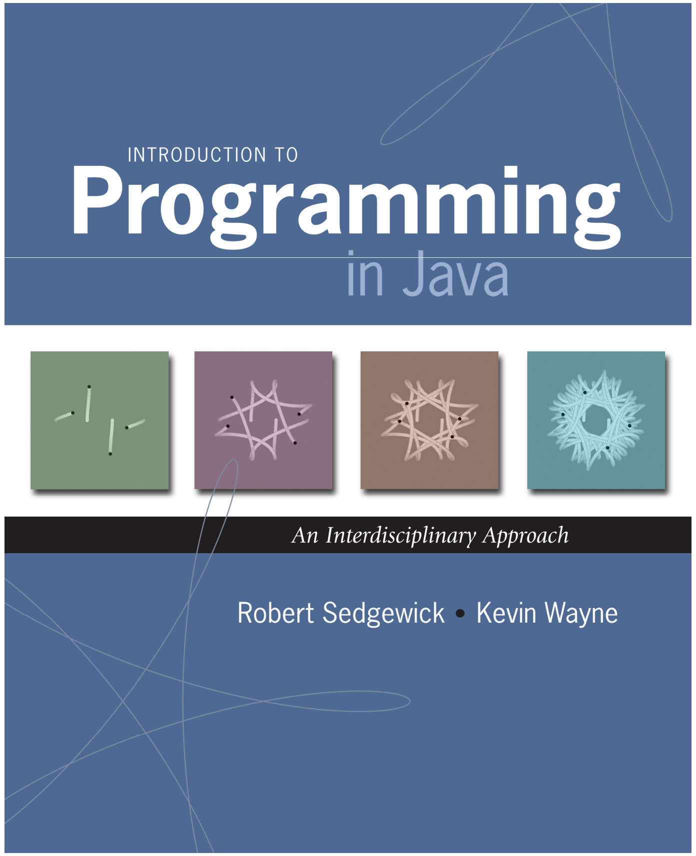
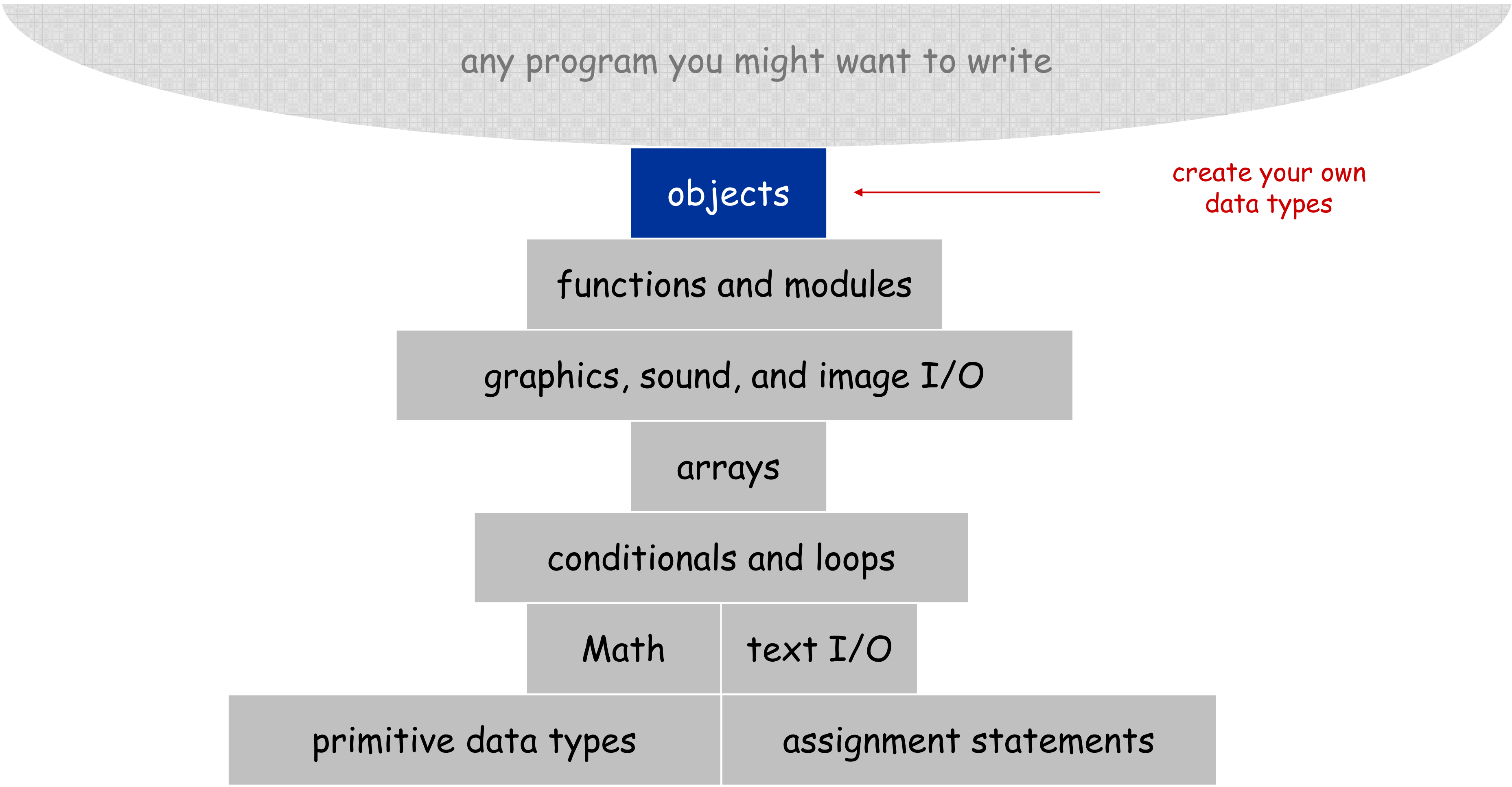
Classes and Objects



A Foundation for Programming



Data Types

Data type. Set of values and operations on those values.

Primitive types. Values directly map to machine representation; ops directly map to machine instructions.

Data Type	Set of Values	Operations
`boolean`	**`true`**, **`false`**	not, and, or, xor
`int`	-2^{31} to $2^{31} - 1$	add, subtract, multiply
`double`	any of 2^{64} possible reals	add, subtract, multiply

We want to write programs that process other types of data.

- Colors, pictures, strings, input streams, …
- Complex numbers, vectors, matrices, polynomials, …
- Points, polygons, charged particles, celestial bodies, …

Objects

Object. Holds a data type value; variable name refers to object.

Object-oriented programming.

- Create your own data types (set of values and ops on them).
- Use them in your programs (manipulate objects that hold values).

Data Type	Set of Values	Operations
`Color`	24 bits	get red component, brighten
`Picture`	2D array of colors	get/set color of pixel (i, j)
`String`	sequence of characters	length, substring, compare

This lecture. Use existing data types.

Next lecture. Create your own data types.

Constructors and Methods

To construct a new object:

- Use keyword **new** (to invoke constructor).
- Use name of data type (to specify which type of object).

To apply an operation:

- Use name of object (to specify which object).
- Use the dot operator (to invoke method).
- Use the name of the method (to specify which operation).

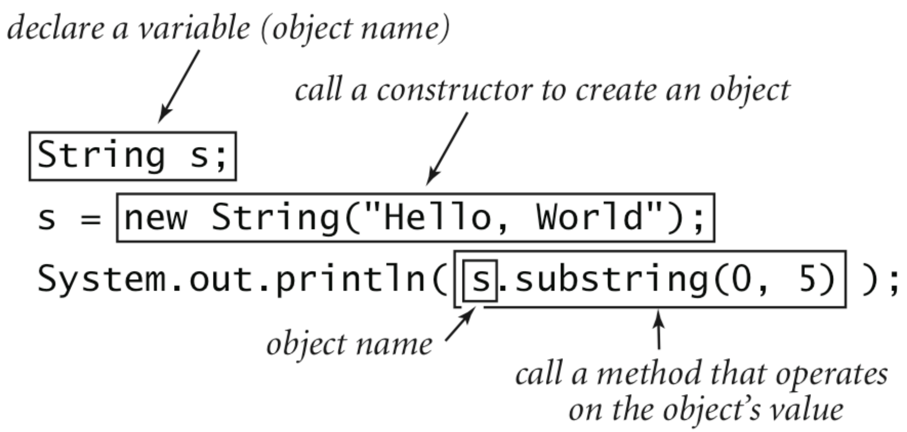


Image Processing

Color Data Type

Color. A sensation in the eye from electromagnetic radiation.

Set of values. [RGB representation] 256^3 possible values, which quantify the amount of red, green, and blue, each on a scale of 0 to 255.

R	G	B	Color
255	0	0	
0	255	0	
0	0	255	
255	255	255	
0	0	0	
255	0	255	
105	105	105	

Color Data Type

Color. A sensation in the eye from electromagnetic radiation.

Set of values. [RGB representation] 256^3 possible values, which quantify the amount of red, green, and blue, each on a scale of 0 to 255.

API. Application Programming Interface.

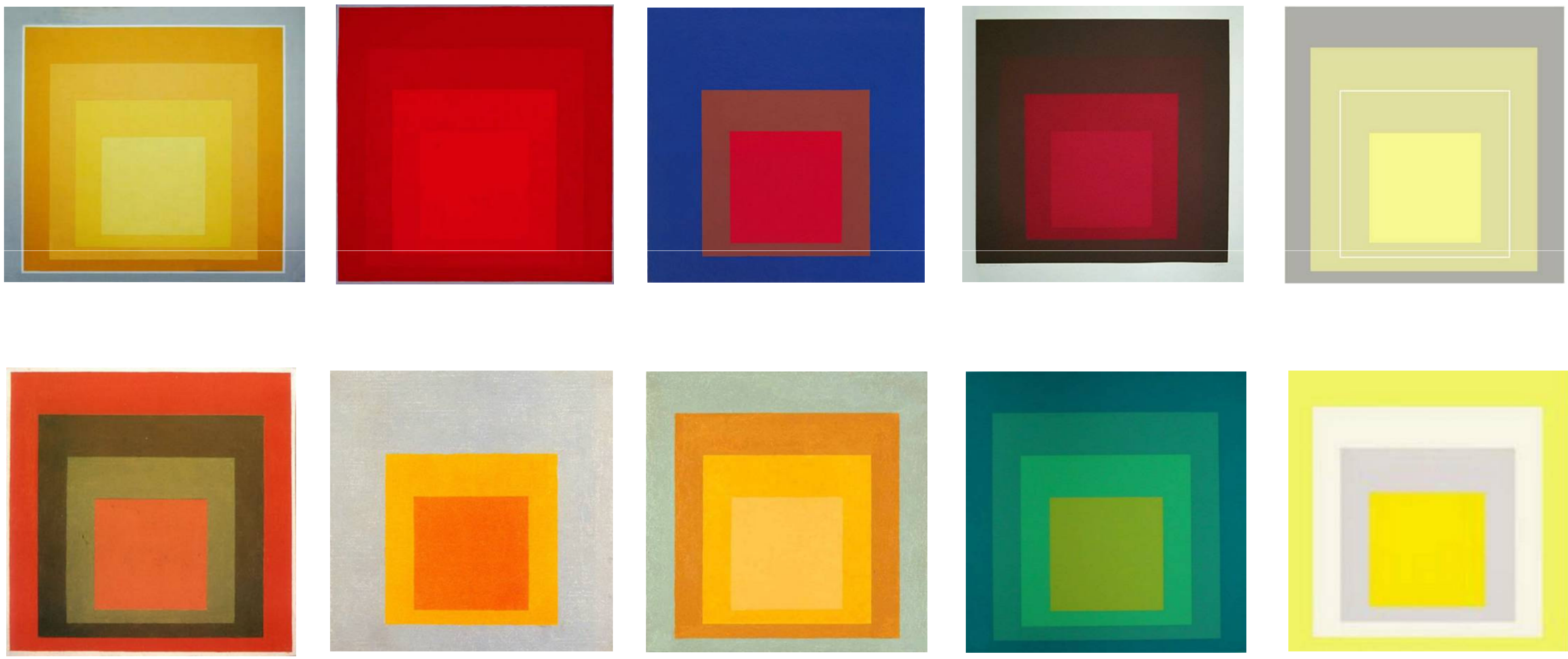
`public class java.awt.Color`

	`Color(int r, int g, int b)`	
`int`	`getRed()`	*red intensity*
`int`	`getGreen()`	*green intensity*
`int`	`getBlue()`	*blue intensity*
`Color`	`brighter()`	*brighter version of this color*
`Color`	`darker()`	*darker version of this color*
`String`	`toString()`	*string representation of this color*
`boolean`	`equals(Color c)`	*is this color's value the same as* `c`*'s?*

http://download.oracle.com/javase/6/docs/api/java/awt/Color.html

Albers Squares

Josef Albers. Revolutionized the way people think about color.

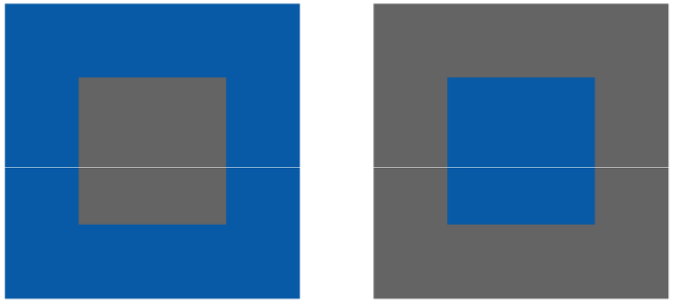


Homage to the Square by Josef Albers (1949-1975)

Albers Squares

Josef Albers. Revolutionized the way people think about color.


```
% java AlbersSquares 9 90 166  100 100 100
```



Using Colors in Java

```
import java.awt.Color;                                    to access Color library

public class AlbersSquares {
   public static void main(String[] args) {
      int r1 = Integer.parseInt(args[0]);
      int g1 = Integer.parseInt(args[1]);                  first color
      int b1 = Integer.parseInt(args[2]);
      Color c1 = new Color(r1, g1, b1);

      int r2 = Integer.parseInt(args[3]);
      int g2 = Integer.parseInt(args[4]);                  second color
      int b2 = Integer.parseInt(args[5]);
      Color c2 = new Color(r2, g2, b2);

      StdDraw.setPenColor(c1);
      StdDraw.filledSquare(.25, .5, .2);                   first square
      StdDraw.setPenColor(c2);
      StdDraw.filledSquare(.25, .5, .1);

      StdDraw.setPenColor(c2);
      StdDraw.filledSquare(.75, .5, .2);                   second square
      StdDraw.setPenColor(c1);
      StdDraw.filledSquare(.75, .5, .1);
   }
}
```

Monochrome Luminance

Monochrome luminance. Effective brightness of a color.

NTSC formula. $Y = 0.299r + 0.587g + 0.114b.$

```
import java.awt.Color;

public class Luminance {

   public static double lum(Color c) {
      int r = c.getRed();
      int g = c.getGreen();
      int b = c.getBlue();
      return .299*r + .587*g + .114*b;
   }

}
```

Color Compatibility

Q. Which font colors will be most readable with which background colors on computer and cell phone screens?

A. Rule of thumb: difference in luminance should be ≥ 128.

255	208	105	47	28	14

```
public static boolean compatible(Color a, Color b) {
   return Math.abs(lum(a) - lum(b)) >= 128.0;
}
```

Grayscale

Grayscale. When all three R, G, and B values are the same, resulting color is on grayscale from 0 (black) to 255 (white).

Convert to grayscale. Use luminance to determine value.

```
public static Color toGray(Color c) {
   int y = (int) Math.round(lum(c));
   Color gray = new Color(y, y, y);
   return gray;
}
```

round `double` to nearest `long`

red	*green*	*blue*	
9	90	166	*this color*
74	74	74	*grayscale version*
0	0	0	*black*

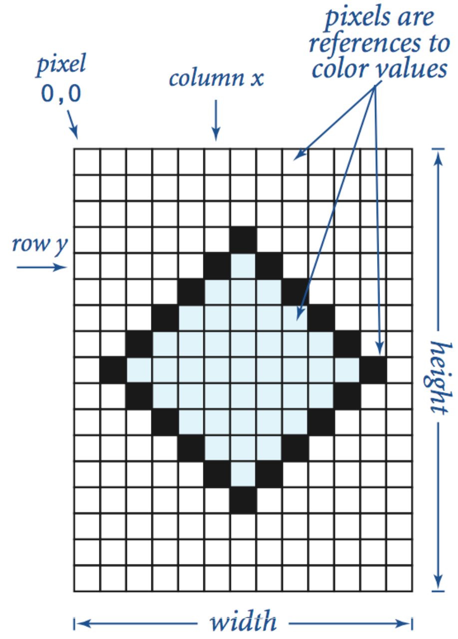
$$0.299 * 9 + 0.587 * 90 + 0.114 * 166 = 74.445$$

Bottom line. We are writing programs that manipulate color.

Picture Data Type

Raster graphics. Basis for image processing.

Set of values. 2D array of `Color` objects (pixels).



API.

public class Picture		
	`Picture(String filename)`	*create a picture from a file*
	`Picture(int w, int h)`	*create a blank* `w`*-by-*`h` *picture*
`int`	`width()`	*return the width of the picture*
`int`	`height()`	*return the height of the picture*
`Color`	`get(int x, int y)`	*return the color of pixel (x, y)*
`void`	`set(int x, int y, Color c)`	*set the color of pixel (x, y) to* `c`
`void`	`show()`	*display the image in a window*
`void`	`save(String filename)`	*save the image to a file*

Image Processing: Grayscale Filter

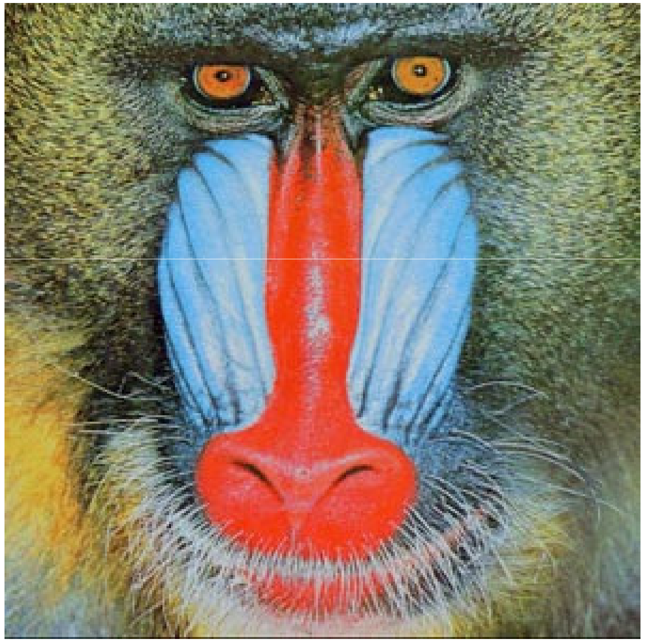
Goal. Convert color image to grayscale according to luminance formula.

```
import java.awt.Color;

public class Grayscale {
   public static void main(String[] args) {
      Picture pic = new Picture(args[0]);
      for (int x = 0; x < pic.width(); x++) {
         for (int y = 0; y < pic.height(); y++) {
            Color color = pic.get(x, y);
            Color gray  = Luminance.toGray(color);
            pic.set(x, y, gray);
         }
      }
      pic.show();
   }
}
```

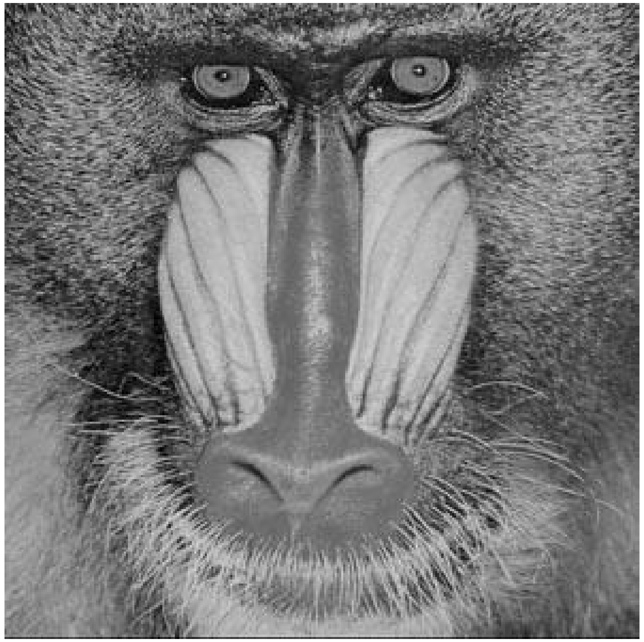
← from before

Image Processing: Grayscale Filter

Goal. Convert color image to grayscale according to luminance formula.



`mandrill.jpg`



`% java Grayscale mandrill.jpg`

Image Processing: Scaling Filter

Goal. Shrink or enlarge an image to desired size.

Downscaling. To shrink, delete half the rows and columns.
Upscaling. To enlarge, replace each pixel by 4 copies.

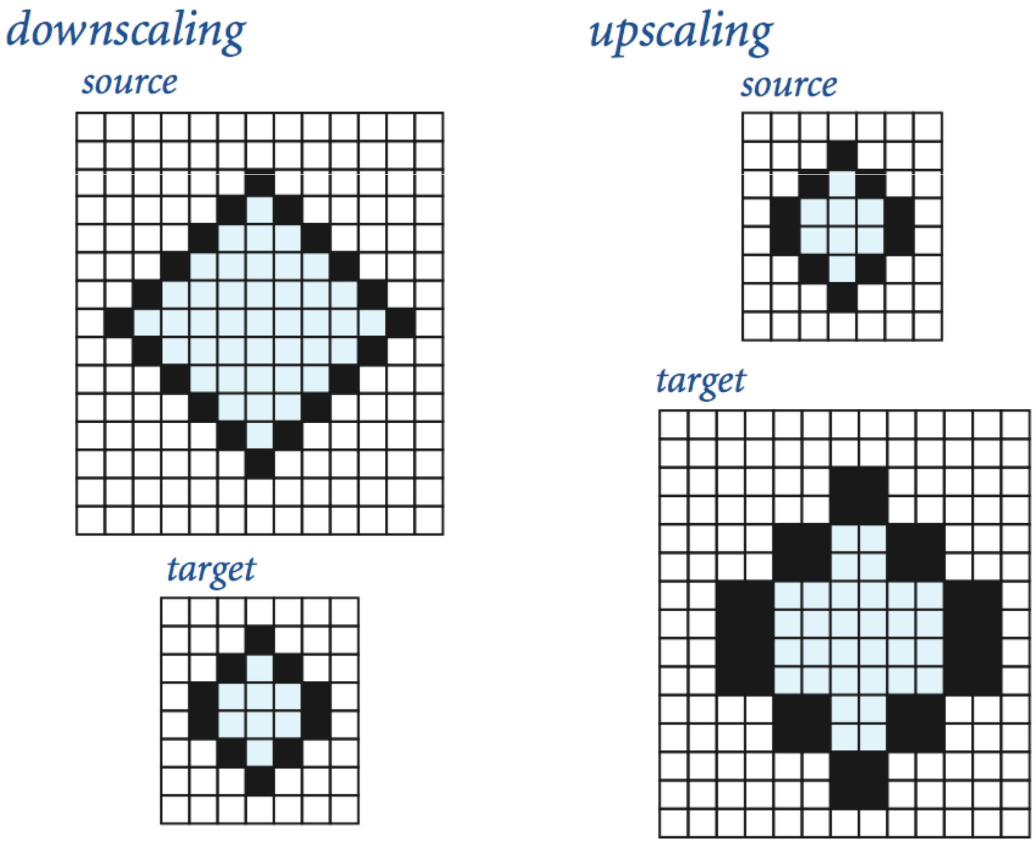


Image Processing: Scaling Filter

Goal. Shrink or enlarge an image to desired size.

Uniform strategy. To convert from w_s-by-h_s to w_t-by-h_t:

- Scale column index by w_s / w_t.
- Scale row index by h_s / h_t.
- Set color of pixel (x, y) in target image to color of pixel $(x \times w_s / w_t,\ y \times h_s / h_t)$ in source image.

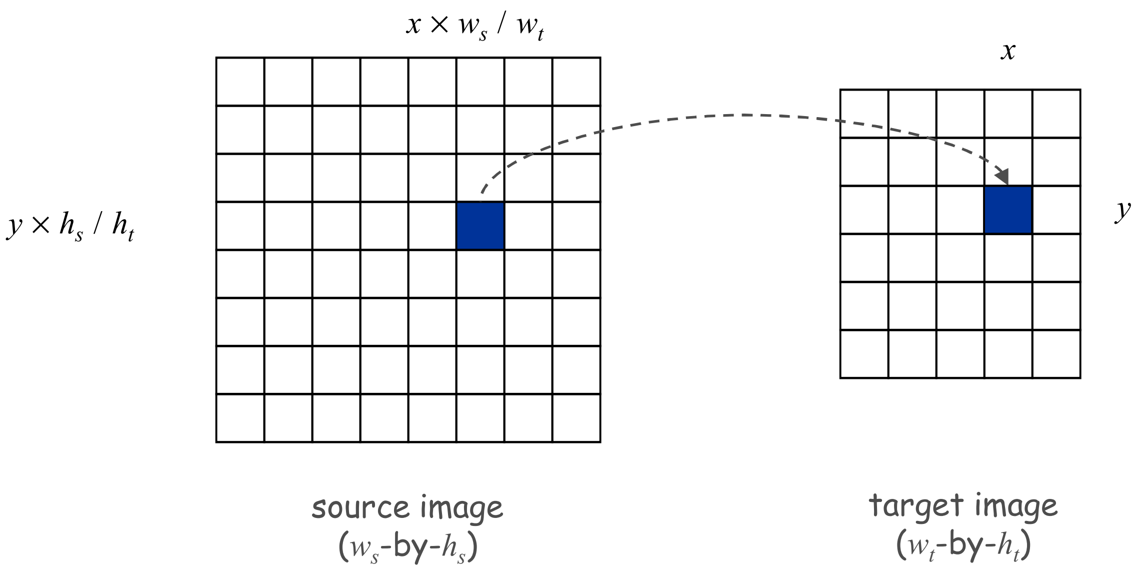
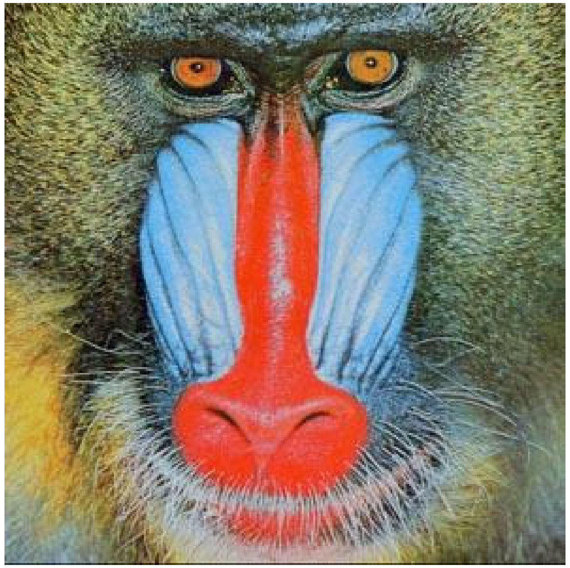


Image Processing: Scaling Filter

```
import java.awt.Color;

public class Scale {
   public static void main(String[] args) {
      String filename = args[0];
      int w = Integer.parseInt(args[1]);
      int h = Integer.parseInt(args[2]);
      Picture source = new Picture(filename);
      Picture target = new Picture(w, h);
      for (int tx = 0; tx < target.width(); tx++) {
         for (int ty = 0; ty < target.height(); ty++) {
            int sx = tx * source.width()  / target.width();
            int sy = ty * source.height() / target.height();
            Color color = source.get(sx, sy);
            target.set(tx, ty, color);
         }
      }
      source.show();
      target.show();
   }
}
```

Image Processing: Scaling Filter

Scaling filter. Creates two `Picture` objects and two windows.

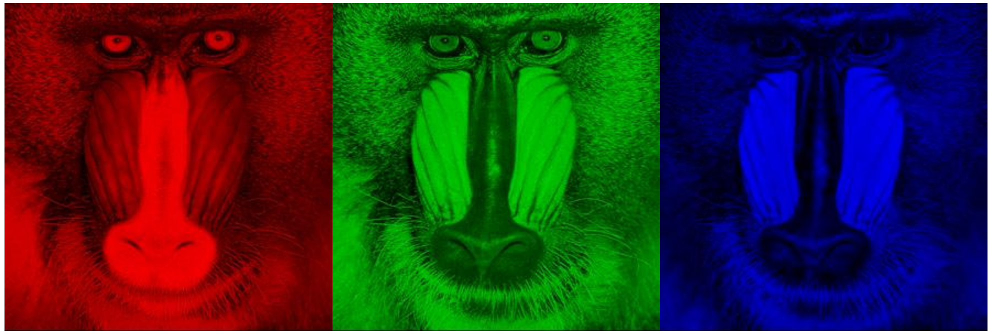


`mandrill.jpg`

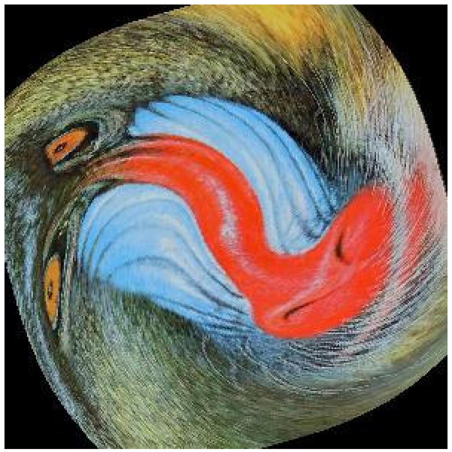
`(298-by-298)`

`% java Scale mandrill.jpg 400 200`

More Image Processing Effects



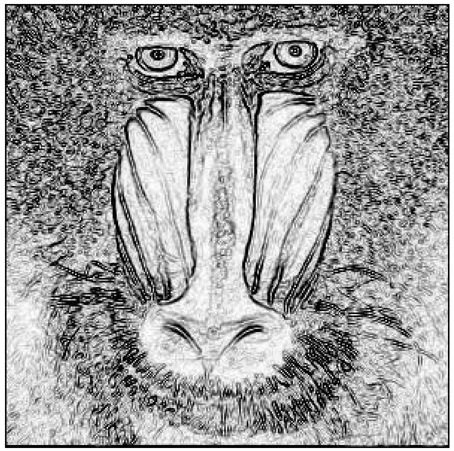
RGB color separation



swirl filter

wave filter

glass filter



Sobel edge detection

Text Processing

String Data Type

String data type. Basis for text processing.
Set of values. Sequence of Unicode characters.

API.

public class String (Java string data type)

	`String(String s)`	*create a string with the same value as* `s`
`int`	`length()`	*string length*
`char`	`charAt(int i)`	`i`*th character*
`String`	`substring(int i, int j)`	`i`*th through (*`j-1`*)st characters*
`boolean`	`contains(String sub)`	*does string contain* `sub` *as a substring?*
`boolean`	`startsWith(String pre)`	*does string start with* `pre`*?*
`boolean`	`endsWith(String post)`	*does string end with* `post`*?*
`int`	`indexOf(String p)`	*index of first occurrence of* `p`
`int`	`indexOf(String p, int i)`	*index of first occurrence of* `p` *after* `i`
`String`	`concat(String t)`	*this string with* `t` *appended*
`int`	`compareTo(String t)`	*string comparison*
`String`	`replaceAll(String a, String b)`	*result of changing* `a`*s to* `b`*s*
`String[]`	`split(String delim)`	*strings between occurrences of* `delim`
`boolean`	`equals(String t)`	*is this string's value the same as* `t`*'s?*

http://download.oracle.com/javase/6/docs/api/java/lang/String.html

Typical String Processing Code

is the string a palindrome?	`public static boolean isPalindrome(String s)`
	`{`
	`   int N = s.length();`
	`   for (int i = 0; i < N/2; i++)`
	`      if (s.charAt(i) != s.charAt(N-1-i))`
	`         return false;`
	`   return true;`
	`}`

extract file name and extension from a command-line argument	`String s = args[0];`
	`int dot = s.indexOf(".");`
	`String base      = s.substring(0, dot);`
	`String extension = s.substring(dot + 1, s.length());`

print all lines in standard input that contain a string specified on the command line	`String query = args[0];`
	`while (!StdIn.isEmpty())`
	`{`
	`   String s = StdIn.readLine();`
	`   if (s.contains(query)) StdOut.println(s);`
	`}`

print all the hyperlinks (to educational institutions) in the text file on standard input	`while (!StdIn.isEmpty())`
	`{`
	`   String s = StdIn.readString();`
	`   if (s.startsWith("http://") && s.endsWith(".edu"))`
	`      StdOut.println(s);`
	`}`

Gene Finding

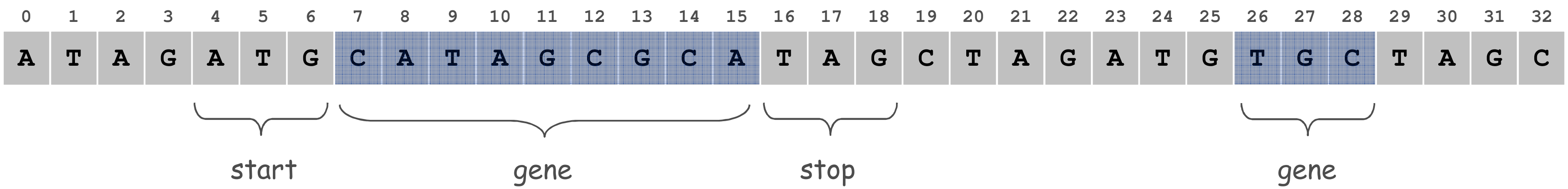
Pre-genomics era. Sequence a human genome.
Post-genomics era. Analyze the data and understand structure.

Genomics. Represent genome as a string over { A, C, T, G } alphabet.

Gene. A substring of genome that represents a functional unit.

- Preceded by **ATG**. [start codon]
- Multiple of 3 nucleotides. [codons other than start/stop]
- Succeeded by **TAG**, **TAA**, or **TGA**. [stop codons]

Goal. Find all genes.



Gene Finding: Algorithm

Algorithm. Scan left-to-right through genome.

- If start codon, then set `beg` to index `i`.
- If stop codon and substring is a multiple of 3
  - output gene
  - reset `beg` to -1

i	codon: start	codon: stop	beg	gene	*remaining portion of input string*
0			-1		ATAGATGCATAGCGCATAGCTAGATGTGCTAGC
1		TAG	-1		ATAGATGCATAGCGCATAGCTAGATGTGCTAGC
4	ATG		4		ATAGATGCATAGCGCATAGCTAGATGTGCTAGC
9		TAG	4		ATAGATGCATAGCGCATAGCTAGATGTGCTAGC
16		TAG	4	CATAGCGCA (multiple of 3)	ATAGATGCATAGCGCATAGCTAGATGTGCTAGC
20		TAG	-1		ATAGATGCATAGCGCATAGCTAGATGTGCTAGC
23	ATG		23		ATAGATGCATAGCGCATAGCTAGATGTGCTAGC
29		TAG	23	TGC	ATAGATGCATAGCGCATAGCTAGATGTGCTAGC

Gene Finding: Implementation

```
public class GeneFind {
   public static void main(String[] args) {
      String start  = args[0];
      String stop   = args[1];
      String genome = StdIn.readAll();

      int beg = -1;
      for (int i = 0; i < genome.length() - 2; i++) {
         String codon = genome.substring(i, i+3);
         if (codon.equals(start)) beg = i;
         if (codon.equals(stop) && beg != -1 && beg+3 < i) {
            String gene = genome.substring(beg+3, i);
            if (gene.length() % 3 == 0) {
               StdOut.println(gene);
               beg = -1;
            }
         }
      }
   }
}
```

```
% more genomeTiny.txt
ATAGATGCATAGCGCATAGCTAGATGTGCTAGC

% java GeneFind ATG TAG < genomeTiny.txt
CATAGCGCA
TGC
```

OOP Context for Strings

Possible memory representation of a string.

- `genome = "aacaagtttacaagc";`

D0	D1	D2	D3	D4	D5	D6	D7	D8	D9	DA	DB	DC	DD	DE
a	a	c	a	a	g	t	t	t	a	c	a	a	g	c

genome

A0	A1
D0	15

memory address (D0) — length (15)

- `s = genome.substring(1, 5);`
- `t = genome.substring(9, 13);`

s

B0	B1
D1	4

t

B2	B3
D9	4

s and t refer to different strings that have the same value "**acaa**"

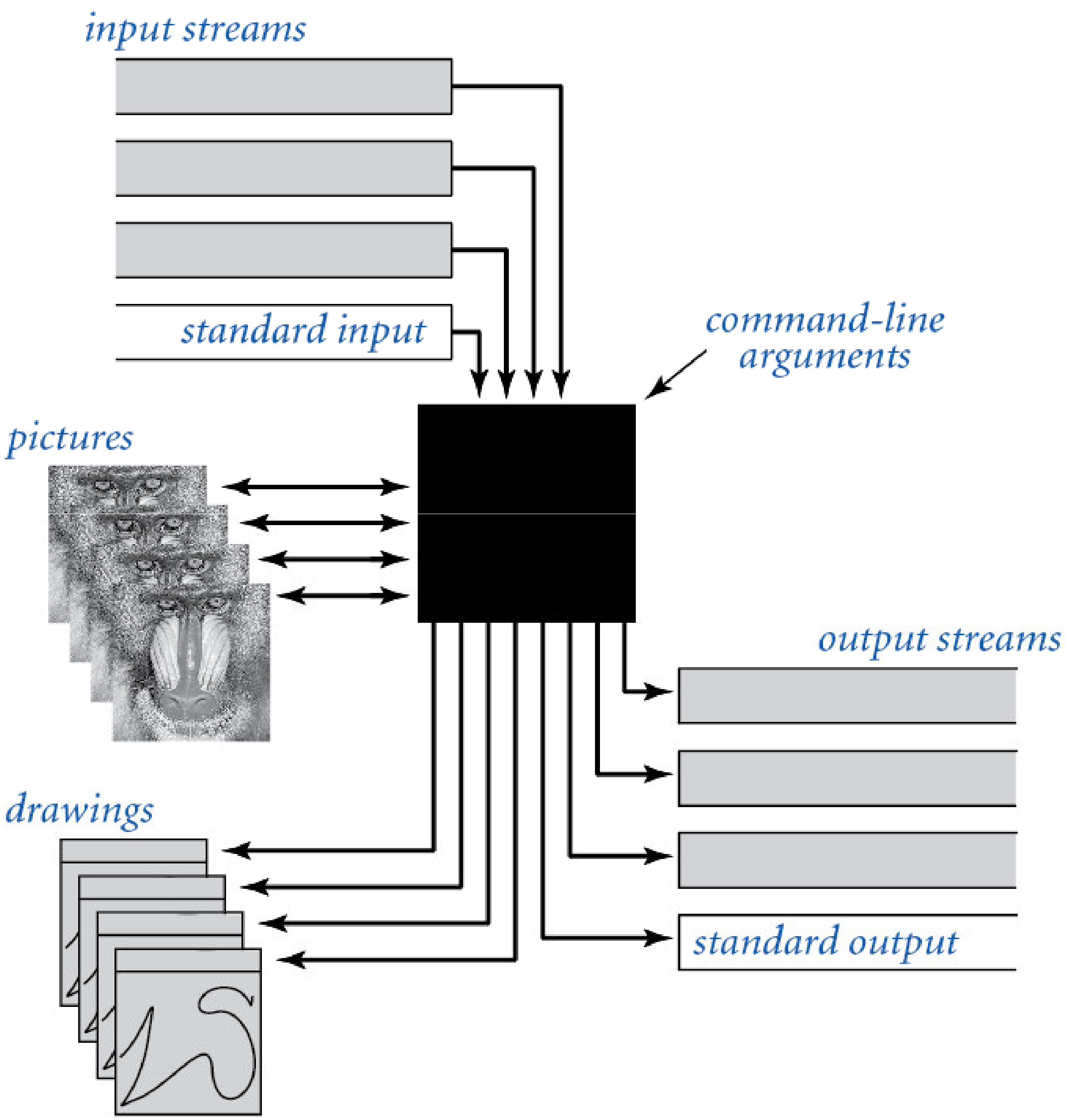
- `(s == t)` is **false**, but `(s.equals(t))` is **true**.

`(s == t)`: compares pointers

`(s.equals(t))`: compares character sequences

In and Out

Bird's Eye View (Revisited)



Non-Standard Input

or use OS to redirect from one file

Standard input. Read from terminal window.
Goal. Read from several different input streams.

`In` data type. Read text from stdin, a file, a web site, or network.

Ex: Are two text files identical?

```
public class Diff {
   public static void main(String[] args) {
      In in0 = new In(args[0]);    // read from one file
      In in1 = new In(args[1]);    // read from another file
      String s = in0.readAll();
      String t = in1.readAll();
      StdOut.println(s.equals(t));
   }
}
```

Screen Scraping

Goal. Find current stock price of Google.

```
…
<tr>
<td class="yfnc_tablehead1" width="48%">
Last Trade:
</td>
<td class="yfnc_tabledata1">
<big>
<b>576.50</b>
</big>
</td>
</tr>
<tr>
<td class="yfnc_tablehead1" width="48%">
Trade Time:
</td>
<td class="yfnc_tabledata1">
11:45AM ET
</td>
</tr>
…
```

`http://finance.yahoo.com/q?s=goog` ← NYSE symbol

Screen Scraping

Goal. Find current stock price of Google.

- `s.indexOf(t, i)`: index of first occurrence of pattern `t` in string `s`, starting at offset `i`.
- Read raw html from `http://finance.yahoo.com/q?s=goog`.
- Find first string delimited by `<b>` and `</b>` after `Last Trade`.

```
public class StockQuote {
   public static void main(String[] args) {
      String name = "http://finance.yahoo.com/q?s=";
      In in = new In(name + args[0]);
      String input = in.readAll();
      int start    = input.indexOf("Last Trade:", 0);
      int from     = input.indexOf("<b>",  start);
      int to       = input.indexOf("</b>", from);
      String price = input.substring(from + 3, to);
      StdOut.println(price);
   }
}
```

```
% java StockQuote goog
576.50
```

OOP Summary

Object. Holds a data type value; variable name refers to object.

In Java, programs manipulate references to objects.

- Exception: primitive types, e.g., `boolean`, `int`, `double`.
- Reference types: `String`, `Picture`, `Color`, arrays, everything else.
- OOP purist: language should not have separate primitive types.

Bottom line. We wrote programs that manipulate colors, pictures, and strings.

Next time. We'll write programs that manipulate our own abstractions.

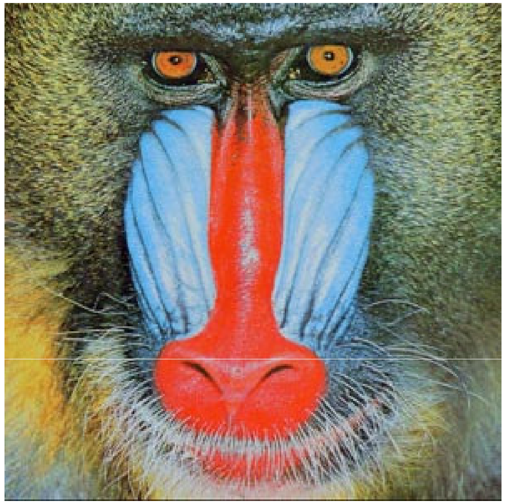
Extra Slides

Color Separation

```
import java.awt.Color;
public class ColorSeparation {
    public static void main(String args[]) {
        Picture pic = new Picture(args[0]);
        int width  = pic.width();
        int height = pic.height();

        Picture R = new Picture(width, height);
        Picture G = new Picture(width, height);
        Picture B = new Picture(width, height);

        for (int x = 0; x < width; x++) {
            for (int y = 0; y < height; y++) {
                Color c = pic.get(x, y);
                int r = c.getRed();
                int g = c.getGreen();
                int b = c.getBlue();
                R.set(x, y, new Color(r, 0, 0));
                G.set(x, y, new Color(0, g, 0));
                B.set(x, y, new Color(0, 0, b));
            }
        }
        R.show();
        G.show();
        B.show();
    }
}
```

Color Separation

ColorSeparation.java. Creates three `Picture` objects and windows.



Memory Management

Value types.

- Allocate memory when variable is declared.
- Can reclaim memory when variable goes out of scope.

Reference types.

- Allocate memory when object is created with `new`.
- Can reclaim memory when last reference goes out of scope.
- Significantly more challenging if several references to same object.

Garbage collector. System automatically reclaims memory; programmer relieved of tedious and error-prone activity.

Defining Data Types in Java

To define a data type, specify:

- Set of values.
- Operations defined on those values.

Java class. Defines a data type by specifying:

- Instance variables. (set of values)
- Methods. (operations defined on those values)
- Constructors. (create and initialize new objects)

Point Charge Data Type

Goal. Create a data type to manipulate point charges.

Set of values. Three real numbers. [position and electrical charge]

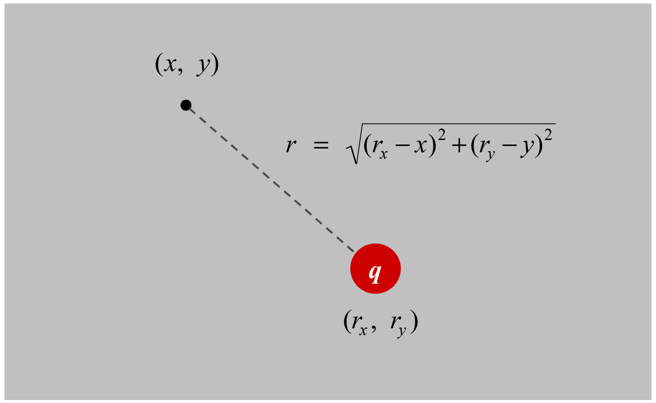
Operations.

- Create a new point charge at (r_x, r_y) with electric charge q.
- Determine electric potential V at (x, y) due to point charge.
- Convert to string.

$$V = k \frac{q}{r}$$

r = distance between (x, y) and (r_x, r_y)

k = electrostatic constant = $8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$



Point Charge Data Type

Goal. Create a data type to manipulate point charges.

Set of values. Three real numbers. [position and electrical charge]

API.

```
public class Charge
---------------------------------------------------------------
        Charge(double x0, double y0, double q0)
double  potentialAt(double x, double y)   electric potential at (x, y) due to charge
String  toString()                        string representation
```

Charge Data Type: A Simple Client

Client program. Uses data type operations to calculate something.

```
public static void main(String[] args) {
   double x = Double.parseDouble(args[0]);
   double y = Double.parseDouble(args[1]);
   Charge c1 = new Charge(.51, .63, 21.3);
   Charge c2 = new Charge(.13, .94, 81.9);
   double v1 = c1.potentialAt(x, y);
   double v2 = c2.potentialAt(x, y);
   StdOut.println(c1);
   StdOut.println(c2);
   StdOut.println(v1 + v2);
}
```

automagically invokes the `toString()` method

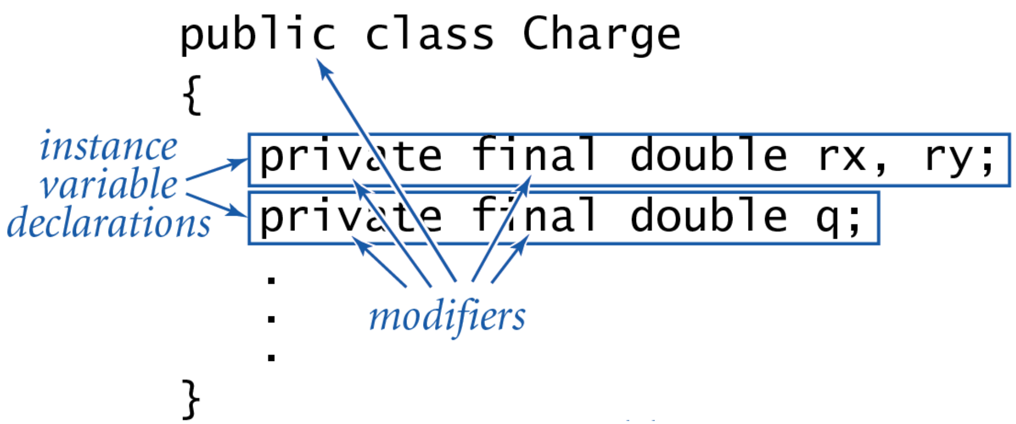
```
% java Charge .50 .50
21.3 at (0.51, 0.63)
81.9 at (0.13, 0.94)
2.74936907085912e12
```

Anatomy of Instance Variables

Instance variables. Specifies the set of values.

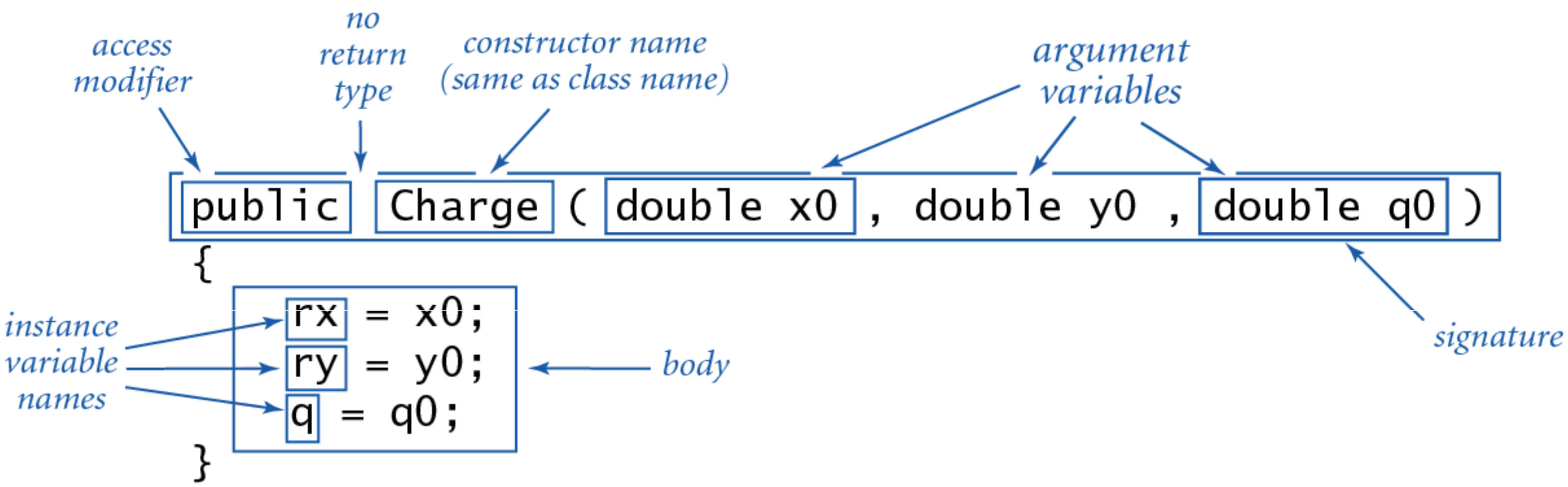
- Declare outside any method.
- Always use access modifier **`private`**.
- Use modifier **`final`** with instance variables that never change.

stay tuned

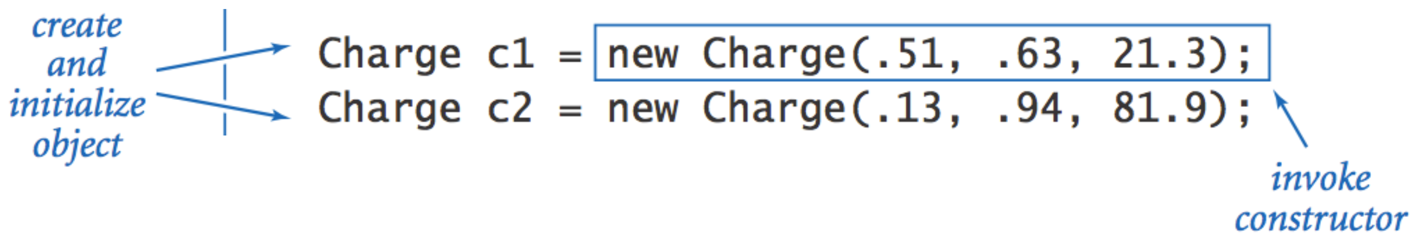


Anatomy of a Constructor

Constructor. Specifies what happens when you create a new object.

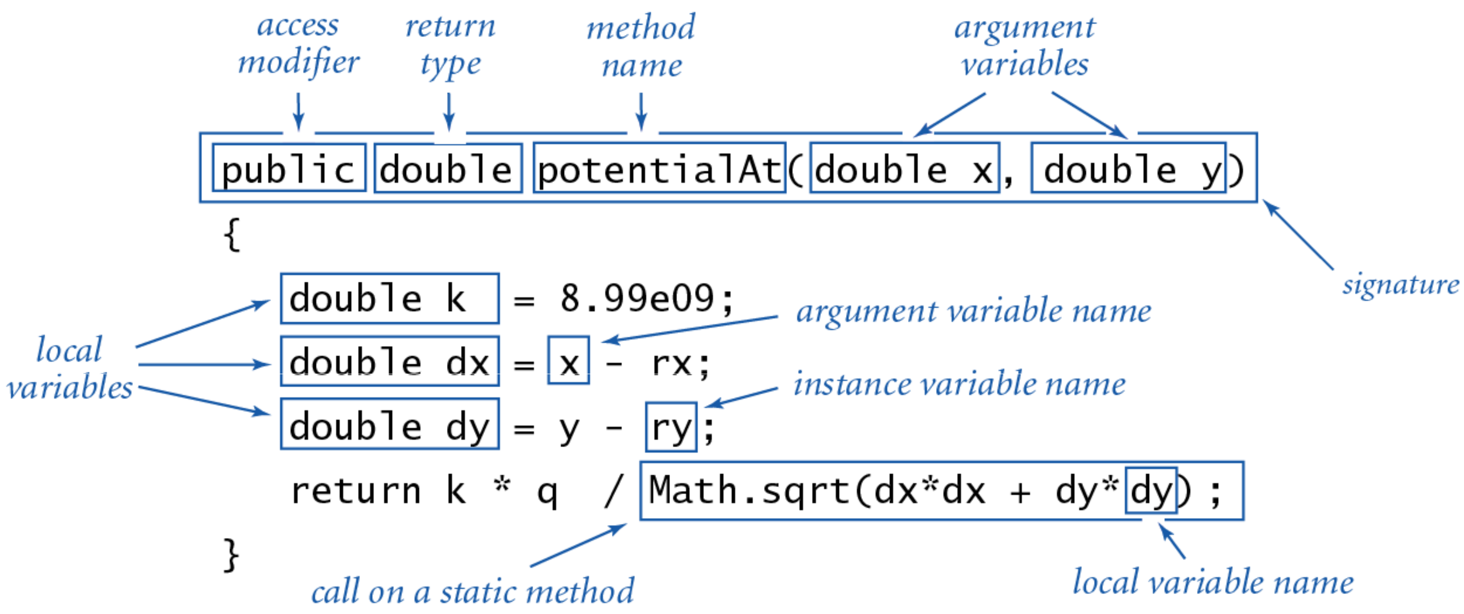


Calling a constructor. Use `new` operator to create a new object.

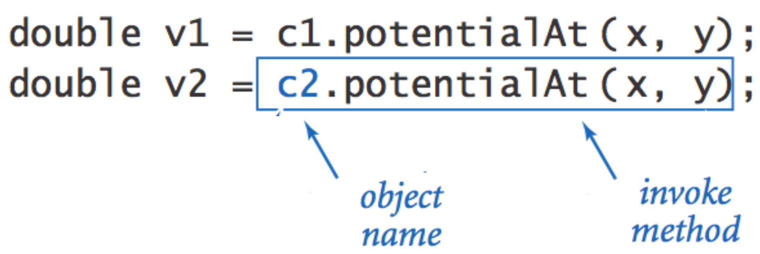


Anatomy of an Instance Method

Instance method. Define operations on instance variables.



Invoking an instance method. Use dot operator to invoke a method.



Anatomy of a Class

```
public class Charge
{
   private final double rx, ry;
   private final double q;

   public Charge(double x0, double y0, double q0)
   {   rx = x0; ry = y0; q = q0;   }

   public double potentialAt(double x, double y)
   {
      double k = 8.99e09;
      double dx = x - rx ;
      double dy = y - ry ;
      return k * q / Math.sqrt(dx*dx + dy*dy);
   }

   public String toString()
   {   return q +" at " + "("+ rx + ", " + ry +")";   }

   public static void main(String[] args)
   {
      double x = Double.parseDouble(args[0]);
      double y = Double.parseDouble(args[1]);
      Charge c1 = new Charge(.51, .63, 21.3);
      Charge c2 = new Charge(.13, .94, 81.9);
      double v1 = c1.potentialAt (x, y);
      double v2 = c2.potentialAt (x, y);
      StdOut.printf("%.1e\n", (v1 + v2));
   }
}
```

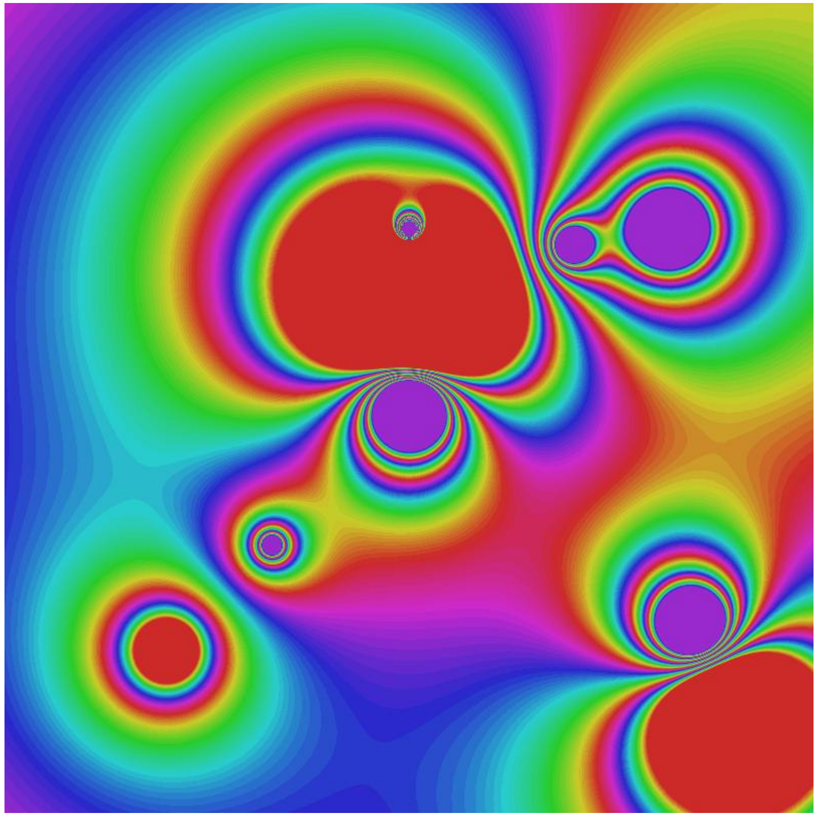
class name · *instance variables* · *constructor* · *instance variable names* · *instance methods* · *test client* · *create and initialize object* · *invoke constructor* · *object name* · *invoke method*

Potential Visualization

Potential visualization. Read in N point charges from standard input; compute total potential at each point in unit square.

```
% more charges.txt
9
.51 .63 -100
.50 .50   40
.50 .72   10
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50
```

```
% java Potential < charges.txt
```



Potential Visualization

Arrays of objects. Allocate memory for the array with **new**; then allocate memory for each individual object with **new**.

```
% more charges.txt
9
.51 .63 -100
.50 .50   40
.50 .72   10
.33 .33    5
.20 .20  -10
.70 .70   10
.82 .72   20
.85 .23   30
.90 .12  -50
```

```
// read in the data
int N = StdIn.readInt();
Charge[] a = new Charge[N];
for (int i = 0; i < N; i++) {
   double x0 = StdIn.readDouble();
   double y0 = StdIn.readDouble();
   double q0 = StdIn.readDouble();
   a[i] = new Charge(x0, y0, q0);
}
```

Potential Visualization

```
// plot the data
int SIZE = 512;
Picture pic = new Picture(SIZE, SIZE);
for (int i = 0; i < SIZE; i++) {
   for (int j = 0; j < SIZE; j++) {
      double V = 0.0;
      for (int k = 0; k < N; k++) {
         double x = 1.0 * i / SIZE;
         double y = 1.0 * j / SIZE;
         V += a[k].potentialAt(x, y);
      }
      Color color = getColor(V);
      pic.set(i, SIZE-1-j, color);
   }
}
pic.show();
```

$$V = \sum_k (k\, q_k / r_k)$$

compute color as a function of potential V

(0, 0) is upper left

Turtle Graphics

Turtle Graphics

Goal. Create a data type to manipulate a turtle moving in the plane.
Set of values. Location and orientation of turtle.

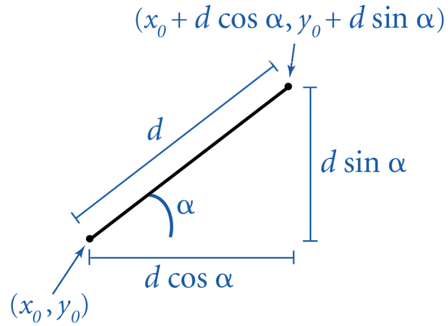
API. `public class Turtle`

	`Turtle(double x0, double y0, double a0)`	*create a new turtle at* (x_0, y_0) *facing* a_0 *degrees counterclockwise from the x-axis*
`void`	`turnLeft(double delta)`	*rotate* `delta` *degrees counterclockwise*
`void`	`goForward(double step)`	*move distance* `step`, *drawing a line*

```
// draw a square
Turtle turtle = new Turtle(0.0, 0.0, 0.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
turtle.goForward(1.0);
turtle.turnLeft(90.0);
```

Turtle Graphics

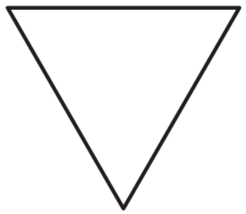
```
public class Turtle {
   private double x, y;    // turtle is at (x, y)
   private double angle;   // facing this direction

   public Turtle(double x0, double y0, double a0) {
      x = x0;
      y = y0;
      angle = a0;
   }

   public void turnLeft(double delta) {
      angle += delta;
   }

   public void goForward(double d) {
      double oldx = x;
      double oldy = y;
      x += d * Math.cos(Math.toRadians(angle));
      y += d * Math.sin(Math.toRadians(angle));
      StdDraw.line(oldx, oldy, x, y);
   }
}
```



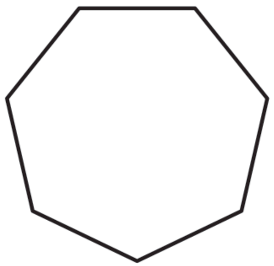
Turtle trigonometry

N-gon

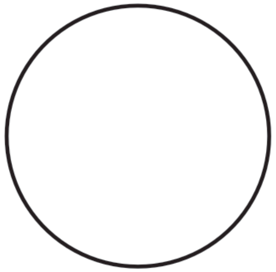
```
public class Ngon {
   public static void main(String[] args) {
      int N          = Integer.parseInt(args[0]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < N; i++) {
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```



3



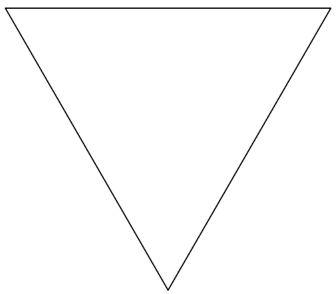
7



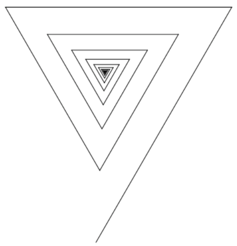
1440

Spira Mirabilis

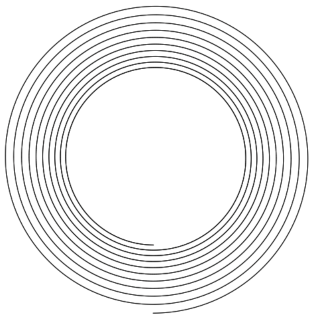
```
public class Spiral {
   public static void main(String[] args) {
      int N          = Integer.parseInt(args[0]);
      double decay = Double.parseDouble(args[1]);
      double angle = 360.0 / N;
      double step  = Math.sin(Math.toRadians(angle/2.0));
      Turtle turtle = new Turtle(0.5, 0, angle/2.0);
      for (int i = 0; i < 10 * N; i++) {
         step /= decay;
         turtle.goForward(step);
         turtle.turnLeft(angle);
      }
   }
}
```



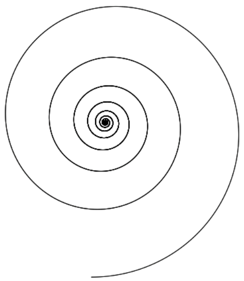
3 1.0



3 1.2

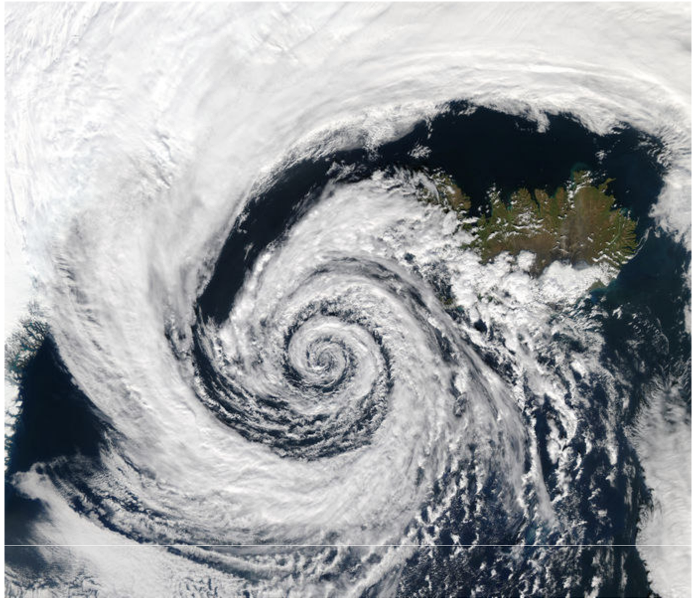


1440 1.00004



1440 1.0004

Spira Mirabilis in Nature



Complex Numbers

Complex Number Data Type

Goal. Create a data type to manipulate complex numbers.
Set of values. Two real numbers: real and imaginary parts.

API.

public class Complex		
	`Complex(double real, double imag)`	
`Complex`	`plus(Complex b)`	*sum of this number and* `b`
`Complex`	`times(Complex b)`	*product of this number and* `b`
`double`	`abs()`	*magnitude*
`String`	`toString()`	*string representation*

$a = 3 + 4i,\ \ b = -2 + 3i$

$a + b = 1 + 7i$

$a \times b = -18 + i$

$|\, a \,| = 5$

Applications of Complex Numbers

Relevance. A quintessential mathematical abstraction.

Applications.
- Fractals.
- Impedance in RLC circuits.
- Signal processing and Fourier analysis.
- Control theory and Laplace transforms.
- Quantum mechanics and Hilbert spaces.
- ...

Complex Number Data Type: *A Simple Client*

Client program. Uses data type operations to calculate something.

```
public static void main(String[] args) {
   Complex a = new Complex( 3.0, 4.0);
   Complex b = new Complex(-2.0, 3.0);
   Complex c = a.times(b);
   StdOut.println("a = " + a);
   StdOut.println("b = " + b);
   StdOut.println("c = " + c);
}
```

result of `c.toString()`

```
% java TestClient
a = 3.0 + 4.0i
b = -2.0 + 3.0i
c = -18.0 + 1.0i
```

Remark. Can't write `c = a * b` since no operator overloading in Java.

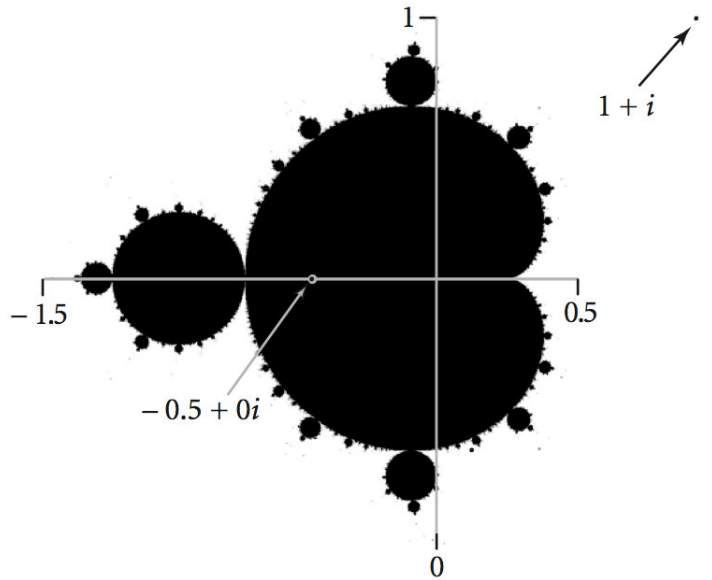
Complex Number Data Type: Implementation

```
public class Complex {
   private final double re;
   private final double im;                    instance variables

   public Complex(double real, double imag) {
      re = real;
      im = imag;
   }                                                  constructor

   public String toString() { return re + " + " + im + "i"; }

   public double abs() { return Math.sqrt(re*re + im*im); }

   public Complex plus(Complex b) {
      double real = re + b.re;
      double imag = im + b.im;            creates a Complex object,
      return new Complex(real, imag);     and returns a reference to it
   }
                                          refers to b's instance variable
   public Complex times(Complex b) {
      double real = re * b.re – im * b.im;
      double imag = re * b.im + im * b.re;
      return new Complex(real, imag);
   }                                                            methods
}
```

Mandelbrot Set

Mandelbrot set. A set of complex numbers.

Plot. Plot (x, y) black if $z = x + y\,i$ is in the set, and white otherwise.

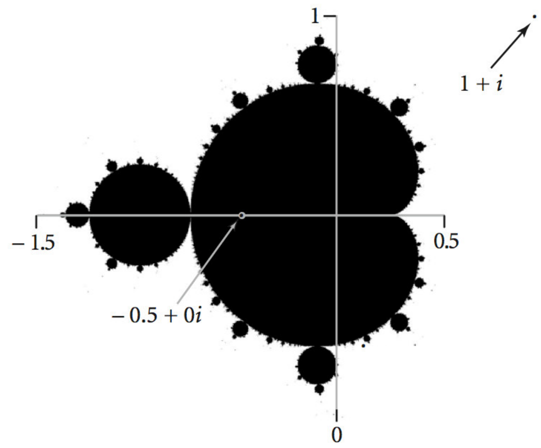


- No simple formula describes which complex numbers are in set.
- Instead, describe using an algorithm.

Mandelbrot Set

Mandelbrot set. Is complex number z_0 in the set?

- Iterate $z_{t+1} = (z_t)^2 + z_0$.
- If $|z_t|$ diverges to infinity, then z_0 is not in set; otherwise z_0 is in set.



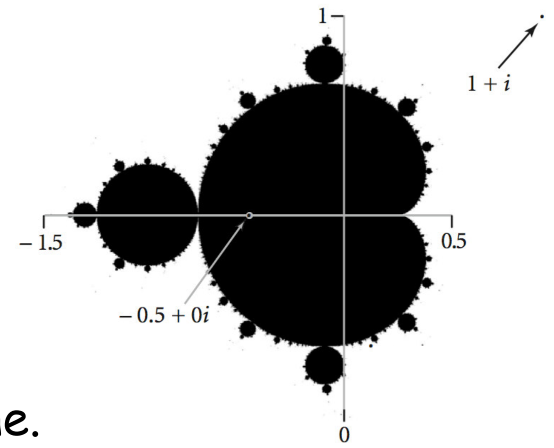
t	z_t
0	-1/2 + 0i
1	-1/4 + 0i
2	-7/16 + 0i
3	-79/256 + 0i
4	-26527/65536 + 0i
5	-1443801919/4294967296 + 0i

z = -1/2 is in Mandelbrot set

t	z_t
0	1 + i
1	1 + 3i
2	-7 + 7i
3	1 - 97i
4	-9407 - 193i
5	88454401 + 3631103i

z = 1 + i not in Mandelbrot set

Plotting the Mandelbrot Set

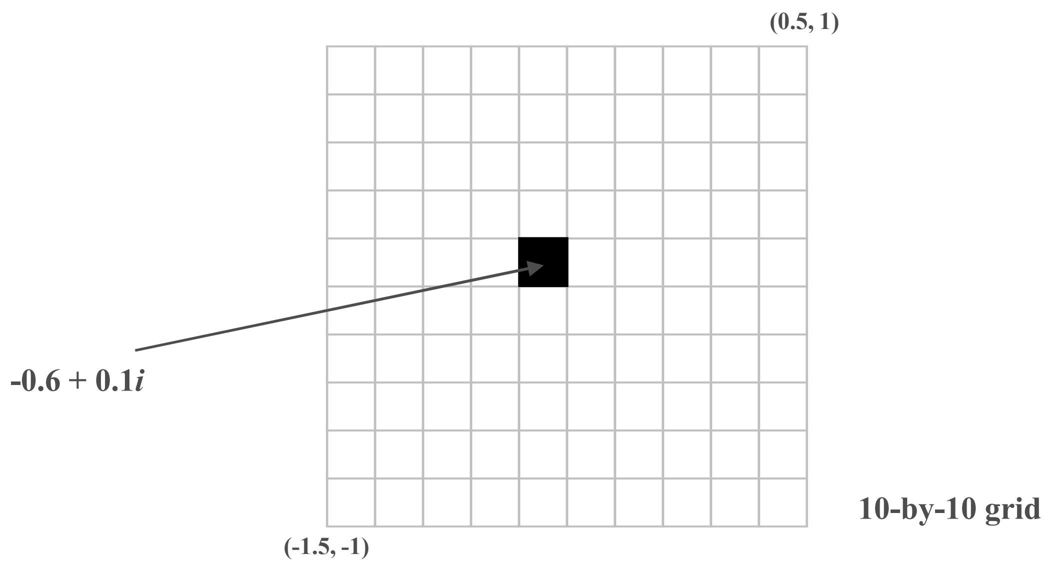


Practical issues.

- Cannot plot infinitely many points.
- Cannot iterate infinitely many times.

Approximate solution.

- Sample from an N-by-N grid of points in the plane.
- Fact: if $|z_t| > 2$ for any t, then z not in Mandelbrot set.
- Pseudo-fact: if $|z_{255}| \leq 2$ then z "likely" in Mandelbrot set.



Complex Number Data Type: Another Client

Mandelbrot function with complex numbers.

- Is z_0 in the Mandelbrot set?
- Returns white (definitely no) or black (probably yes).

```
public static Color mand(Complex z0) {
   Complex z = z0;
   for (int t = 0; t < 255; t++) {
      if (z.abs() > 2.0) return StdDraw.WHITE;
      z = z.times(z);
      z = z.plus(z0);
   }
   return StdDraw.BLACK;
}
```

$z = z^2 + z_0$

More dramatic picture: replace `StdDraw.WHITE` with grayscale or color.

`new Color(255-t, 255-t, 255-t)`

Complex Number Data Type: Another Client

Plot the Mandelbrot set in gray scale.

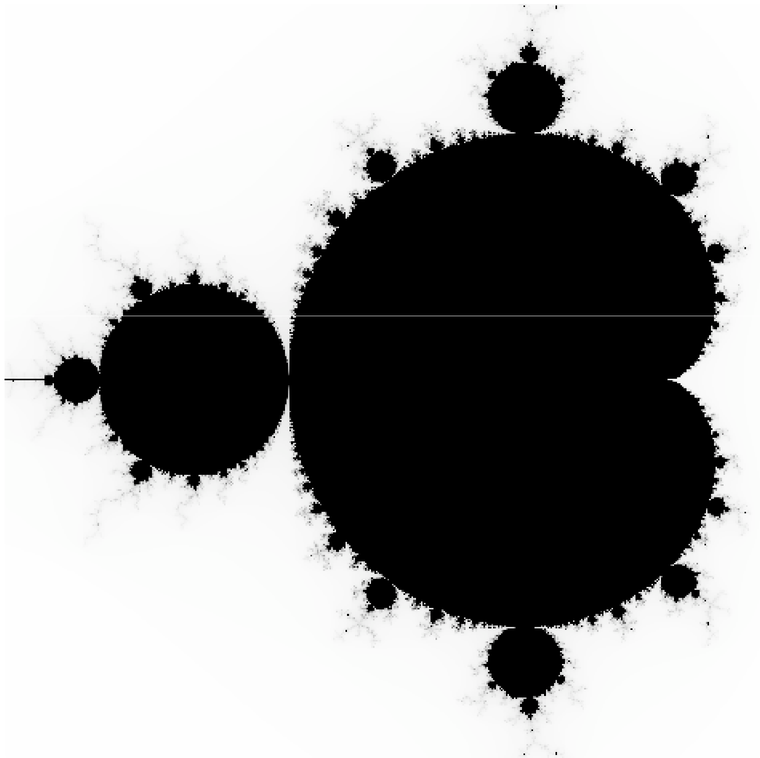
```
public static void main(String[] args) {
   double xc   = Double.parseDouble(args[0]);
   double yc   = Double.parseDouble(args[1]);
   double size = Double.parseDouble(args[2]);
   int N = 512;
   Picture pic = new Picture(N, N);

   for (int i = 0; i < N; i++) {
      for (int j = 0; j < N; j++) {
         double x0 = xc - size/2 + size*i/N;
         double y0 = yc - size/2 + size*j/N;
         Complex z0 = new Complex(x0, y0);
         Color color = mand(z0);
         pic.set(i, N-1-j, color);
      }
   }
   pic.show();
}
```

scale to screen coordinates

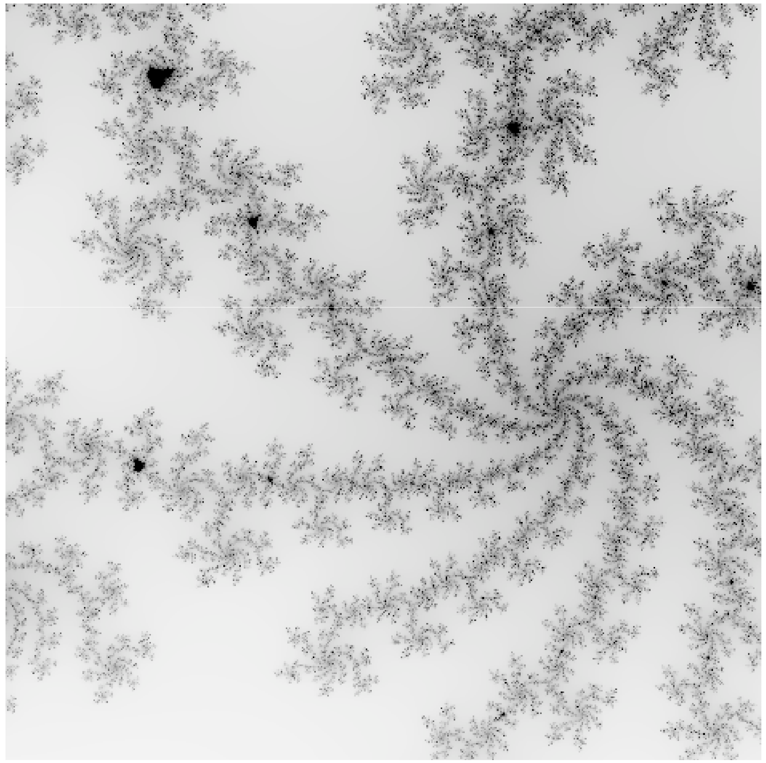
(0, 0) is upper left

Mandelbrot Set

`% java Mandelbrot -.5 0 2`

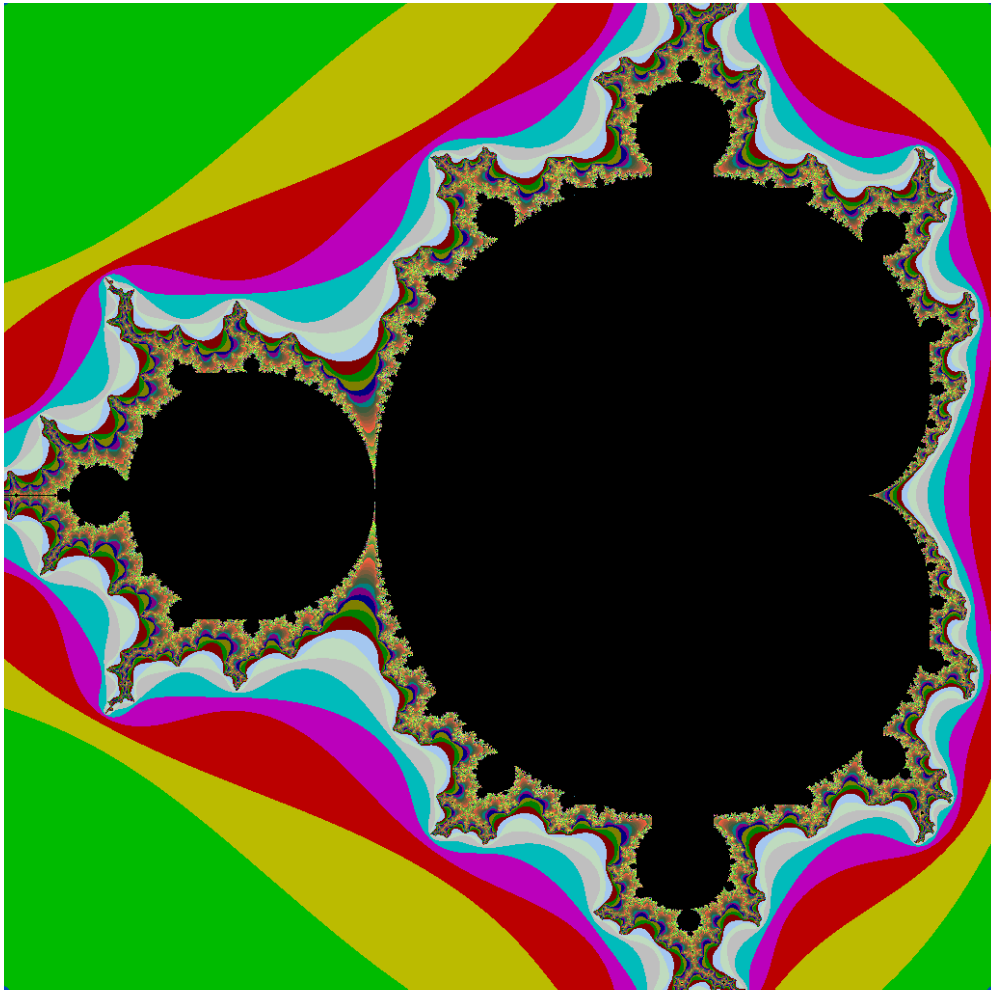


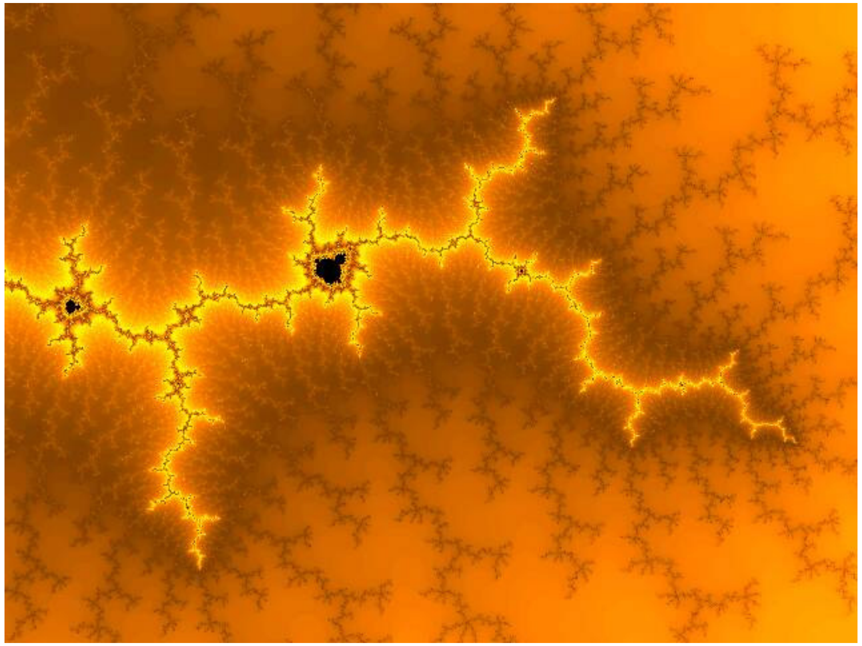
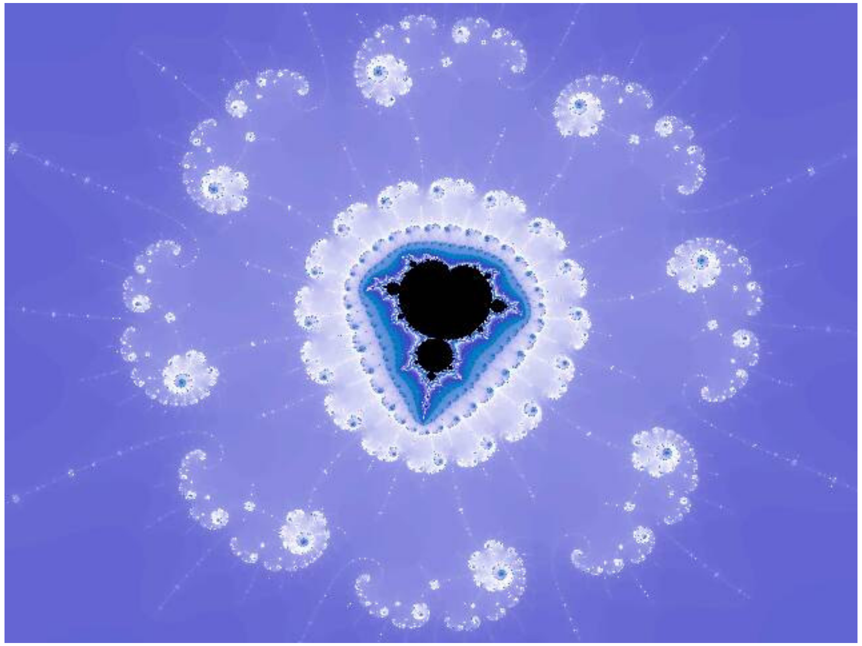
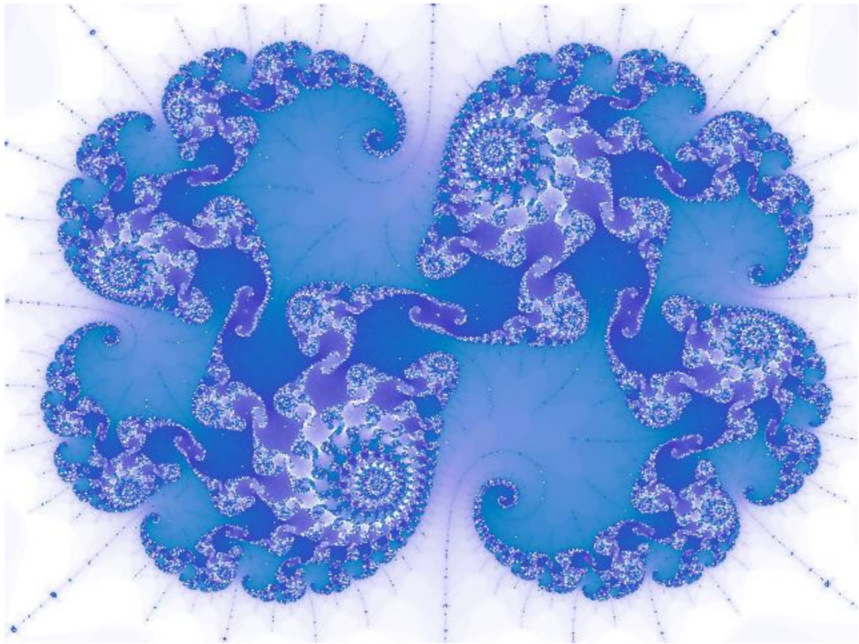
`% java Mandelbrot .1045 -.637 .01`

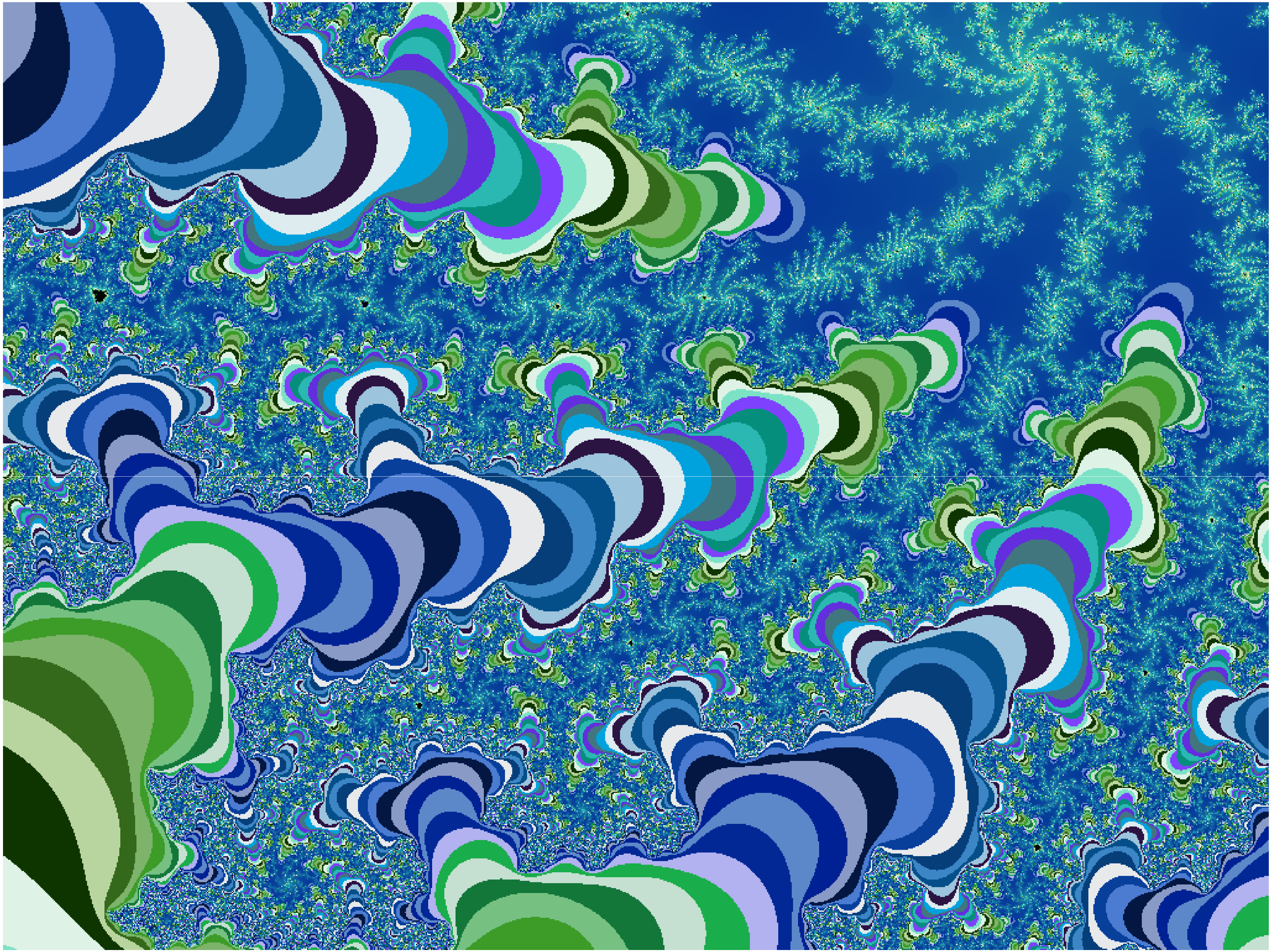


Mandelbrot Set

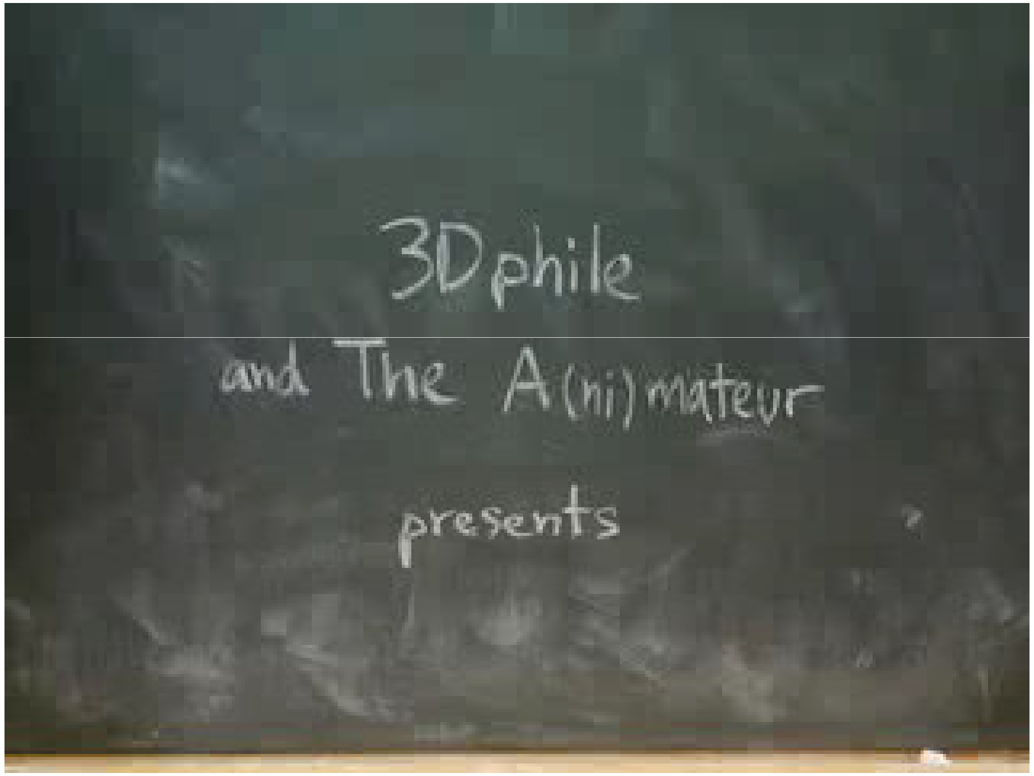
```
% java ColorMandelbrot -.5 0 2 < mandel.txt
```



Mandelbrot Set Music Video



http://www.jonathancoulton.com/songdetails/Mandelbrot Set

Applications of Data Types

Data type. Set of values and collection of operations on those values.

Simulating the physical world.

- Java objects model real-world objects.
- Not always easy to make model reflect reality.
- Ex: charged particle, molecule, COS 126 student, ….

Extending the Java language.

- Java doesn't have a data type for every possible application.
- Data types enable us to add our own abstractions.
- Ex: complex, vector, polynomial, matrix, ...

3.2 Extra Slides

Example: Bouncing Ball in Unit Square

Bouncing ball. Model a bouncing ball moving in the unit square with constant velocity.

Example: Bouncing Ball in Unit Square

Ball.java

```
public class Ball {
   private double rx, ry;
   private double vx, vy;
   private double radius;

   public Ball() {
      rx = ry = 0.5;
      vx     = 0.015 - Math.random() * 0.03;
      vy     = 0.015 - Math.random() * 0.03;
      radius = 0.01  + Math.random() * 0.01;
   }

   public void move() {
      if ((rx + vx > 1.0) || (rx + vx < 0.0)) vx = -vx;
      if ((ry + vy > 1.0) || (ry + vy < 0.0)) vy = -vy;
      rx = rx + vx;
      ry = ry + vy;
   }

   public void draw() {
      StdDraw.filledCircle(rx, ry, radius);
   }
}
```

← instance variables

constructor

↑ bounce

methods

Object References

Object reference.

- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

addr	value
C0	0
C1	0
C2	0
C3	0
C4	0
C5	0
C6	0
C7	0
C8	0
C9	0
CA	0
CB	0
CC	0

main memory (64-bit machine)

Object References

Object reference.

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```
Ball b1 = new Ball();
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b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

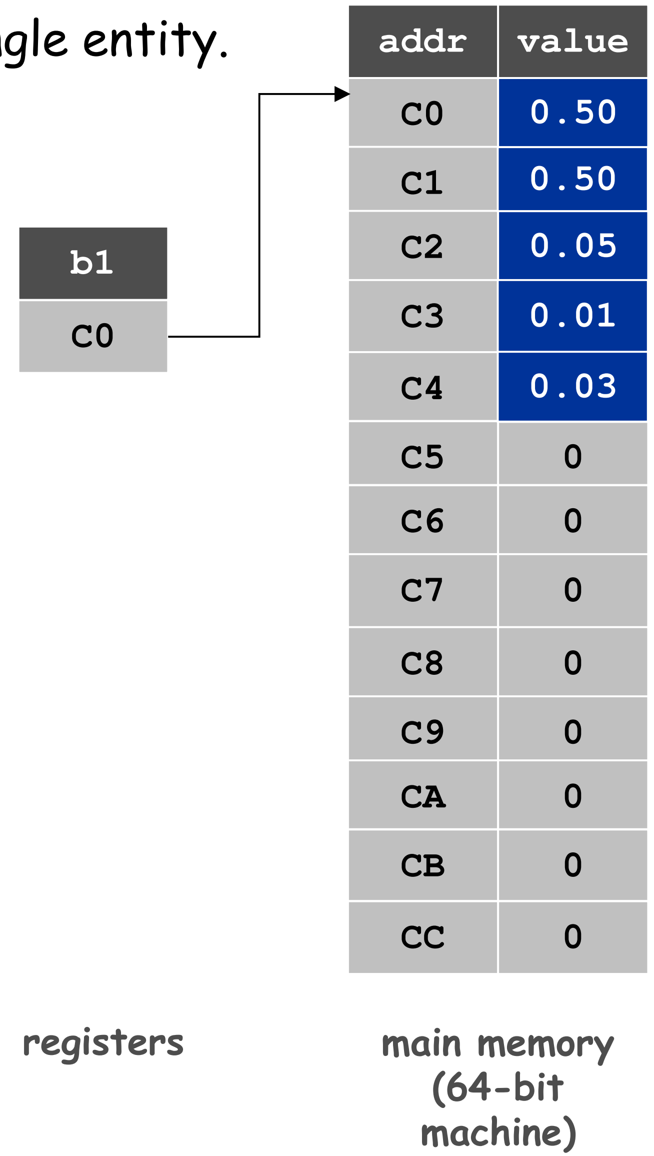

Object References

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```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

b1
C0

registers

addr	value
C0	0.55
C1	0.51
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0
C8	0
C9	0
CA	0
CB	0
CC	0

main memory (64-bit machine)

Object References

Object reference.

- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

b1
C0

registers

addr	value
C0	0.60
C1	0.52
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0
C8	0
C9	0
CA	0
CB	0
CC	0

main memory (64-bit machine)

Object References

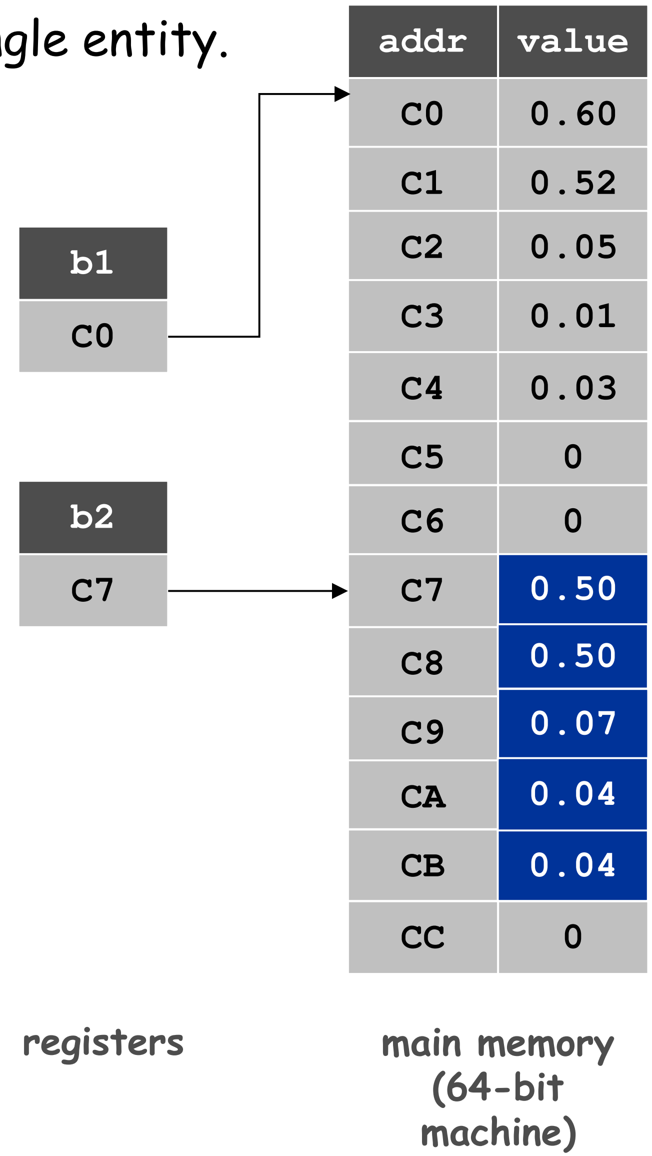
Object reference.

- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```



Object References

Object reference.

- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

b1
C0

b2
C7

registers

addr	value
C0	0.60
C1	0.52
C2	0.05
C3	0.01
C4	0.03
C5	0
C6	0
C7	0.57
C8	0.54
C9	0.07
CA	0.04
CB	0.04
CC	0

main memory (64-bit machine)

Object References

Object reference.

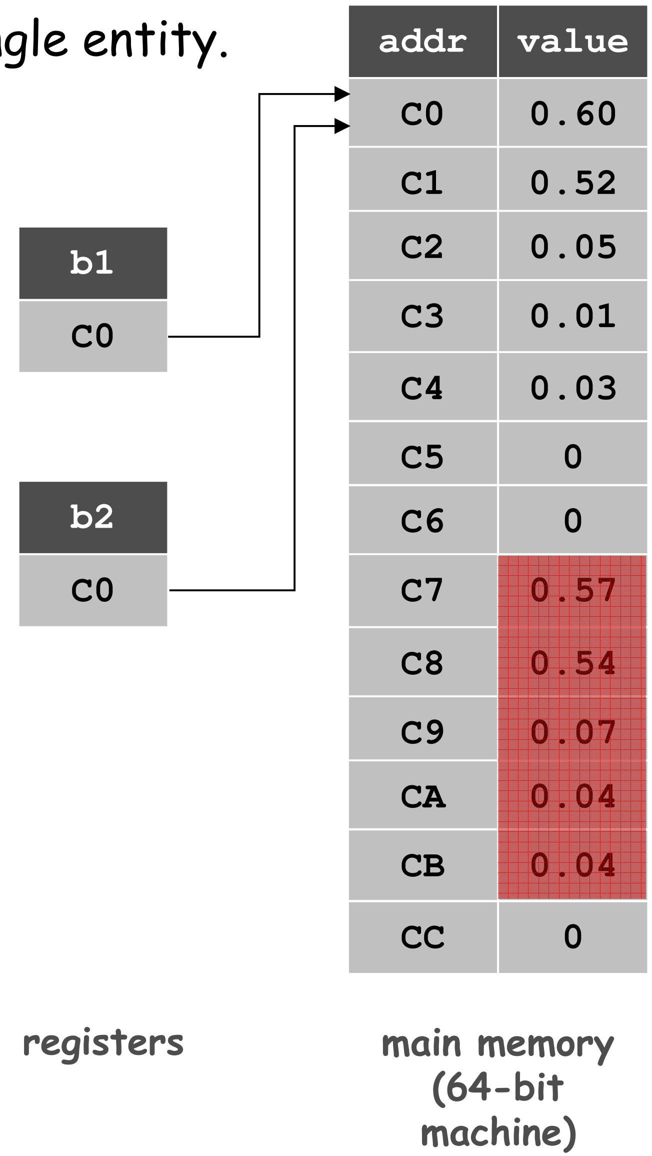
- Allow client to manipulate an object as a single entity.
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```

Data stored in C7 - CB for abstract bit recycler.



Object References

Object reference.

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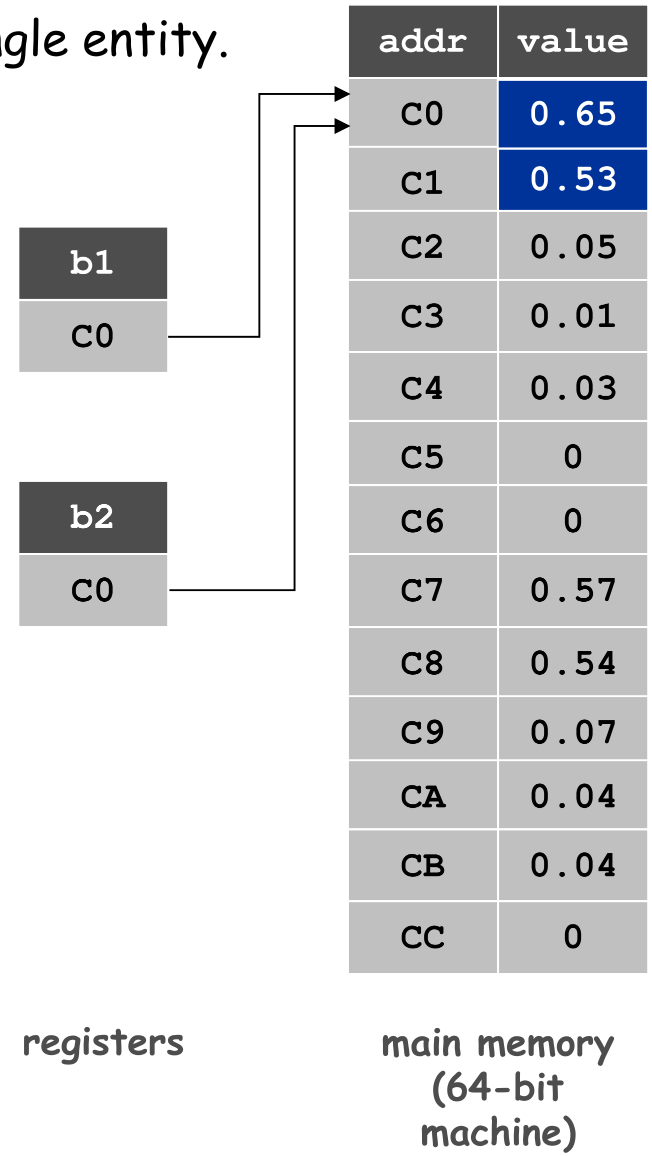
- Essentially a machine address (pointer).

```
Ball b1 = new Ball();
b1.move();
b1.move();

Ball b2 = new Ball();
b2.move();

b2 = b1;
b2.move();
```



Moving `b2` also moves `b1` since they are aliases that reference the same object.

Creating Many Objects

Each object is a data type value.

- Use `new` to invoke constructor and create each one.
- Ex: create N bouncing balls and animate them.

```
public class BouncingBalls {
   public static void main(String[] args) {
      int N = Integer.parseInt(args[0]);
      Ball balls[] = new Ball[N];
      for (int i = 0; i < N; i++)
         balls[i] = new Ball();

      while(true) {
         StdDraw.clear();
         for (int i = 0; i < N; i++) {
            balls[i].move();
            balls[i].draw();
         }
         StdDraw.show(20);
      }
   }
}
```

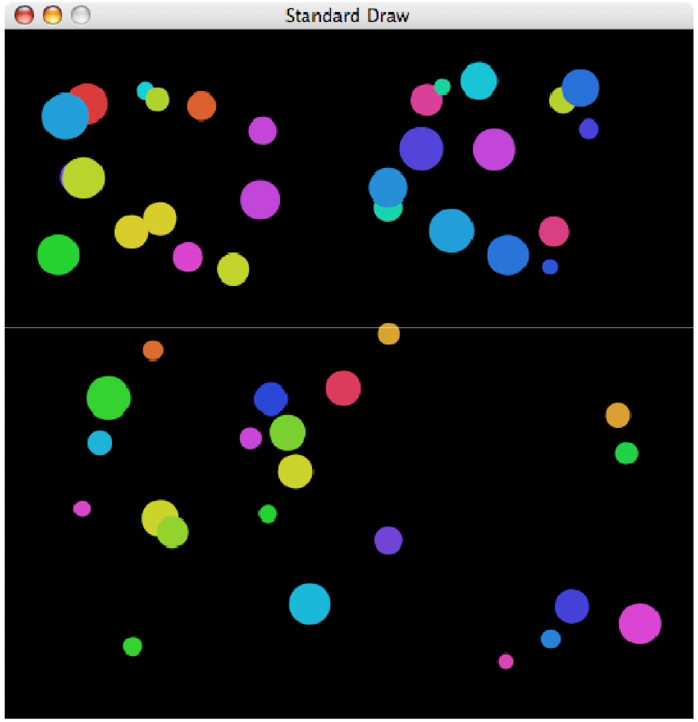
create and initialize N objects

animation loop

50 Bouncing Balls

Color. Associate a color with each ball; paint background black.

```
% java BouncingBalls 50
```



Scientific variations. Account for gravity, spin, collisions, drag, …

OOP Context

Reference. Variable that stores the name of a thing.

Thing	Name
Web page	`www.princeton.edu`
Bank account	`45-234-23310076`
Word of TOY memory	`1C`
Byte of computer memory	`00FACADE`
Home	`35 Olden Street`

Some consequences.

- Assignment statements copy references (not objects).
- The == operator tests if two references refer to same object.
- Pass copies of references (not objects) to functions.
  - efficient since no copying of data
  - function can change the object

Using a Data Type in Java

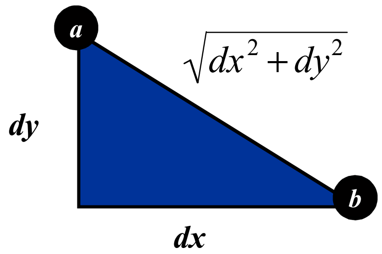
Client. A sample client program that uses the `Point` data type.

```
public class PointTest {
   public static void main(String[] args) {
      Point a = new Point();
      Point b = new Point();
      double distance = a.distanceTo(b);
      StdOut.println("a =  " + a);
      StdOut.println("b =  " + b);
      StdOut.println("distance =  " + distance);
   }
}
```

```
% java PointTest
a = (0.716810971264761, 0.0753539063358446)
b = (0.4052136795358151, 0.033848435224524076)
distance = 0.31434944941098036
```

Data type. Points in the plane.

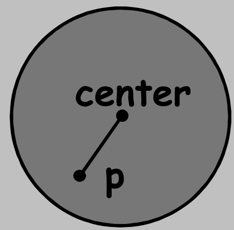
```
public class Point {
   private double x;
   private double y;

   public Point() {
      x = Math.random();
      y = Math.random();
   }

   public String toString() {
      return "(" + x + ", " + y + ")";
   }

   public double distanceTo(Point p) {
      double dx = x - p.x;
      double dy = y - p.y;
      return Math.sqrt(dx*dx + dy*dy);
   }
```

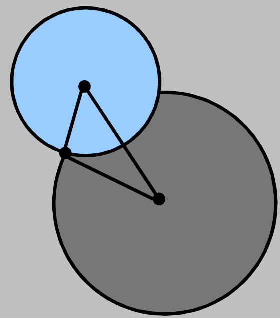


A Compound Data Type: Circles

Goal. Data type for circles in the plane.

```
public class Circle {
   private Point center;
   private double radius;

   public Circle(Point center, double radius) {
      this.center = center;
      this.radius = radius;
   }

   public boolean contains(Point p) {
      return p.dist(center) <= radius;
   }

   public double area() {
      return Math.PI * radius * radius;
   }

   public boolean intersects(Circle c) {
      return center.dist(c.center) <= radius + c.radius;
   }
}
```





Pass-By-Value

Arguments to methods are always passed by value.

- Primitive types: passes copy of value of actual parameter.
- Objects: passes copy of reference to actual parameter.

```
public class PassByValue {
   static void update(int a, int[] b, String c) {
      a    = 7;
      b[3] = 7;
      c    = "seven";
      StdO.println(a + " " + b[3] + " " + c);
   }
   public static void main(String[] args) {
      int a = 3;
      int[] b = { 0, 1, 2, 3, 4, 5 };
      String c = "three";
      StdOut.println(a + " " + b[3] + " " + c);
      update(a, b, c);
      StdOut.println(a + " " + b[3] + " " + c);
   }
}
```

```
% java PassByValue
3 3 three
7 7 seven
3 7 three
```

Object Oriented Programming

Procedural programming. [verb-oriented]

- Tell the computer to do this.
- Tell the computer to do that.

OOP philosophy. Software is a simulation of the real world.

- We know (approximately) how the real world works.
- Design software to model the real world.

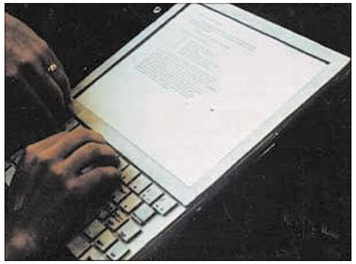
Objected oriented programming (OOP). [noun-oriented]

- Programming paradigm based on data types.
- Identify objects that are part of the problem domain or solution.
- Identity: objects are distinguished from other objects (references).
- State: objects know things (instance variables).
- Behavior: objects do things (methods).

Alan Kay

Alan Kay. [Xerox PARC 1970s]

- Invented Smalltalk programming language.
- Conceived Dynabook portable computer.
- Ideas led to: laptop, modern GUI, OOP.



" The computer revolution hasn't started yet. "

" The best way to predict the future is to invent it. "

" If you don't fail at least 90 per cent of the time, you're not aiming high enough. "

— Alan Kay

Alan Kay
2003 Turing Award

Encapsulation

Encapsulation

Data type. Set of values and operations on those values.

Ex. `int`, `String`, `Complex`, `Vector`, `Document`, `GuitarString`, …

Encapsulated data type. Hide internal representation of data type.

Separate implementation from design specification.

- Class provides data representation and code for operations.
- Client uses data type as black box.
- API specifies contract between client and class.

Bottom line. You don't need to know how a data type is implemented in order to use it.

Intuition

Client

API
- volume
- change channel
- adjust picture
- decode NTSC signal

Implementation
- cathode ray tube
- electron gun
- Sony Wega 36XBR250
- 241 pounds

client needs to know
how to use API

implementation needs to know
what API to implement

Implementation and client need to agree on API ahead of time.

Intuition

Client

API
- volume
- change channel
- adjust picture
- decode NTSC signal

Implementation
- gas plasma monitor
- Samsung FPT-6374
- wall mountable
- 4 inches deep

client needs to know how to use API

implementation needs to know what API to implement

Can substitute better implementation without changing the client.

Counter Data Type

Counter. Data type to count electronic votes.

```
public class Counter {
   public int count;
   public final String name;

   public Counter(String id) { name = id;    }
   public void increment()   { count++;      }
   public int value()        { return count; }

}
```

Legal Java client.

```
Counter c = new Counter("Volusia County");
c.count = -16022;
```

Oops. Al Gore receives -16,022 votes in Volusia County, Florida.

Counter Data Type

Counter. Encapsulated data type to count electronic votes.

```
public class Counter {
   private int count;
   private final String name;

   public Counter(String id) { name = id;     }
   public void increment()   { count++;       }
   public int value()        { return count;  }

}
```

Does not compile.

```
Counter c = new Counter("Volusia County");
c.count = -16022;
```

Benefit. Can guarantee that each data type value remains in a consistent state.

Changing Internal Representation

Encapsulation.

- Keep data representation hidden with **private** access modifier.
- Expose API to clients using **public** access modifier.

```
public class Complex {
   private final double re, im;

   public Complex(double re, double im) { … }
   public double abs()                   { … }
   public Complex plus(Complex b)        { … }
   public Complex times(Complex b)       { … }
   public String toString()              { … }
}
```

e.g., to polar coordinates

Advantage. Can switch internal representation without changing client.

Note. All our data types are already encapsulated!

Time Bombs

Internal representation changes.

- [Y2K] Two digit years: January 1, 2000.
- [Y2038] 32-bit seconds since 1970: January 19, 2038.
- [VIN numbers] We'll run out by 2010.

www.cartoonstock.com/directory/m/millenium_time-bomb.asp

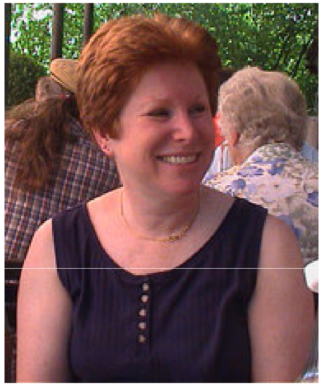
Lesson. By exposing data representation to client, might need to sift through millions of lines of code in client to update.

Ask, Don't Touch

Encapsulated data types.

- Don't touch data and do whatever you want.
- Instead, ask object to manipulate its data.

"Ask, don't touch."



Adele Goldberg
Former president of ACM
Co-developed Smalltalk

Lesson. Limiting scope makes programs easier to maintain and understand.

"principle of least privilege"

Immutability

Immutability

Immutable data type. Object's value cannot change once constructed.

mutable	*immutable*
`Picture`	`Charge`
`Histogram`	`Color`
`Turtle`	`Stopwatch`
`StockAccount`	`Complex`
`Counter`	`String`
Java arrays	primitive types

Immutability: Advantages and Disadvantages

Immutable data type. Object's value cannot change once constructed.

Advantages.

- Avoid aliasing bugs.
- Makes program easier to debug.
- Limits scope of code that can change values.
- Pass objects around without worrying about modification.

Disadvantage. New object must be created for every value.

Final Access Modifier

Final. Declaring an instance variable to be **final** means that you can assign it a value only once, in initializer or constructor.

```
public class Counter {
   private final String name;
   private int count;
...
}
```

this value doesn't change once the object is constructed

this value changes by invoking instance method

Advantages.

- Helps enforce immutability.
- Prevents accidental changes.
- Makes program easier to debug.
- Documents that the value cannot not change.

Spatial Vectors

Vector Data Type

Set of values. Sequence of real numbers. [Cartesian coordinates]

API.

`public class Vector`

	`Vector(double[] a)`	*create a vector with the given Cartesian coordinates*
`Vector`	`plus(Vector b)`	*sum of this vector and* `b`
`Vector`	`minus(Vector b)`	*difference of this vector and* `b`
`Vector`	`times(double t)`	*scalar product of this vector and* `t`
`double`	`dot(Vector b)`	*dot product of this vector and* `b`
`double`	`magnitude()`	*magnitude of this vector*
`Vector`	`direction()`	*unit vector with same direction as this vector*

$x = (0, 3, 4, 0), \quad y = (0, -3, 1, -4)$

$x + y = (0, 0, 5, -4)$

$3x = (0, 9, 12, 0)$

$x \bullet y = (0 \times 0) + (3 \times -3) + (4 \times 1) + (0 \times -4) = -5$

$|x| = (0^2 + 3^2 + 4^2 + 0^2)^{1/2} = 5$

$x = x \,/\, |x| = (0, 0.6, 0.8, 0)$

Vector Data Type Applications

Relevance. A quintessential mathematical abstraction.

Applications.

- Statistics.
- Linear algebra.
- Clustering and similarity search.
- Force, velocity, acceleration, momentum, torque.
- …

Vector Data Type: Implementation

```
public class Vector {
    private int N;
    private double[] coords;
```

instance variables

```
    public Vector(double[] a) {
        N = a.length;
        coords = new double[N];
        for (int i = 0; i < N; i++)
            coords[i] = a[i];
    }
```

constructor

```
    public double dot(Vector b) {
        double sum = 0.0;
        for (int i = 0; i < N; i++)
            sum += (coords[i] * b.coords[i]);
        return sum;
    }

    public Vector plus(Vector b) {
        double[] c = new double[N];
        for (int i = 0; i < N; i++)
            c[i] = coords[i] + b.coords[i];
        return new Vector(c);
    }
```

methods

Vector Data Type: Implementation

```
public Vector times(double t) {
   double[] c = new double[N];
   for (int i = 0; i < N; i++)
      c[i] = t * coords[i];
   return new Vector(c);
}

public double magnitude() {
   return Math.sqrt(this.dot(this));
}

public Vector direction() {
   return this.times(1.0 / this.magnitude());
}
...
```

This. The keyword `this` is a reference to the invoking object.

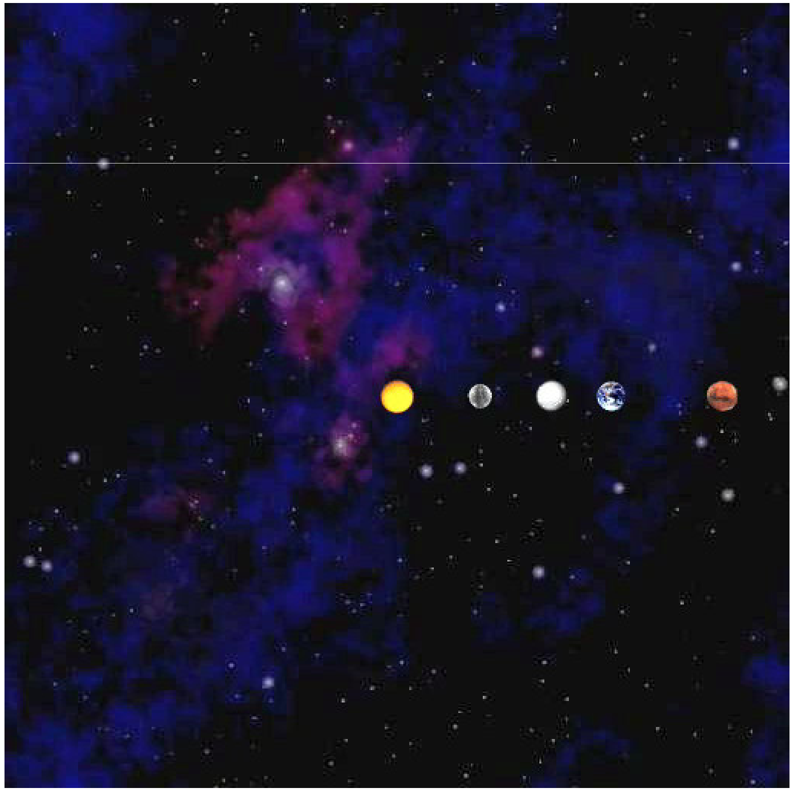
Ex. When you invoke `a.magnitude()`, `this` is an alias for `a`.

N-body Simulation

N-Body Problem

Goal. Determine the motion of N particles, moving under their mutual Newtonian gravitational forces.

Ex. Planets orbit the sun.



N-Body: Applications

Applications to astrophysics.

- Orbits of solar system bodies.
- Stellar dynamics at the galactic center.
- Stellar dynamics in a globular cluster.
- Stellar dynamics during the collision of two galaxies.
- Formation of structure in the universe.
- Dynamics of galaxies during cluster formation.

N-Body Problem

Goal. Determine the motion of N particles, moving under their mutual Newtonian gravitational forces.

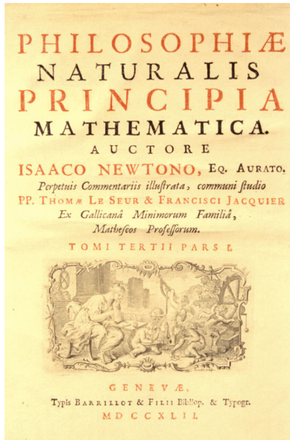
Context. Newton formulated the physical principles in Principia.

$$F = m\,a$$

Newton's second law of motion

$$F = \frac{G\,m_1\,m_2}{r^2}$$

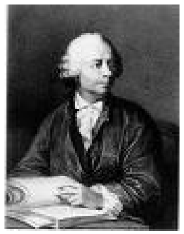
Newton's law of universal gravitation

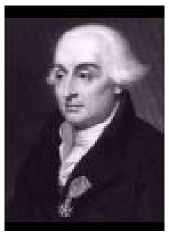

PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.
AUCTORE
ISAACO NEWTONO, EQ. AURATO.
Perpetuis Commentariis illustrata, communi studio
PP. THOMÆ LE SEUR & FRANCISCI JACQUIER
Ex Gallicanâ Minimorum Familiâ,
Matheseos Professorum.
TOMI TERTII PARS I.
GENEVÆ,
Typis BARRILLOT & FILII Bibliop. & Typogr.
MDCCXLII.


Kepler


Bernoulli


Newton


Euler


Lagrange

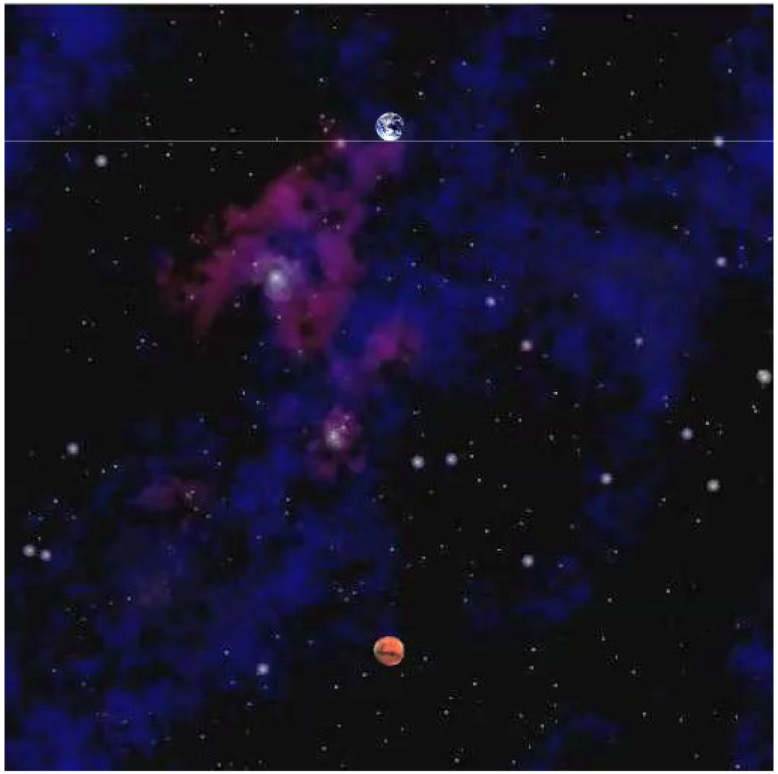

Delaunay


Poincaré

2-Body Problem

2 body problem.

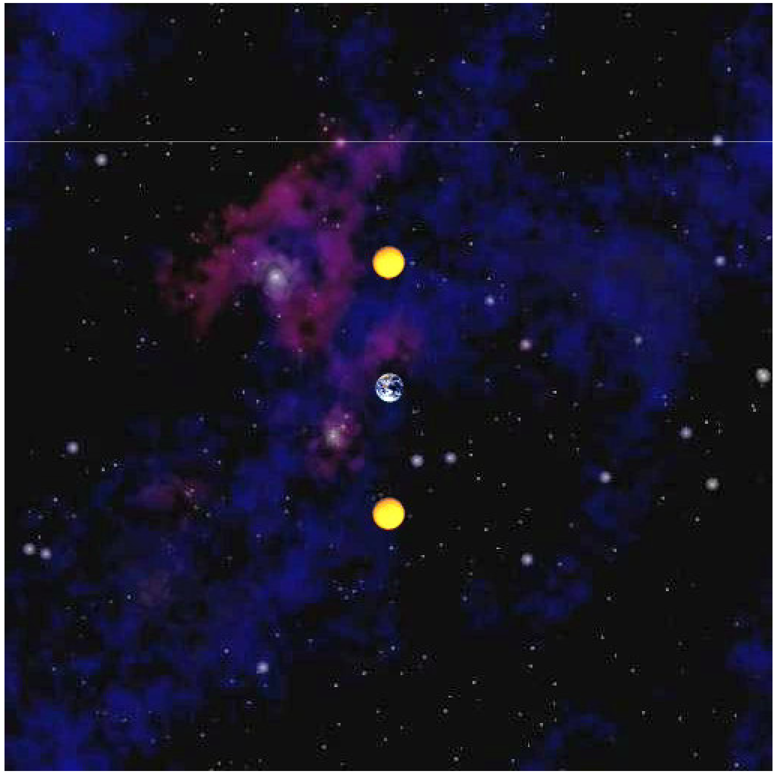
- Can be solved analytically via Kepler's 3rd law.
- Bodies move around a common barycenter (center-of-mass) with elliptical orbits.



3-Body Problem

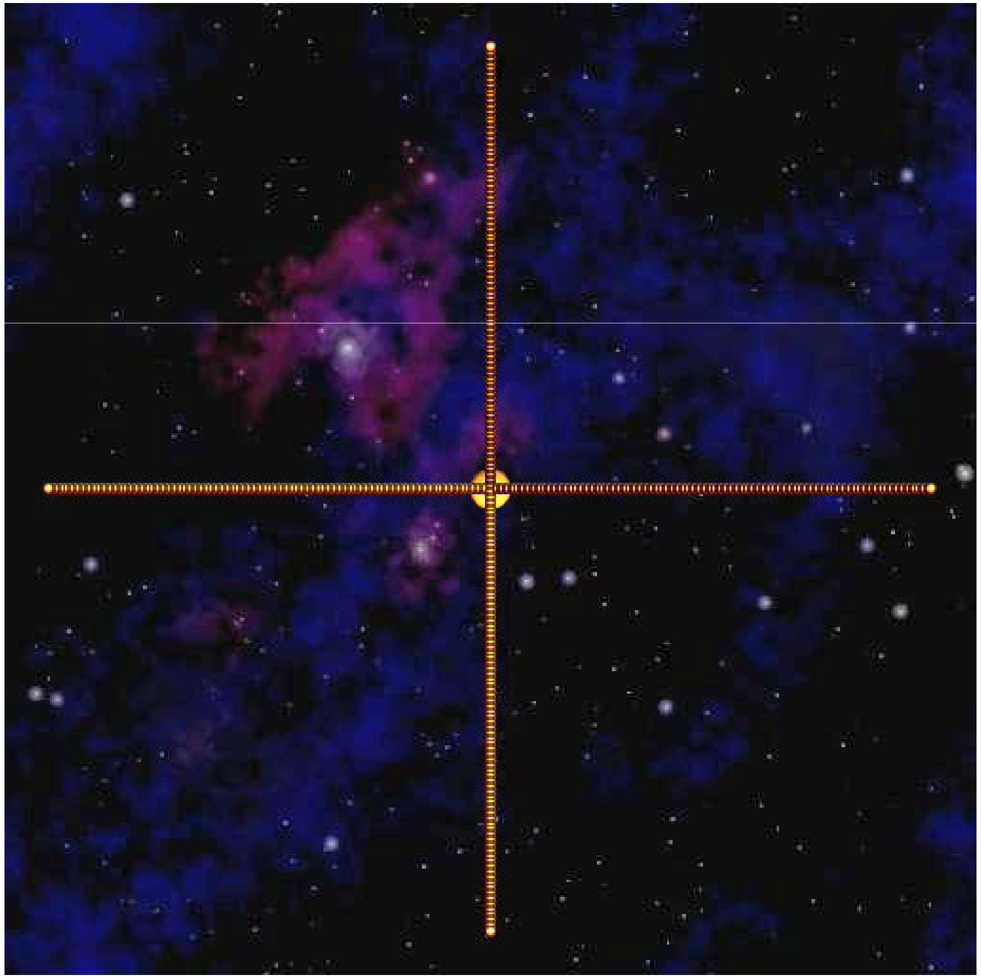
3-body problem. No solution possible in terms of elementary functions; moreover, orbits may not be stable or periodic!

Consequence. Must resort to computational methods.



N-Body Simulation

N-body simulation. The ultimate object-oriented program: simulate the universe.



Body Data Type

Body data type. Represent a particle.

public class Body		
	`Body(Vector r, Vector v, double mass)`	
`void`	`move(Vector f, double dt)`	*apply force* `f`*, move body for* `dt` *seconds*
`void`	`draw()`	*draw the ball*
`Vector`	`forceFrom(Body b)`	*force vector between this body and* `b`

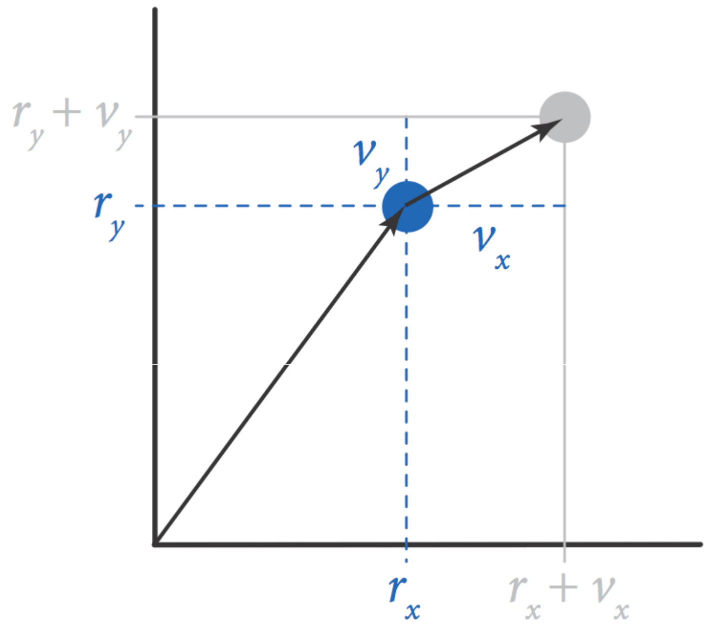
Vector notation. Represent position, velocity, and force using `Vector`.

```
public class Body {
   private Vector r;        // position
   private Vector v;        // velocity
   private double mass;     // mass
```

instance variables

Moving a Body

Moving a body. Assuming no other forces, body moves in straight line.



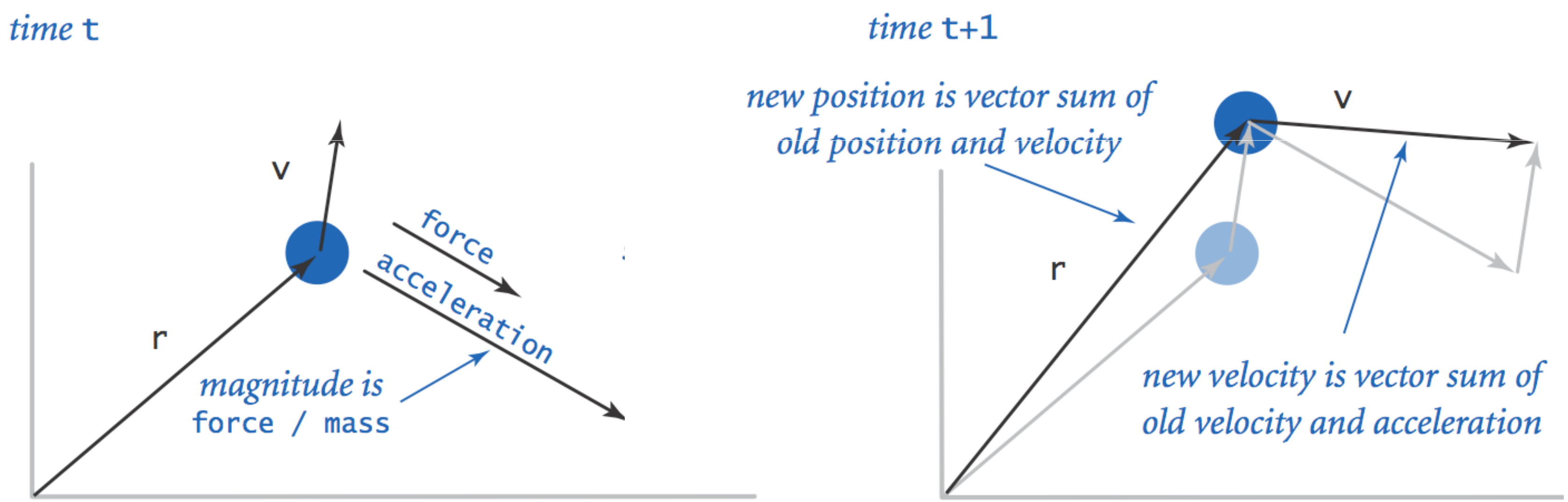

```
r = r.plus(v.times(dt));
```

$$r_x = r_x + dt \cdot v_x$$
$$r_y = r_y + dt \cdot v_y$$

Moving a Body

Moving a body.

- Given external force F, acceleration a = F/m.
- Use acceleration (assume fixed) to compute change in velocity.
- Use velocity to compute change in position.

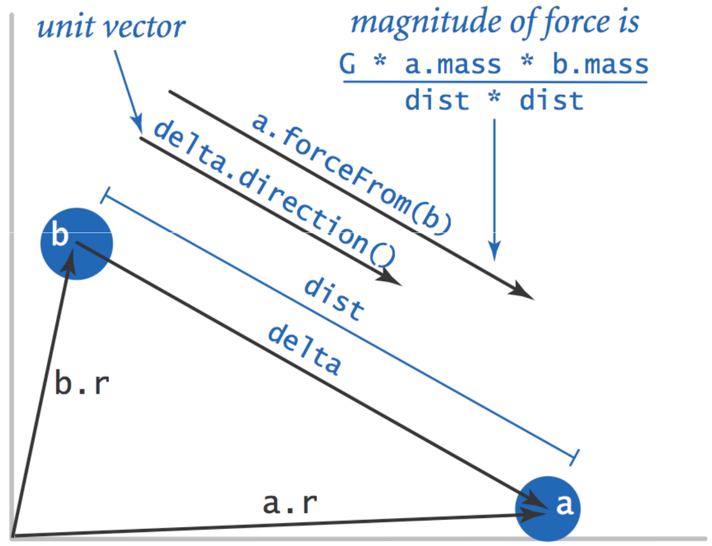



```
Vector a = f.times(1/mass);
v = v.plus(a.times(dt));
r = r.plus(v.times(dt));
```

Force Between Two Bodies

Newton's law of universal gravitation.

- $F = G\, m_1\, m_2 \,/\, r^2$.
- Direction of force is line between two particles.




```
double G = 6.67e-11;
Vector delta = a.r.minus(b.r);
double dist = delta.magnitude();
double F = (G * a.mass * b.mass) / (dist * dist);
Vector force = delta.direction().times(F);
```

Body Data Type: Java Implementation

```
public class Body {
   private Vector r;       // position
   private Vector v;       // velocity
   private double mass;    // mass

   public Body(Vector r, Vector v, double mass) {
      this.r = r;
      this.v = v;
      this.mass = mass;
   }

   public void move(Vector f, double dt) {
      Vector a = f.times(1/mass);
      v = v.plus(a.times(dt));
      r = r.plus(v.times(dt));
   }

   public Vector forceFrom(Body that) {
      double G = 6.67e-11;
      Vector delta = that.r.minus(this.r);
      double dist = delta.magnitude();
      double F = (G * this.mass * that.mass) / (dist * dist);
      return delta.direction().times(F);
   }

   public void draw() {
      StdDraw.setPenRadius(0.025);
      StdDraw.point(r.cartesian(0), r.cartesian(1));
   }
}
```

Universe Data Type

Universe data type. Represent a universe of N particles.

`public class Universe`

	`Universe()`	
`void`	`increaseTime(double dt)`	*simulate the passing of* `dt` *seconds*
`void`	`draw()`	*draw the universe*

```
public static void main(String[] args) {
   Universe newton = new Universe();
   double dt = Double.parseDouble(args[0]);
   while (true) {
      StdDraw.clear();
      newton.increaseTime(dt);
      newton.draw();
      StdDraw.show(10);
   }
}
```

main simulation loop

Universe Data Type

Universe data type. Represent a universe of N particles.

public class Universe		
	Universe()	
void	increaseTime(double dt)	*simulate the passing of* dt *seconds*
void	draw()	*draw the universe*

```
public class Universe {
   private double radius; // radius of universe
   private int N          // number of particles
   private Body[] orbs;   // the bodies
```

instance variables

Data-Driven Design

File format.

```
% more 4body.txt
4
5.0e10
-3.5e10 0.0e00 0.0e00  1.4e03 3.0e28
-1.0e10 0.0e00 0.0e00  1.4e04 3.0e28
 1.0e10 0.0e00 0.0e00 -1.4e04 3.0e28
 3.5e10 0.0e00 0.0e00 -1.4e03 3.0e28
```

N (4), *radius* (5.0e10), *velocity*, *mass*, *position*

Constructor.

```
public Universe() {
   N = StdIn.readInt();
   radius = StdIn.readDouble();
   StdDraw.setXscale(-radius, +radius);
   StdDraw.setYscale(-radius, +radius);

   // read in the N bodies
   orbs = new Body[N];
   for (int i = 0; i < N; i++) {
      double rx   = StdIn.readDouble();
      double ry   = StdIn.readDouble();
      double vx   = StdIn.readDouble();
      double vy   = StdIn.readDouble();
      double mass = StdIn.readDouble();
      double[] position = { rx, ry };
      double[] velocity = { vx, vy };
      Vector r = new Vector(position);
      Vector v = new Vector(velocity);
      orbs[i] = new Body(r, v, mass);
   }
}
```

Principle of Superposition

Principle of superposition. Net gravitational force acting on a body is the sum of the individual forces.

```
// compute the forces
for (int i = 0; i < N; i++) {
   for (int j = 0; j < N; j++) {
      if (i != j) {
         f[i] = f[i].plus(orbs[i].forceFrom(orbs[j]));
      }
   }
}
```

$$F_i = \sum_{i \neq j} \frac{G m_i m_j}{|r_i - r_j|^2}$$

Universe Data Type: Java Implementation

```
public class Universe {
   private final double radius;      // radius of universe
   private final int N;              // number of bodies
   private final Body[] orbs;        // array of N bodies

   public Universe() { /* see previous slide */ }

   public void increaseTime(double dt) {
      Vector[] f = new Vector[N];
      for (int i = 0; i < N; i++)
          f[i] = new Vector(new double[2]);
      for (int i = 0; i < N; i++)
          for (int j = 0; j < N; j++)
              if (i != j)
                  f[i] = f[i].plus(orbs[j].forceTo(orbs[i]));

      for (int i = 0; i < N; i++)
         orbs[i].move(f[i], dt);
   }

   public void draw() {
      for (int i = 0; i < N; i++)
         orbs[i].draw();
   }

   public static void main(String[] args) { /* see previous slide */ }

}
```

create universe

update the bodies

draw the bodies

simulate the universe

Odds and Ends

Accuracy. How small to make `dt`? How to avoid floating-point inaccuracies from accumulating?

Efficiency.

- Direct sum: takes time proportional to N^2
  $\Rightarrow$ not usable for large N.
- Appel / Barnes-Hut: takes time proportional to N log N time
- $\Rightarrow$ can simulate large universes.

3D universe. Use a 3D vector (only drawing code changes!).

Collisions.

- Model inelastic collisions.
- Use a softening parameter to avoid collisions.

$$F_i = \sum_{i \neq j} \frac{G\, m_i\, m_j}{|r_i - r_j|^2 + \varepsilon^2}$$

Extra Slides

N-Body Simulation

1. Setup initial distribution of particles.
   - Need accurate data and model of mass distribution.

2. Compute forces between particles.
   - Direct sum: N^2.
   - Appel / Barnes-Hut" N log N.

$$\mathbf{F}_i = \sum_{i \neq j} \frac{G m_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|^2 + \epsilon^2}$$

ε = softening parameter
eliminates binary stars with r <
ε
hard binaries can be important
source of energy

3. Evolve particles using ODE solver.
   - Leapfrog method balances efficiency and accuracy.
   - Truncation error = O(dt^2).
   - Symplectic.

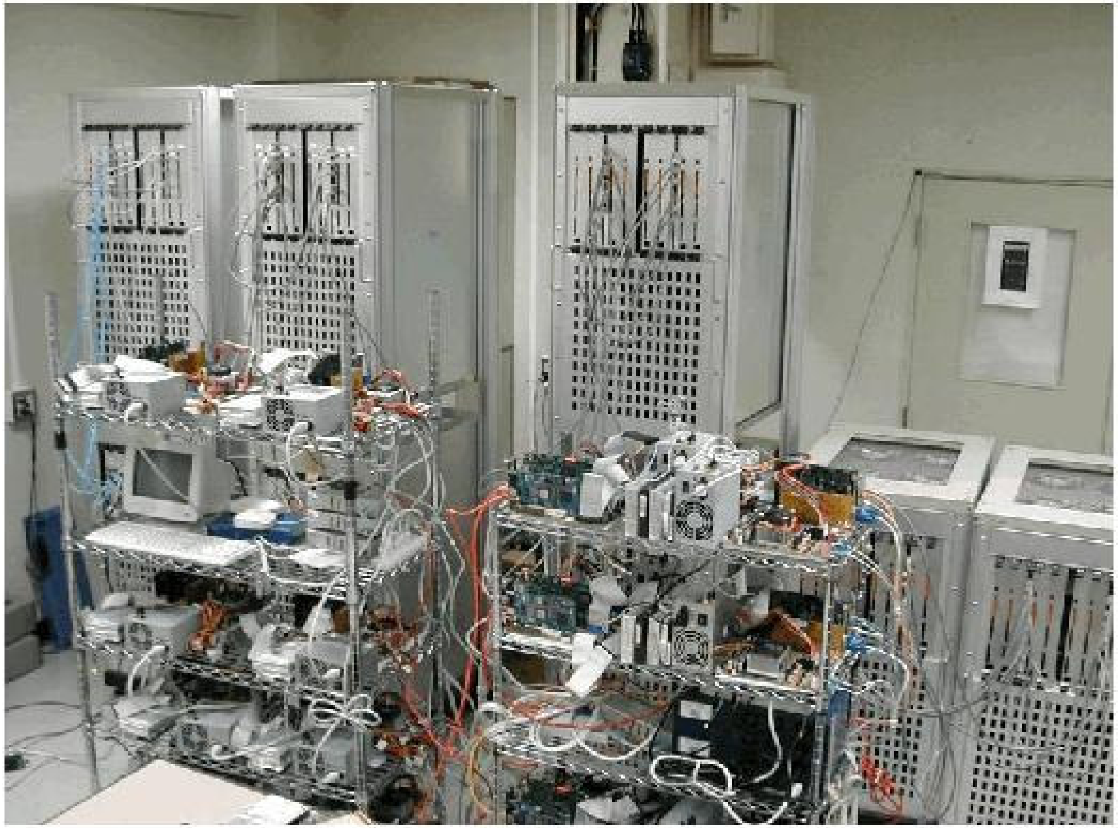
$$\frac{d\mathbf{X_i}}{dt} = \mathbf{V_i}$$

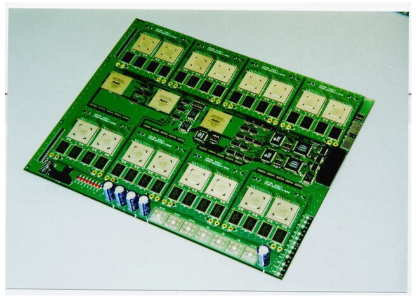
$$m_i \frac{d\mathbf{V_i}}{dt} = \mathbf{F_i}$$

4. Display and analyze results.

Solving the force problem with hardware.

GRAPE-6. Special purpose hardware to compute force.

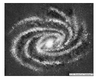




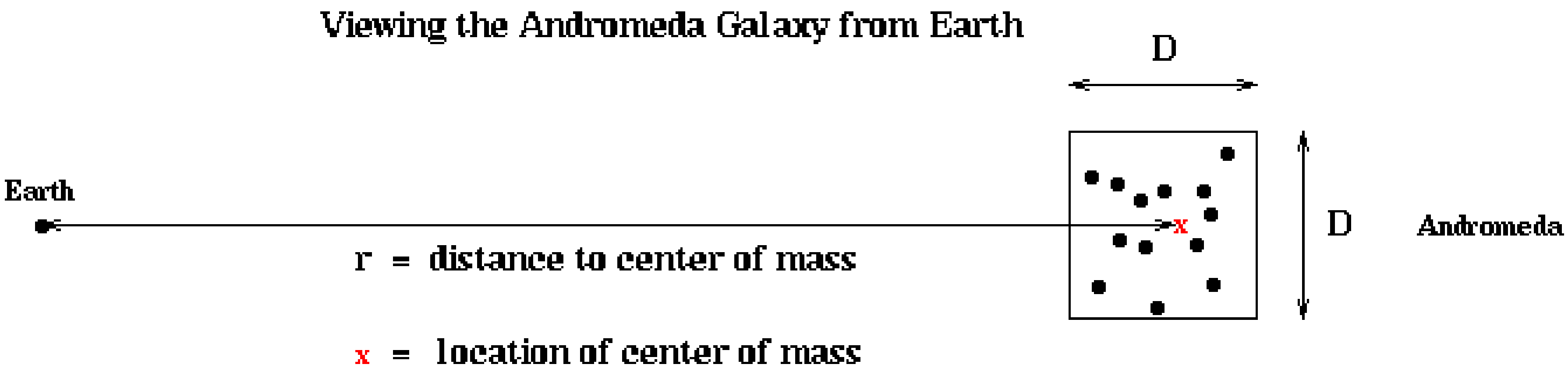
Jun Makino, U. Tokyo

Do we really need to compute force from every star for distant objects

Solving the force problem with software – tree codes



Distance = 25 times size



Organize particles into a tree. In Barnes-Hut algorithm, use a quadtree in 2D