Preface

It is true that there exist many books dedicated to linear algebra and somewhat fewer to multilinear algebra, written in several languages, and perhaps one can think that no more books are needed. However, it is also true that in algebra many new results are continuously appearing, different points of view can be used to see the mathematical objects and their associated structures, and different orientations can be selected to present the material, and all of them deserve publication.

Under the leadership of Juan Ramón Ruíz-Tolosa, Professor of multilinear algebra, and the collaboration of Enrique Castillo, Professor of applied mathematics, both teaching at an engineering school in Santander, a tensor textbook has been born, written from a practical point of view and free from the esoteric language typical of treatises written by algebraists, who are not interested in descending to numerical details. The balance between following this line and keeping the rigor of classical theoretical treatises has been maintained throughout this book.

The book assumes a certain knowledge of linear algebra, and is intended as a textbook for graduate and postgraduate students and also as a consultation book. It is addressed to mathematicians, physicists, engineers, and applied scientists with a practical orientation who are looking for powerful tensor tools to solve their problems.

The book covers an existing chasm between the classic theory of tensors and the possibility of solving tensor problems with a computer. In fact, the computational algebra is formulated in matrix form to facilitate its implementation on computers.

The book includes 197 examples and end-of-chapter exercises, which makes it specially suitable as a textbook for tensor courses. This material combines classic matrix techniques together with novel methods and in many cases the questions and problems are solved using different methods. They confirm the applied orientation of the book.

A computer package, written in Mathematica, accompanies this book, available on: http://personales.unican.es/castie/tensors. In it, most of the novel methods developed in the book have been implemented. We note that existing general computer software packages (Mathematica, Mathlab, etc.) for tensors are very poor, up to the point that some problems cannot be dealt

with using computers because of the lack of computer programs to perform these operations.

The main contributions of the book are:

1. The book employs a new technique that permits one to extend (stretch) the tensors, as one-column matrices, solve on these matrices the desired problems, and recover the initial format of the tensor (condensation). This technique, applied in all chapters, is described and used to solve matrix equations in Chapter 1.
2. An important criterion is established in Chapter 2 for all the components of a tensor to have a given ordering, by the definition of a unique canonical tensor basis. This permits the mentioned technique to be applied.
3. In Chapter 3, factors are illustrated that have led to an important confusion in tensor books due to inadequate notation of tensors or tensor operations.
4. In addition to dealing with the classical topics of tensor books, new tensor concepts are introduced, such as the rotation of tensors, the transposer tensor, the eigentensors, and the permutation tensor structure, in Chapter 5.
5. A very detailed study of generalized Kronecker deltas is presented in Chapter 8.
6. Chapter 10 is devoted to mixed exterior algebras, analyzing the problem of change-of-basis and the exterior product of this kind of tensors.
7. In Chapter 11 the rules for the "Euclidean contraction" are given in detail. This chapter ends by introducing the geometric concepts to tensors.
8. The orientation and polar tensors in Euclidean spaces are dealt with in Chapter 12.
9. In Chapter 13 the Gram matrices $G(r)$ are established to connect exterior tensors.
10. Chapter 14 is devoted to Euclidean tensors in $E^n(\mathbb{R})$, affine geometric tensors (homographies), and some important tensors in physics and mechanics, such as the stress and strain tensors, the elastic tensor and the inertial moment tensor. It is shown how tensors allow one to solve very interesting practical problems.

In summary, the book is not a standard book on tensors because of its orientation, the many novel contributions included in it, the careful notation and the stretching–condensing techniques used for most of the transformations used in the book. We hope that our readers enjoy reading this book, discover a new world, and acquire stimulating ideas for their applications and new contributions and research.

The authors want to thank an anonimous French referee for the careful reading of the initial manuscript, and to Jeffrey Boys for the copyediting of the final manuscript.

Santander, September 30, 2004

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