

Algorithms for the computation of classical Gaussian quadratures and associated barycentric interpolation

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We present MATLAB algorithms for computing the three classical Gaussian rules based on the asymptotic methods described in [1] (Gauss-Hermite and Gauss-Laguerre), as well as in the globally convergent iterative methods of [3] (Gauss-Hermite and Gauss-Laguerre) and [4] (Gauss-Jacobi). The Gauss-Radau and Gauss-Lobatto variants are also considered, alongside the computation of barycentric weights for Lagrange interpolation. The asymptotic and iterative algorithms offer distinct advantages: asymptotic methods are highly precise for large degrees, while iterative methods are generally faster and valid for a broader range of parameters. The combination of both methods provides the fastest and most accurate double precision methods to date, with an extended range of validity compared to previous methods. We also discuss how the iterative methods can be used for arbitrary precision computations, and this is illustrated with some Maple implementations of the algorithms.

Work in collaboration with **Nico M. Temme**.

References

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