SOLUTION TO PRACTICE PROBLEM

The following data are given:

	Inputs Needed per Unit of Output		
Purchased from:	**Goods**	**Services**	**External Demand**
Goods	.2	.4	20
Services	.5	.3	30

The Leontief input–output model is $\mathbf{x} = C\mathbf{x} + \mathbf{d}$, where

$$C = \begin{bmatrix} .2 & .4 \\ .5 & .3 \end{bmatrix}, \qquad \mathbf{d} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$$

2.7 APPLICATIONS TO COMPUTER GRAPHICS

Computer graphics are images displayed or animated on a computer screen. Applications of computer graphics are widespread and growing rapidly. For instance, computer-aided design (CAD) is an integral part of many engineering processes, such as the aircraft design process described in the chapter introduction. The entertainment industry has made the most spectacular use of computer graphics—from the special effects in *Amazing Spider-Man 2* to PlayStation 4 and Xbox One.

Most interactive computer software for business and industry makes use of computer graphics in the screen displays and for other functions, such as graphical display of data, desktop publishing, and slide production for commercial and educational presentations. Consequently, anyone studying a computer language invariably spends time learning how to use at least two-dimensional (2D) graphics.

This section examines some of the basic mathematics used to manipulate and display graphical images such as a wire-frame model of an airplane. Such an image (or picture) consists of a number of points, connecting lines or curves, and information about how to fill in closed regions bounded by the lines and curves. Often, curved lines are approximated by short straight-line segments, and a figure is defined mathematically by a list of points.

Among the simplest 2D graphics symbols are letters used for labels on the screen. Some letters are stored as wire-frame objects; others that have curved portions are stored with additional mathematical formulas for the curves.

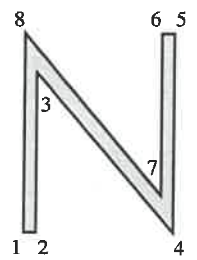


FIGURE 1 Regular *N*.

EXAMPLE 1 The capital letter N in Figure 1 is determined by eight points, or *vertices*. The coordinates of the points can be stored in a data matrix, D.

$$\begin{array}{r} \\ \\ x\text{-coordinate} \\ y\text{-coordinate} \end{array} \begin{array}{c} \text{Vertex:} \\ \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & .5 & .5 & 6 & 6 & 5.5 & 5.5 & 0 \\ 0 & 0 & 6.42 & 0 & 8 & 8 & 1.58 & 8 \end{bmatrix} \end{array} = D$$

In addition to D, it is necessary to specify which vertices are connected by lines, but we omit this detail. ■

The main reason graphical objects are described by collections of straight-line segments is that the standard transformations in computer graphics map line segments onto other line segments. (For instance, see Exercise 27 in Section 1.8.) Once the vertices

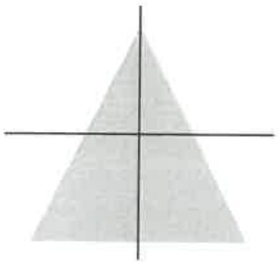
EXAMPLE 5 Any linear transformation on $\mathbb{R}^2$ is represented with respect to homogeneous coordinates by a partitioned matrix of the form $\begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$, where A is a 2×2 matrix. Typical examples are

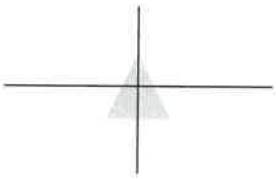
$$\begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} s & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

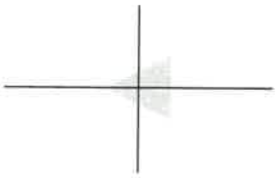
Counterclockwise rotation about the origin, angle φ — Reflection through $y = x$ — Scale x by s and y by t ■

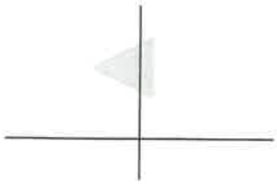
Composite Transformations

The movement of a figure on a computer screen often requires two or more basic transformations. The composition of such transformations corresponds to matrix multiplication when homogeneous coordinates are used.


Original Figure


After Scaling


After Rotating


After Translating

EXAMPLE 6 Find the 3×3 matrix that corresponds to the composite transformation of a scaling by .3, a rotation of 90° about the origin, and finally a translation that adds $(-.5, 2)$ to each point of a figure.

SOLUTION If $\varphi = \pi/2$, then $\sin\varphi = 1$ and $\cos\varphi = 0$. From Examples 4 and 5, we have

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \xrightarrow{\text{Scale}} \begin{bmatrix} .3 & 0 & 0 \\ 0 & .3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\xrightarrow{\text{Rotate}} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} .3 & 0 & 0 \\ 0 & .3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\xrightarrow{\text{Translate}} \begin{bmatrix} 1 & 0 & -.5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} .3 & 0 & 0 \\ 0 & .3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

The matrix for the composite transformation is

$$\begin{bmatrix} 1 & 0 & -.5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} .3 & 0 & 0 \\ 0 & .3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -.5 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} .3 & 0 & 0 \\ 0 & .3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -.3 & -.5 \\ .3 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ ■

3D Computer Graphics

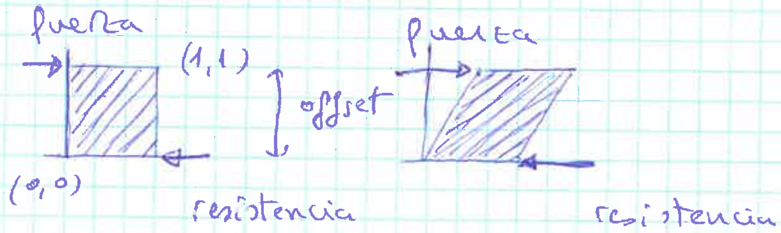
Some of the newest and most exciting work in computer graphics is connected with molecular modeling. With 3D (three-dimensional) graphics, a biologist can examine a simulated protein molecule and search for active sites that might accept a drug molecule. The biologist can rotate and translate an experimental drug and attempt to attach it to the protein. This ability to *visualize* potential chemical reactions is vital to modern drug and cancer research. In fact, advances in drug design depend to some extent upon progress

Tema . Transformaciones afines.

Antes/estudiamos/algunas transformaciones lineales.
revisamos

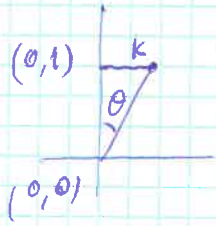
Shear transformation o cizalladura

shear force



$A = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ es la matriz estandar asociada en este caso.

$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} x + ky \\ y \end{bmatrix}$



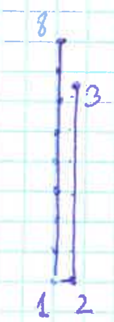
$\tan\theta = \frac{k}{1} = k$

es una cizalladura horizontal con coeficiente $k > 0$.

$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} k \\ 1 \end{pmatrix}$

LAY 2.7 $\quad$ Linear Algebra and its Applications
5ª edición Pearson 2016

Ejemplos 1, 2, 3 páginas 140-141

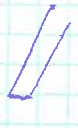


$$\begin{array}{cccc} \begin{bmatrix} 0 & 0.50 & 0.50 & 0 \\ 0 & 0 & 6.42 & 8 \end{bmatrix} \\ \begin{matrix} 1 & \quad 2 & \quad 3 & \quad 8 \end{matrix} \end{array}$$

1ª transformación $\begin{bmatrix} 1 & 0.25 \\ 0 & 1.0 \end{bmatrix}$ inclina (slanted)
figura 2

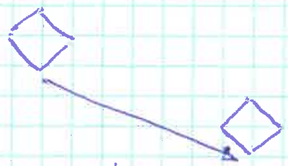
2) Contracción no uniforme $k = 0.75$ en eje x

$\begin{bmatrix} 0.75 & 0 \\ 0 & 1 \end{bmatrix}$ estrecha según eje x

1º  2º 

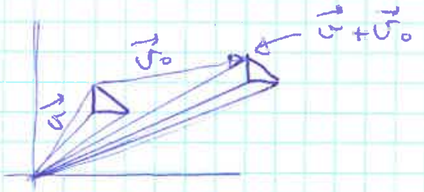
Ninguna de las dos transformaciones es un movimiento rígido, pues en ningún caso la matriz es ortogonal.
Además resulta obvio que las transformaciones no conservan la norma de los vectores.

<u>Traslación</u> La traslación en $\mathbb{R}^n$ es una aplicación que no es lineal, pero sí es un movimiento rígido, pues conserva la forma de los objetos.



vector de traslación $\vec{v}_0$

$$f(\vec{v}) = \vec{v} + \vec{v}_0$$



$$f(x,y) = (x,y) + (x_0, y_0)$$

$$f(x,y) = (x,y) + (5,1)$$

en el segundo dibujo

Esta transformación se puede expresar matricialmente mediante <u>coordenadas homogéneas</u>

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

no se puede reducir a una expresión $A\vec{x} = \vec{x}'$

pero veremos que con las "coordenadas homogéneas" sí podremos.

$$\left[\begin{array}{cc|c} 1 & 0 & v_{0x} \\ 0 & 1 & v_{0y} \\ \hline 0 & 0 & 1 \end{array}\right] \qquad \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \\ \hline 0 & 0 & 1 \end{array}\right]$$

Matriz Transformación afín estándar

En $\mathbb{R}^2$ las coordenadas homogéneas tienen la forma $(x_1, x_2, 1)$
" $\mathbb{R}^3$ " " " " " " $(x_1, x_2, x_3, 1)$

Sólo analizaremos transformaciones afines en $\mathbb{R}^2$

Para averiguar el transformado de $(2,2)$ efectuaremos:

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \\ \hline 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} 2 \\ 2 \\ \hline 1 \end{array}\right] = \left[\begin{array}{c} 2+0+5 \\ 0+2+1 \\ \hline 0+0+1 \end{array}\right] = \left[\begin{array}{c} 7 \\ 3 \\ \hline 1 \end{array}\right]$$

El transformado de $(2,2)$ mediante esta traslación es $(7,3)$

$$T_{Af} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{I} & v_{01} \\ & & v_{02} \\ \hline 0 & 0 & 1 \end{array}\right]$$ es la matriz de la traslación dada.

T_{Af} se refiere a transformación afín.

¿Es posible combinar matricialmente traslaciones con transformaciones lineales? Sí, porque éstas admiten expresión en coordenadas homogéneas.

Ejemplo en $\mathbb{R}^2$, giro de $30°$

$$T_{af} = \left(\begin{array}{cc|c} \cos 30 & -\operatorname{sen} 30 & 0 \\ \operatorname{sen} 30 & \cos 30 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

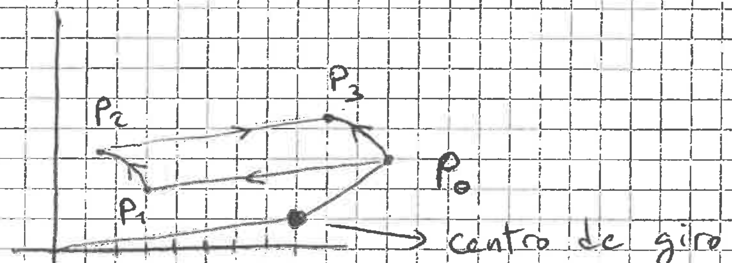
Ejemplo en $\mathbb{R}^2$, simetría respecto de recta $x+2y=0$

$$T_{af} = \left(\begin{array}{cc|c} -3/5 & -4/5 & 0 \\ -4/5 & -3/5 & 0 \\ \hline 0 & 0 & 1 \end{array}\right)$$

Hemos (expresado) (representado) el giro y la simetría en el formato de las coordenadas homogéneas.

a) Matriz de la transformación afín del giro de 30° de centro (8,1)

b) Calcular el transformado del punto (11,3)

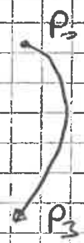


P_0 (11,3) original

P_1 (3,2) trasladado

P_2 girado

P_3 trasladado de vuelta



giro efectivo por "composición"

a)

$$T_{af} = \left(\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & 1 \\ \hline 0 & 0 & 1 \end{array}\right) \left(\begin{array}{cc|c} \cos 30 & -\operatorname{sen} 30 & 0 \\ \operatorname{sen} 30 & \cos 30 & 0 \\ \hline 0 & 0 & 1 \end{array}\right) \left(\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & -1 \\ \hline 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{cc|c} \sqrt{3}/2 & -1/2 & 1.57 \\ 1/2 & \sqrt{3}/2 & 3.87 \\ \hline 0 & 0 & 1 \end{array}\right)$$

b)

$$T_{af} \begin{bmatrix} 11 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 9.598 \\ 4.232 \\ 1 \end{bmatrix}$$

el transformado de (11, 3) es (9.598, 4.232)

4