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Validated Evaluation of Special Mathematical Functions

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Abstract

Special functions are pervasive in all fields of science. Because of their importance, several books and websites and a large collection of papers have been devoted to their use and computation, the most well-known being the Abramowitz and Stegun handbook [1] and its successor [3]. Due to their regular appearance in mathematical models of scientific proiblems, special functions are also pervasive in numerical computations. Consequently, there is no shortage of numerical routines for evaluating many of the special functions in widely used mathematical software packages, systems and libraries such as Maple, Mathematica, MATLAB, IMSL, CERN and NAG. But up to this date, no environment offers routines for the provable correct evaluation of these special functions. A reliable, or validated, routine must do more than just compute an accurate approximation. In addition to this, it must provide a guaranteed bound on the error of the computed numerical value. In the case of a real-valued function, this error bound determines an interval within which the exact function value is guaranteed to lie. Our goal is to present our approach to the development of reliable algorithms for special functions. It is based on the observation that functions that are termed special have explicitly known and simple infinite series and/or continued fraction representations. Together the convergence domains of these representations often cover the full area of interest for users of these functions. Hence they lend themselves easily for a variable precision implementation. While the use of series to approximate a function in numeric software is well established, that of continued fractions is far from traditional [4].

References

[1] M. Abramowitz and I.A. Stegun. *Handbook of mathematical functions with formulas, graphs and mathematical tables.* U.S. Government Printing Office, NBS, Washington, D. C., 1964.

[2] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark. NIST Digital Library of Mathematical Functions. http://dlmf.nist.gov/, online companion to [3] with errata list and other updates.

[3] F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark. *NIST Handbook of Mathematical Functions.* Cambridge University Press, New York, NY, 2010. Print companion to [2].

[4] A. Cuyt, V. Brevik Petersen, B. Verdonk, H. Waadeland, and W.B. Jones. *Handbook of Continued Fractions for Special Functions.* Springer, Berlin, 2008.