Automated steepest descent contour deformation for oscillatory integrals with multiple coalescing saddles

Andrew Gibbs[(a)]

[(a)]Department of Mathematics, University College London, (UK)

e-mail `andrew.gibbs@ucl.ac.uk`

Oscillatory integrals arise in a broad range of models for wave-based phenomena. The method of Numerical Steepest Descent (NSD) combines complex contour deformation with numerical quadrature to provide an efficient and accurate approach for evaluating such integrals.

Unless the phase function governing the oscillation is particularly simple, the application of Numerical Steepest Descent requires a significant amount of a priori analysis and expert user input. In particular, naive implementations of NSD are known to break down in the presence of coalescing saddle points [3], which occur in various applications — for example, approximation of special functions, such as the Airy Functions [1, §9] and the Catastrophe Integrals [1, §36.2].

PathFinder is an algorithm presented in [2], implemented in Matlab/Octave on top of C code, which automatically deforms the integration range/contour and applies a modified version of NSD to evaluate integrals of the form

$$I = \int_a^b f(x) \exp(\mathrm{i}\omega g(x))\ \mathrm{d}x,$$

where the endpoints a and b can be complex-valued, even infinite; $\omega > 0$ is the frequency parameter, f is an entire function and g is a polynomial.

PathFinder remains accurate with a bounded computational cost for all $\omega > 0$ and is robust in the presence of coalescing saddle points. It can be easily used by non-experts, without understanding the underlying complex analysis.

In this talk, I will begin by introducing NSD, explaining when it works well and when it breaks down. I will then explain why the PathFinder algorithm works when NSD fails, and I will provide a live demonstration with applications to special functions.

Work in collaboration with **Daan Huybrechs** and **David P. Hewett**.

References

[1] *NIST Digital Library of Mathematical Functions*, 2023. http://dlmf.nist.gov/, release 1.1.12 of 2023-12-15.

[2] A. Gibbs, D. Hewett, and D. Huybrechs, *Numerical evaluation of oscillatory integrals via automated steepest descent contour deformation*, Journal of Computational Physics, 501 (2024).

[3] D. Huybrechs, A. K., and N. Lejon, *A numerical method for oscillatory integrals with coalescing saddle points*, SIAM J. Numer. Anal., 57 (2019), pp. 2707–2729.