutions with finite energy of the forced fractional Navier Stokes equations where the dissipative term is given by $|\nabla|^\alpha$ for any $\alpha \in [0, \alpha_0)$ ($\alpha_0 = 0.09\cdots$).

- **Michele Coti Zelati**
  Stability and entropy maximization in the two-dimensional Euler equations

  Abstract:
  We investigate certain questions arising in two-dimensional statistical hydrodynamics, by relying on principles of entropy maximization for the vorticity of a two-dimensional perfect fluid in a disc. In analogy with the entropy functions used in statistical mechanics and thermodynamics, we show that similar concavity properties hold for the 2d Euler equations when maximizing entropies at fixed energy levels. The proofs rely on rearrangement inequalities, a modification of the classical min-max principle, and the properties of

the Euler-Lagrange equations for the corresponding constrained optimization. As a byproduct, we obtain Lyapunov stability for Onsager solutions arising from a system of point-vortices.

- **Gianluca Crippa**
  Lack of selection for passive-scalar advection and for the forced Euler equations

  Abstract:
  Consider the advection of a passive-scalar under a velocity field in the Hölder class C^α. It is known that, in general, for $\alpha < 1$ weak solutions may be nonunique. I will show that in the same setting two canonical regularization mechanisms (regularization of the velocity field, and limit of vanishing diffusivity) are also ineffective to select a unique solution in the limit. Moreover, I will show that the same holds for the forced Euler equations in three spatial dimensions. I will also describe the relation to the question of anomalous dissipation for both equations. These results are contained in joint paper with E. Bruè, M. Colombo, C. De Lellis, and M. Sorella.

- **Raphaël Danchin**
  On the large time asymptotics of global solutions to the Vlasov-Navier-Stokes equations in the whole space

  Abstract:
  The incompressible Vlasov-Navier-Stokes equations are a toy model for describing the dynamics of a cloud of particles that are immersed in an incompressible viscous fluid. Here we are concerned with the behavior of global strong solutions when the time goes to infinity, in the case where the equations are posed in the whole space. For small enough initial data with sufficient integrability at infinity, it is shown that the velocity decays to zero, with the same rate as the classical Navier-Stokes equations, and that the kinetic distribution of the particles is well approximated by a monokinetic distribution with velocity which is the same as that of the viscous fluid. We first establish this result in the "smooth" case of H^1 velocity fields, then show that it remains true for critical regularity $H^{1/2}$.
  Our results rely on the use of a higher order energy functional that controls the regularity H^1 of the velocity and seems to have been first introduced by Li, Shou and Zhang in the context of the nonhomogeneous Vlasov-Navier-Stokes system, and on the propagation of suitable negative regularity.

- **Francesco De Anna**
  Prandtl-type equations in boundary layers of the Beris-Edwards theory of liquid crystals

  Abstract:
  The Prandtl equations are fundamental in describing the dynamics of boundary layers in Newtonian fluids. Over the past fifty years, research has shown that, without structural assumptions, the well-posedness of these equations is influenced by turbulent and multiscale dynamics, potentially leading to ill-posedness in Sobolev spaces. One might expect that these instabilities could be exacerbated in non-Newtonian fluids due to the complexities introduced by multi-structure interactions between the fluid and additional state variables. However, this is not always the case; the added structure can provide a stabilizing effect on the boundary layer over short periods.
  This talk first reviews collaborative research with S. Scrobogna (University of Trieste) and J. Kortum (University of Würzburg) on the well-posedness and ill-posedness in Gevrey classes for simplified models of viscoelastic fluids. Secondly, it presents some recent findings in collaboration with J. Kortum and A. Zarnescu (BCAM) regarding boundary layers in the hydrodynamics of liquid crystals using the Q-tensor theory. Specifically, we introduce a novel decomposition of the order tensor for the non-Newtonian component of the Beris-Edwards equations. At high Reynolds numbers, this decomposition preserves the traceless property and generates equations that asymptotically retain the principal terms of the original model, ensuring analytical tractability.

- **Eduard Feireisl**
  Incompressible limits of thermally driven compressible fluid flows under stratification

  Abstract:
  We consider the motion of a compressible, viscous, and heat conducting fluid in the low Mach number regime. The motion of the fluid is driven by the interaction of a strong gravitational force and the temperature gradient imposed through the Dirichlet boundary conditions. The case of mild and strong stratification are considered yielding different systems of limit equations. In both cases the conclusion is rather unexpected in view of the existing results for problems with other boundary conditions.

- **Isabelle Gallagher**
  On the Cauchy problem for a class of parabolic quasilinear systems

  Abstract:
  We study a class of parabolic quasilinear systems, in which the diffusion matrix is not uniformly elliptic, but satisfies a Petrovskii condition of positivity of the real part of the eigenvalues. Local wellposedness is known since the work of Amann in the 90s, by a semi-group method. We shall revisit these results in the context of Sobolev spaces modelled on L^2: in particular, we will see that if the Petrovskii condition is known not to be sufficient to ensure exponential decay in time for systems of ordinary differential equations, the quasilinear structure here nevertheless ensures the well-posedness of the system. This is a collaboration with Ayman Moussa.

- **Francisco Gancedo**
  On evolution of vortex filaments

  Abstract:
  In this talk we show two new results of vortex filament evolution for incompressible Navier-Stokes and Euler equations. For Navier-Stokes, we prove global-in-time regularity for initial helical vortex filament. For Euler, we give existence of weak dissipative solutions with initial vorticity concentrated in a circle.

- **Alexander Kiselev**
  Suppression of chemotactic blow up by active scalar

  Abstract:
  There exist many regularization mechanisms in nonlinear PDE that help make solutions more regular or prevent formation of singularity: diffusion, dispersion, damping. A relatively less understood regularization mechanism is transport. There is evidence that in the fundamental PDE of fluid mechanics such as Euler or Navier-Stokes, transport can play a regularizing role. In this talk, I will discuss another instance where this phenomenon appears: the Patlak-Keler-Segel equation of chemotaxis. Chemotactic blow up in the context of the Patlak-Keller-Segel equation is an extensively studied phenomenon. In recent years, it has been shown that the presence of a given fluid advection can arrest singularity formation given that the fluid flow possesses mixing or diffusion enhancing properties and its amplitude is sufficiently strong. This talk will focus on the case when the fluid advection is active: the Patlak-Keller-Segel equation coupled with fluid that obeys Darcy's law for incompressible porous media flow via gravity. Surprisingly, in this context, in contrast with the passive advection, active fluid is capable of suppressing chemotactic blow up at arbitrary small coupling strength: namely, the system always has globally regular solutions. The talk is based on work joint with Zhongtian Hu and Yao Yao.

- **Xian Liao**
  Global-in-time regularity for the two-dimensional incompressible Navier-Stokes equations with freely transported viscosity coefficient

  Abstract:
  In this talk we consider the two-dimensional incompressible Navier-Stokes equations, where the viscosity coefficient may exhibit a big jump across a regular interface that is freely transported by the fluidâs velocity field. We investigate the interaction between growth due to transport and decay due to dissipation effects, by making use of two "good" unknowns. The first good unknown is related to the vorticity in a non-local manner and satisfies H^1-energy estimates, while the second good unknown is related to the velocity gradient in a local manner and remains continuous globally in space. Under the smallness assumption on the initial velocity field, we achieve global-in-time regularity through a bootstrap argument. This is a joint work with Rebekka Zimmermann (KIT).

- **Alberto Maspero**
  One dimensional energy cascades in a fractional quasilinear NLS

  Abstract:
  We consider the problem of transfer of energy to high frequencies in a quasilinear Schrödinger equation with sublinear dispersion, on the one-dimensional torus. We exhibit initial data undergoing finite but arbitrary large Sobolev norm explosion: their initial norm is arbitrary small in Sobolev spaces of high regularity, but at a later time becomes arbitrary large.

We develop a novel mechanism producing instability, which is based on extracting, via paradifferential normal forms, an effective equation driving the dynamics whose leading term is a non-trivial transport operator with non-constant coefficients. We prove that such operator is responsible for energy cascades via a positive commutator method, inspired by Mourre's commutator theory.
This is a joint work with Federico Murgante.

- **Riccardo Montalto**
  Nonlinear quasi-periodic oscillations in Fluid Mechanics

  Abstract:
  In this talk I shall discuss some recent results about the construction of quasi-periodic waves in Euler equations and other hydro-dynamical models in dimension greater or equal than two. I shall discuss quasi-peridic solutions and vanishing viscosity limit for forced Euler and Navier-Stokes equations and the problem of constructing quasi-periodic traveling waves bifurcating from Couette flow (and connections with inviscid damping). Time permitting, I also discuss some results concerning the construction of large amplitude quasi-periodic waves in MHD system and rotating fluids. The techniques are of several kinds: Nash-Moser iterations, micro-local analysis, analysis of resonances in higher dimension, normal form constructions and spectral theory.

- **Šárka Nečasová**
  On existence of weak solutions to a Baer–Nunziato type system

  Abstract:
  In this lecture, a dissipative version of a compressible one velocity Baer–Nunziato type system for a mixture of two compressible heat conducting gases is considered. The complete existence proof for weak solutions to this system was addressed as an open problem in Kwon, Y.-S., Novoty, A. and Arthur Cheng, C.H, 2020. We showed the global in time existence of weak solutions to the one velocity Baer–Nunziato type system for arbitrary large initial data. It is done in collaboration with M. Kalousek.

- **Michela Procesi**
  KAM theory in infinite dimensions

  Abstract:
  I shall discuss the existence of infnite-dimensional invariant tori in a mechanical system made of infinitely many rotators weakly interacting with each other. I shall concentrate on interactions depending only on the angles, with the aim of discussing in a simple case the analyticity properties to be required on the perturbation of the integrable system in order to ensure the persistence of a large measure set of invariant tori with finite energy.

- **Alexis Vasseur**
  From Navier-Stokes to discontinuous solutions of compressible Euler

  Abstract:
  The compressible Euler equation can lead to the emergence of shock discontinuities in finite time, notably observed behind supersonic planes. A very natural way to justify these singularities involves studying solutions as inviscid limits of Navier-Stokes solutions with evanescent viscosities.The mathematical study of this problem is however very difficult because of the destabilization effect of the viscosities.
  Bianchini and Bressan proved the inviscid limit to small BV solutions using the so-called artificial viscosities in 2004. However, until very recently, achieving this limit with physical viscosities remained an open question. In this presentation, we will provide the basic ideas of classical mathematical theories to compressible fluid mechanics and introduce the recent a-contraction with shifts method. This method is employed to describe the physical inviscid limit in the context of the barotropic Euler equation. This is a joint work with Geng Chen and Moon-Jin Kang.

- **Arghir Zarnescu**
  On a variational approach to the study of incompressible fluids

  Abstract:
  We consider a construction proposed by A. Acharya in QAM 2023, LXXXI(1) that builds on the notion of weak solutions for incompressible fluids to provide a scheme that generates variationally a certain type of

dual solutions. If these dual solutions are regular enough one can use them to recover standard solutions. The scheme provides a generalisation of a construction of Y. Brenier for Euler. We rigorously analyze the scheme, extending the work of Brenier for Euler, and also providing an extension of it to the case of Navier-Stokes equations. This is joint work with A. Acharya and B. Stroffolini.

- **Ewelina Zatorska**
  On the dissipative Aw-Rascle model

  Abstract:
  I will talk about a multi-dimensional generalization of the Aw-Rascle model of vehicular traffic. It is a system of conservation laws for the mass and linear momentum associated with the desired velocity of motion. It differs from the actual velocity of motion by the so called offset, or cost function, which in our case is a gradient of some function of the density. After explaining the connection between this model and the compressible (pressureless) Euler, Euler-Alignment and Navier-Stokes equations, I will discuss the existence and singular limits results that we obtained so far.