

Triangulations of point sets  
— Open problems (more will be added) —

## Dimension 2

1. Is there a number  $N$  such that every set of points in the plane which has a non-regular triangulation, contains a subset of at most  $N$  points which already have a non-regular triangulation? (The conjecture is “Yes,  $N = 8$ ”).
2. Is the graph of triangulations of every 2-dimensional point set  $(n - 3)$ -connected? (We at least know that all triangulations have at least  $n - 3$  flips.
3. Let  $A$  be a set of  $n$  points in the plane, and let us look at the triangulations of  $A$  **that use all the vertices**.

- Prove there are at most  $8^n$  of them. The best known upper bound is  $59^n$ .

Note (or exercise): if you prove an upper bound of  $c^n$  for the triangulations that use all the vertices, this implies an upper bound of  $(c + 1)^n$  for the total number of triangulations of the point set.

- Prove that if  $A$  is in general position (no 3 points collinear) then there are at least  $\sqrt{12}^n$  triangulations, modulo a polynomial factor. The best known lower bound is something like  $2.1^n$ .

## Dimension 3

1. Prove that the graph of triangulations of every point set in dimension 3 is connected, or find an example where it is not.

This is open even if you assume that your points are in convex and general position. That is, they are the vertices of a polytope and no four of them lie in a hyperplane.

2. If you succeeded in proving the above for points in convex position, try to prove that the graph is  $(n - 4)$ -connected. We at least know that every triangulation has  $n - 4$  flips (in copnvox position).