Asymptotic power of goodness of fit tests based on Wasserstein distance

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We present a preliminary study for the power of Wasserstein goodness of fit test. Under H_0, $X_1, \ldots, X_n$ are i.i.d. with distribution function F, density function f and quantile function F^{-1}. The Wasserstein test is based on the statistic:

$$n \int_0^1 \left(F_n^{-1}(t) - F^{-1}(t)\right)^2 dt - a_n,$$

where F_n^{-1} is the empirical quantile function and a_n are some coefficients which depend on n and on F. Under tail conditions, this statistic converges to

$$\int_0^1 \frac{B^2(t) - t(1-t)}{f^2(F^{-1}(t))} dt, \tag{1}$$

where B is a Brownian bridge (see, e.g., del Barrio et al. [1]). Let W be the Brownian motion such that $B(t) = W(t) - tW(1)$. We prove that almost surely, (1) is equal to

$$\int \int \tilde{K}(s,t) dW(s) dW(t),$$

where $\tilde{K}$ is a kernel associated to F. The multiple integral must be understood as in Nualart [2].

Similarly to this expression, we derive a new statistic based on the kernel $\tilde{K}$, which has interesting regularity properties. This statistic can be completely treated under some contiguous alternatives within the frame of Gaussian shifts (see, e.g., Janssen [3]).

References

[1] del Barrio, E., Cuesta-Albertos, J.A., Matrn, C. and Rodrguez-Rodrguez, J.M. (1999). Tests of goodness of fit based on the L_2-Wasserstein distance. *The Annals of Statistics*, **27**(4): 1230–1239

[2] Nualart, D. (1995). The Malliavin calculus and related topics. Probability and its Applications, Springer-Verlag.

[3] Janssen, A. (2003). Which power of goodness of fit tests can really be expected: intermediate versus contiguous alternatives. *Statist. Decisions*, **21**(4): 301–325.