FDTD Analysis of Magnetized Ferrites: Application to the Calculation of Dispersion Characteristics of Ferrite-Loaded Waveguides

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Abstract— **The finite-difference time-domain (FDTD) method is extended to include magnetized ferrites. The treatment of the ferrite material is based on the equation of motion of the magnetization vector. Magnetic losses are also included in the equation of motion by means of Gilbert's approximation of the phenomenological Landau-Lifschitz damping term. The discretization scheme is based on central finite-differences and linear interpolation. This scheme allows the fully explicit nature of the FDTD method to be maintained. This extension of the FDTD method to magnetized ferrites is applied to the full-wave analysis of ferrite-loaded waveguides. The dispersion curves are calculated by using a recently proposed 2D-FDTD formulation for dispersion analysis which has been adapted to the present problem. The results for both the phase and attenuation constants of various transversely and longitudinally magnetized ferrite-loaded waveguides are compared with the exact values and with those obtained by means of Schelkunoff's method.**

I. Introduction

FERRITES are basic materials in the development of non-reciprocal and control devices such as circulators and phase shifters due to the fact that the magnetic constitutive characteristics of ferrites can be controlled by the application of a dc magnetic bias field. However, the analysis of structures with magnetized ferrites is normally very complex and in most cases does not admit an analytical solution. Consequently, the development of new numerical techniques that are capable of analyzing these structures is of great interest.

The finite-difference time-domain (FDTD) method is now a well-established numerical technique for the analysis of a great variety of electromagnetic problems. It is based on the direct discretization of Maxwell's time-dependent curl equations by using central finite-differences [1]. The FDTD method has been gaining in popularity because it has several advantages. For example, it leads to an explicit scheme (avoiding matrix inversion); the time domain solution is obtained directly; and a broadband frequency response can be obtained from a single computer simulation. The reported applications of this method range from radiation and scattering problems [2] to others involving guided waves [3], [4] or eigenvalue computation [5].

The FDTD method was initially proposed to handle isotropic, non-dispersive materials [1]. Later extensions have made it possible to apply the method to anisotropic materials, which are characterized by diagonal tensors [5], and also to dispersive materials [6]–[8]. Recently the FDTD method has been extended to include more complex media such as magnetized ferrites [9]–[11] and magnetized plasmas [12]. These materials, in addition to their highly dispersive nature, are characterized by tensorial constitutive parameters with nonzero off-diagonal elements (tensorial permeability in the case of ferrites and permittivity in that of plasmas). Magnetized ferrites have been treated by means of the equation of motion of the magnetization vector (differential approach) [9]–[11], while magnetized plasmas have been handled by working with the permittivity tensor and applying recursive convolution [12]. Based on the duality of these media, both approaches should be valid for analyzing both media. A formulation based on the spatial network method has been presented in [13] that allows ferrites to be analyzed in the time-domain and is also based on the equation of motion of the magnetization vector. However, the FDTD method is simpler and more efficient than the spatial network method [14].

This paper provides a detailed presentation of the extended FDTD formulation for the treatment of saturated magnetized ferrites, which was briefly introduced in [11]. Furthermore, the new algorithm is applied to the full-wave analysis of waveguides containing ferrites by adapting a recently proposed 2D-FDTD formulation to the present problem. Because of the practical relevance of the distinction, two different cases are considered according to the relative angle between the direction of the dc magnetic field and the wave propagation: the transverse and the longitudinal magnetization cases. The results for both the phase and attenuation constants are compared with the exact values and with those obtained by Schelkunoff's method [15]. For a given phase constant, the frequencies and quality factors (the latter should be calculated to obtain the corresponding attenuation constants) are computed by using Prony's method [16].

II. Formulation

Maxwell's time-dependent curl equations can be expressed as

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \tag{1}$$

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$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\epsilon_r \epsilon_0} \nabla \times \vec{H} \tag{2}$$

where $\vec{E}$ is the electric field, $\vec{H}$ the magnetic field, $\vec{B}$ the magnetic flux density, ϵ_0 is the permittivity of free-space, and ϵ_r the dielectric constant. An electric constitutive equation of the form $\vec{D} = \epsilon_r \epsilon_0 \vec{E}$ has been assumed in (2).

In addition to (1) and (2), three more scalar equations—the magnetic constitutive equations—must be taken into account. These equations describe the interaction of the electromagnetic fields with the ferrite from a macroscopic point of view. It is assumed that the ferrite is saturated by a dc magnetic field applied in the z-direction, $\vec{H}_i = H_i \vec{a}_z$. The interaction of the magnetic field with the ferrite can be described by the equation of motion of the magnetization vector with Gilbert's approximation of the Landau-Lifschitz damping term [17]. Under the small signal approximation, the equation of motion can be written in terms of $\vec{B}$ and $\vec{H}$, and in scalar form, as

$$\frac{\partial B_x}{\partial t} - \mu_0 \frac{\partial H_x}{\partial t} = -\gamma \mu_0 (H_i B_y - (M_s + H_i)\mu_0 H_y) - \alpha \left(\frac{\partial B_y}{\partial t} - \mu_0 \frac{\partial H_y}{\partial t} \right) \tag{3}$$

$$\frac{\partial B_y}{\partial t} - \mu_0 \frac{\partial H_y}{\partial t} = -\gamma \mu_0 ((M_s + H_i)\mu_0 H_x - H_i B_x) + \alpha \left(\frac{\partial B_x}{\partial t} - \mu_0 \frac{\partial H_x}{\partial t} \right) \tag{4}$$

$$B_z = \mu_0 H_z \tag{5}$$

where γ is the gyromagnetic ratio, α the damping constant, μ_0 the permeability of free-space, and M_s is the saturation magnetization.

An essential assumption in the derivation of the equation of motion is that the ferrite is infinite. In actual devices, the ferrite sample is obviously finite. As a consequence, demagnetizing effects appear, hence the dc magnetic field inside the ferrite sample, H_i is always less than (or equal to for some particular sample geometries) the applied dc magnetic field, H_o. Only for some simple ferrite sample geometries can the demagnetizing factors be calculated analytically [17]. The evaluation of H_i in actual complex samples is a problem that is not considered in this paper, where it is assumed that the value of H_i is known.

The dispersive and anisotropic nature of the magnetized ferrite is modeled in the time domain by equations (3)–(5). This is analogous to modelling by means of the Polder permeability tensor in frequency domain. In fact, this tensor can easily be obtained from (3)–(5) by assuming a time dependence of the form $e^{j\omega t}$ for the fields vectors.

Equations (3)–(5), which are the required magnetic constitutive relations, together with (1) and (2) form a system of coupled differential equations. To simulate the electromagnetic wave propagation inside a ferrite material by a finite-difference model, these equations must be discretized by means of a suitable scheme. This discretization provides a system of difference (algebraic) equations that replace the original differential problem.

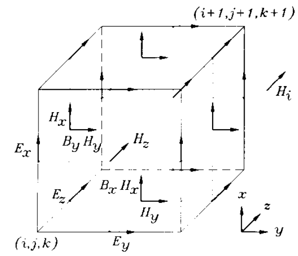

Fig. 1. Three-dimensional extended Yee mesh for the analysis of magnetized ferrites with dc magnetic field applied in the z-direction.

A. Discretization

Since the dc magnetic field is applied in the z-direction, the H_x and H_y components of the magnetic field are coupled. Hence, these two components must be discretized at the same points of the space and at the same instant of time. Furthermore, the $\vec{H}$ field and the $\vec{B}$ field must be discretized at the same instant of time. Following Yee's notation [1], any function of space and time can be discretized as $Fn(i,j,k) = F(i\Delta x, j\Delta y, k\Delta t, n\Delta t) = F(x,y,z,t)$, where Δx, Δy, and Δz are the space increments in the x, y and z coordinate directions; Δt is the time step; and i, j, k and n are integers. Taking these considerations into account, Yee's unit cell [1] is modified as shown in Fig. 1.

Equation (5) is directly incorporated into equation (1). The resulting equations (1) and (2) are discretized as in the isotropic case [1]. Equations (3) and (4) are discretized in time by using central finite-differences and linear interpolation. After (3) and (4) have been discretized, they are decoupled (by solving for $H_x^{n+1/2}$ and $H_y^{n+1/2}$) and we obtain

$$H_x^{n+1/2} = f_0 H_x^{n-1/2} + f_1 B_x^{n+1/2} + f_2 B_x^{n-1/2} + f_3 B_y^{n+1/2} + f_4 B_y^{n-1/2} + f_5 H_y^{n-1/2} \tag{6}$$

$$H_y^{n+1/2} = f_0 H_y^{n-1/2} + f_1 B_y^{n+1/2} + f_2 B_y^{n-1/2} - f_3 B_x^{n+1/2} - f_4 B_x^{n-1/2} - f_5 H_x^{n-1/2} \tag{7}$$

Expressions for evaluating the coefficients f_i (i = 0,..,5) are given in the appendix.

After this discretization in time, the field components in equations (6) and (7) are still continuous functions of the space. Hence, these equations are valid for any number of space dimensions and their evaluation at the required mesh points is sufficient to obtain the H_x and H_y components of the magnetic field at the instant $t = (n + 1/2)\Delta t$.

B. FDTD Algorithm for Magnetized Ferrites

The new FDTD algorithm for magnetized ferrites has the following steps in each time iteration:

1) $B_x^{n+1/2}$, $B_y^{n+1/2}$, and $H_z^{n+1/2}$ are calculated by using the difference form of (1).
2) $H_x^{n+1/2}$ and $H_y^{n+1/2}$ are calculated by using (6) and (7), respectively, where $H_x^{n-1/2}$, $H_y^{n-1/2}$, $B_x^{n-1/2}$ and $B_y^{n-1/2}$ are obtained from the previous iteration, and $B_x^{n+1/2}$ and $B_y^{n+1/2}$ are obtained from the step 1. As

can be seen in Fig. 1, (6) and (7) are discretized at mesh points where both H_x and H_y are available, but only one component of the magnetic flux density (B_y or B_x) is known. The unknown component is calculated by using linear interpolation. For example, at the point $(i, j+1/2, k+1/2)$ (see Fig. 1), B_y is calculated from

$$\begin{aligned} B_y(i,j+1/2,k+1/2) =& \frac{1}{4}(B_y(i+1/2,j,k+1/2) \\ &+ B_y(i+1/2,j+1,k+1/2) \\ &+ B_y(i-1/2,j,k+1/2) \\ &+ B_y(i-1/2,j+1,k+1/2)) \end{aligned} \quad (8)$$

3) E_x^{n+1}, E_y^{n+1}, and E_z^{n+1} are calculated by using the difference form of (2).

These three steps are repeated in each time iteration in order to obtain the time domain electromagnetic response in the ferrite material.

The unit cell shown in Fig. 1, can be simplified by removing $H_x(i+1/2, j, k+1/2)$and $H_y(i, j+1/2, k+1/2)$, which leads to a scheme with a unit cell of eight field components. In order to carry out step 2 of the algorithm, these removed field components can be calculated by interpolation as in the case of the components $B_x(i+1/2, j, k+1/2)$ and $B_y(i, j+1/2, k+1/2)$. This is a more efficient approach; however, a drawback arises when the ferrite sample is next to a metallic wall. For example, if a metallic wall is located in a plane j = constant, the interpolation of H_y at the point $(i, j+1/2, k+1/2)$ requires the value of H_y in the metallic plane, which is unknown. Consequently, extrapolation must be used, which may introduce some inaccuracy. If the metallic wall is perpendicular to the dc magnetic field, H_z is zero at the wall and the eight-component unit cell can be used without any extrapolation.

III. Full-Wave Analysis of Ferrite-Loaded Waveguides

Apart from the practical interest of using FDTD techniques to study propagation characteristics in waveguides and transmission lines, within the FDTD approach there is an important topic: the design of optimal absorbing boundary conditions to terminate guides with matched loads. This design problem requires the propagation constants of the first modes of the terminal guides to be known. The full-wave analysis of guiding structures is a 3-D problem that can be reduced to an equivalent 2-D problem by noticing that for a uniform guide with an arbitrary cross-section, the functional dependence of the modes in the direction of propagation is analytically known. In general, an exponential term must be used; however, in particular cases (isotropic, uniaxial or biaxial materials) sinusoidal functions can be used. Two different approaches have been proposed to derive the 2-D FDTD formulation. In the direct approach, the modal term is first included in the differential form of Maxwell's equations, and then the discretization is carried out in a 2-D space (the transverse section of the guide) [18], [19]. In the indirect approach, Maxwell's curl equations are first discretized in a 3-D space and then the formulation is reduced by substituting for the modal term [20], [21]. Both approaches involve a 2D-mesh. In this paper, the direct approach is adopted because the 3-D discretization introduces a larger numerical dispersion error than the 2-D discretization. Moreover, although the 3D-problem is also reduced to a 2D-problem in the indirect approach, the spatial increment in the direction of propagation appears as an explicit parameter in the formulation, as well as in the stability condition and in the numerical dispersion relation.

In order to calculate the dispersion characteristics, the cross-section of the waveguide under analysis is discretized and the boundary conditions imposed. A desired value of the phase constant β is selected. The time domain response is calculated and, finally, the frequency domain response, i.e. the resonant frequencies and quality factors of the resonant modes of the cross-section of the waveguide, is obtained from the spectral analysis of the time domain response. Each pair of resonant frequencies and quality factors (f_i, Q_i) corresponds to one excited propagating mode, which has the previously fixed value of β at the frequency f_i, and according to [22] an attenuation constant of

$$\alpha_i' = \frac{Power\ loss\ per\ unit\ length}{2 \times Transmitted\ power} = \frac{\pi f_i}{Q_i v_{gi}} \quad (9)$$

where v_{gi} is the group velocity of the mode. By changing the value of β and repeating this process the whole dispersion diagram is obtained. The group velocity is calculated from the β(f) curve.

Frequency domain data are usually obtained by applying the FFT algorithm. Then, the resonant frequencies are calculated from the local maxima of the spectrum, and the quality factors can be calculated from the width of the resonant peaks or by determining the time attenuation factor as described in [23]. However, the FFT approach has two important limitations. First, there is a limitation in the frequency resolution, which is roughly the reciprocal of the observation time. Secondly, there is the windowing of the time domain data; the time domain response is truncated because it is usually excessively long. As a consequence of this windowing, the peaks in the spectrum are widened, the whole spectrum is distorted (resonant frequencies are shifted from their actual values), and some weak (low amplitude) resonances may be masked. In some cases, masking can be avoided and distortion reduced by using special windows [24], but, in general, distortion can be reduced and the resolution increased only by making the window larger, i.e. increasing the simulation time. A number of alternative spectral estimation procedures have recently been proposed [25]–[28] in order to overcome these limitations and improve the efficiency of time-domain methods for providing frequency domain data.

A. Transverse Magnetization

For waveguides containing transversely magnetized ferrites, an exponential term must be used. Hence, it is assumed that the fields have the form

$$\vec{F}(x,y,z,t) = \vec{f}(x,z,t)\ \exp(-j\beta y) \quad (10)$$

where y is the direction of propagation, β the phase constant of the mode, and F (and f) denotes any field component.

Substituting (10) into (1) and (2), we obtain the following equations:

$$\frac{\partial \vec{b}}{\partial t} = \mathrm{j}\beta\vec{a}_z \times \vec{e} - \nabla_{x,y} \times \vec{e} \tag{11}$$

$$\frac{\partial \vec{e}}{\partial t} = \frac{1}{\epsilon}\left(\mathrm{j}\beta\vec{a}_z \times \vec{h} + \nabla_{x,y} \times \vec{h}\right) \tag{12}$$

where $\boldsymbol{j}$ denotes the imaginary unit.

As in 3D-problems, equations (11) and (12) can easily be discretized by using central finite-differences. For example, $b_x^{n+1/2}$ is calculated from

$$\begin{aligned} b_x^{n+1/2}(i,k+1/2) =&\Delta t\Bigg(\frac{e_y^n(i,k+1)-e_y^n(i,k)}{\Delta z} \\ &+\mathrm{j}\beta e_z^n(i,k+1/2)\Bigg) \\ &+ b_x^{n-1/2}(i,k+1/2) \end{aligned} \tag{13}$$

The remaining difference equations related to differential equations (11) and (12) can be calculated similarly. These equations, together with the constitutive equations (6) and (7), allow the application of the FDTD algorithm described in the preceding section.

If the indirect approach is used to derive the 2D-formulation for dispersion analysis, the same difference equations are obtained, but the following substitution must be made

$$\mathrm{j}\beta \leftarrow \frac{1-\exp(-\mathrm{j}\beta\Delta y)}{\Delta y}$$

Moreover, in the indirect approach, although it might seem that the mesh size in the propagation direction, Δy, could take any value, there are forbidden values for which the formulation is not valid. These are given by

$$\Delta y = \frac{2p\pi}{\beta} = p\lambda_g \quad p = 1,2,... \tag{14}$$

where λ_g is the wavelength in the guide.

It should be noted that, due to the choice of an exponential function in (10), all field components become complex quantities. The difference equations can be separated into real and imaginary parts; hence, the implementation of the algorithm requires twice as much memory and CPU time for arithmetic operations as would be needed if the use of sinusoidal functions were possible. The discretization mesh for this reduced 2-D problem is obtained by projecting the 3D-mesh onto the x-z plane (see Fig. 2(a)).

For the 2D-FDTD formulation for dispersion analysis using the direct approach, the stability condition can be expressed as [29]

$$s = \frac{c}{\sqrt{\epsilon_{r_{min}}}}\Delta t\left(\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta z)^2}+\frac{\beta^2}{4}\right)^{1/2} \tag{15}$$

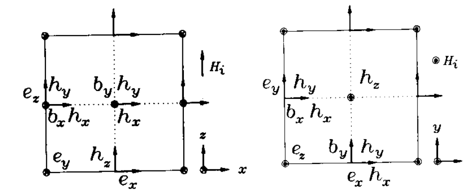

Fig. 2. 2D-mesh for the analysis of (a) transversely and (b) longitudinally magnetized ferrite-loaded waveguides.

where c is the velocity of light, $\epsilon_{r_{min}}$ the minimum value of the dielectric constant of the media contained in the guide, and s is the stability factor, whose value must not exceed unity to guarantee the numerical stability of the algorithm. If the indirect approach is used to derive the 2D-FDTD formulation for dispersion analysis, the corresponding stability condition has the same form as (15), but the following substitution must be made:

$$\frac{\beta^2}{4} \leftarrow \frac{\sin^2(\beta\Delta y/2)}{(\Delta y)^2}$$

B. Longitudinal Magnetization

When the magnetization is purely longitudinal (the anisotropy purely transverse), both propagation constants β and $-\beta$ are solutions of Maxwell's equations and their respective fields have reflectional symmetry. Hence, instead of an exponential term, a sinusoidal one can be used. In other words, a standing wave can be formed from the forward and backward waves, shown at the bottom of the page in (16a), and in

$$\vec{E}(x,y,z,t) = \vec{e}_t(x,y,t)\cos(\beta z) + \vec{e}_z(x,y,t)\sin(\beta z) \tag{16b}$$

where the subscript t denotes the transverse field.

As in the transverse magnetized case, the substitution of equations (16) into (1) and (2) provides a set of 2D-equations, which in combination with the constitutive equations (6) and (7) allow the FDTD algorithm for ferrites to be applied to the analysis of longitudinally magnetized ferrite-loaded waveguides. Furthermore, the choice of sinusoidal functions to describe the behavior of the fields in the z-direction leads to a real formulation, saving half of the memory and CPU time required with the exponential choice. For example, for $b_x^{n+1/2}$

$$\begin{aligned} b_x^{n+1/2}(i,j+1/2) =& -\Delta t\Bigg(\beta e_y^n(i,j+1/2) \\ &+\frac{e_z^n(i,j+1)-e_z^n(i,j)}{\Delta y}\Bigg) \\ &+ b_x^{n-1/2}(i,j+1/2) \end{aligned} \tag{17}$$

If the indirect approach is used, the same difference equation is obtained but the following substitution must be made

$$\beta \leftarrow \frac{2\sin(\beta\Delta z/2)}{\Delta z}$$

$$\begin{Bmatrix}\vec{B}(x,y,z,t)\\ \vec{H}(x,y,z,t)\end{Bmatrix} = \begin{Bmatrix}\vec{b}_t(x,y,t)\\ \vec{h}_t(x,y,t)\end{Bmatrix}\sin(\beta z) + \begin{Bmatrix}\mu_0\vec{h}_z(x,y,t)\\ \vec{h}_z(x,y,t)\end{Bmatrix}\cos(\beta z) \tag{16a}$$

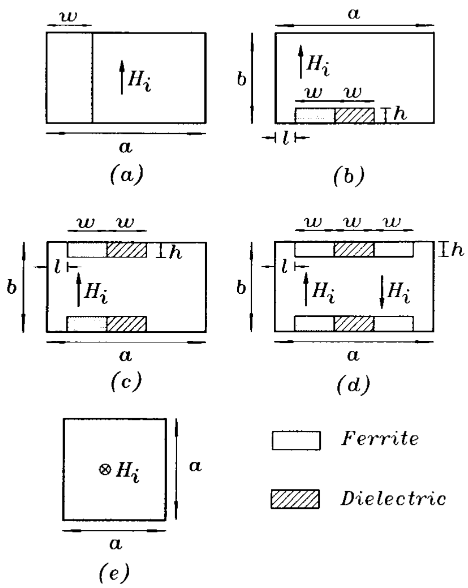

Fig. 3. Cross-section of various ferrite-loaded waveguides.

The discretization mesh is obtained by projecting the 3D-mesh onto the x-y plane (see Fig. 2(b)).

IV. Numerical Results

To demonstrate the validity of the extended FDTD method for ferrite treatment, we have used it for the analysis of various ferrite-loaded waveguides, which are shown in Fig. 3. As an example for which exact results are available, we have considered a rectangular waveguide loaded with a lossy transverse magnetized ferrite slab. Fig. 4 shows the phase constant β and the attenuation constant α' of the TE_{10} mode calculated by FDTD, compared with the exact results. The nonsymmetric localization of the slab allows a forward and a backward wave to propagate with different propagation constants. The frequency-domain results, i.e. resonant frequencies and quality factors, have been calculated by Prony's method [16]. As can be seen, good agreement is obtained. For 1D-problems, like the one under consideration, the term $1/(\Delta z)^2$ must be removed from expression (15).

As examples of 2D-structures, rectangular waveguides loaded with H-plane ferrite slabs have been studied. One important application of these structures is in the construction of four-port differential phase shift circulators. This type of circulator consists of a folded magic T and a 3-dB sidewall hybrid between that is placed a dual section of waveguide containing nonreciprocal 45° ferrite phase shifters. For efficient evacuation of the heat generated within the ferrite, the H-plane geometry is utilized to implement the phase shifters. The differential phase shift of these structures can be increased when dielectric loading is added as shown in Figs. 3(b)–(d). The results for the propagation constant of the dominant mode of a rectangular waveguide loaded with a single H-plane ferrite slab are shown in Fig. 5. These results have been compared with those obtained by Schelkunoff's

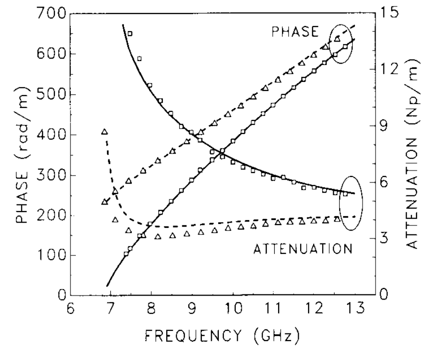

Fig. 4. Phase and attenuation constants of the TE_{10} mode of the rectangular waveguide loaded with a ferrite slab shown in Fig. 3(a) as a function of frequency. $a = 22.86$mm, $w = a/3$, $\epsilon_{rf} = 9.4$, $\pi M_s = 2000$ G, $H_i = 200$ Oe, $\alpha = 0.02$, $\Delta x = a/12$ and $s = 1/2$. Forward wave: ______ exact; ■ FDTD. Backward wave: _ _ _ exact; △ FDTD.

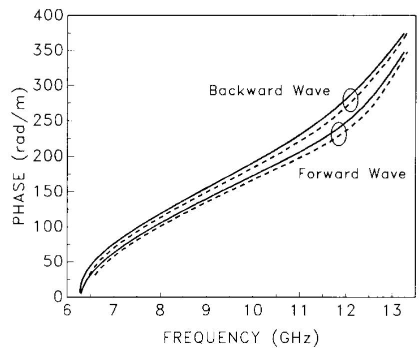

Fig. 5. Phase constant as a function of frequency of the dominant mode of a rectangular waveguide loaded with a single H-plane ferrite slab, as shown in Fig. 3(b). $a = 22.86$ mm, $b = 10.16$ mm, $w = a/4$, $l = a/8$, $h = b/6$. $\epsilon_{rd} = 1$, $\epsilon_{rf} = 12$, $4\pi M_s = 2000$ G, $H_i = 200$ Oe, $\alpha = 0$, $\Delta x = a/40$, $\Delta z = b/30$ and $s = 0.8$. ______ Schelkunoff's method, _ _ _ FDTD method.

method [15]. Good agreement is observed between these methods, although there is a slight displacement between the curves predicted by the two methods.

Fig. 6 shows plots of the differential phase shift corresponding to the dominant mode of a rectangular waveguide loaded with an H-plane ferrite slab and with an H-plane dielectric slab for various values of the dielectric slab permittivity. In this Fig., the onset of the first higher-order mode is marked with an arrow. It can be observed that the differential phase shift actually increases as the permittivity of the dielectric slab rises. Another effect of the dielectric loading is that the bandwidth is reduced: without dielectric loading an almost flat response is obtained. On the other hand, when the dielectric slab is added, the differential phase shift increases rapidly with the frequency in the upper part of the considered band. Moreover, the higher the value of the permittivity of the dielectric slab, the lower the cut-off frequency of the first higher-order mode, which also reduces the bandwidth of the phase shifter.

One way of increasing the differential phase shift while conserving the flat form of the curves is to use waveguides with two or four H-plane ferrite slabs, as shown in Fig. 3(c) and (d). The differential phase shift characteristics of these

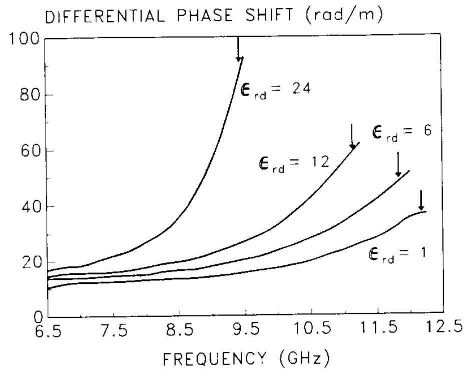

Fig. 6. Differential phase shift for the dominant mode of a rectangular waveguide loaded with an H-plane ferrite slab and an H-plane dielectric slab (Fig. 3(b)) for various values of the dielectric slab permittivity, ϵ_{rd}. Arrows: cut-off of the first higher-order mode. a = 22.86 mm, b = 10.16 mm, $w = a/4, l = a/8, h = b/6, \epsilon_{rf} = 12, 4\pi M_s = 2000$ G, $H_i = 200$ Oe, $\alpha = 0$, $\Delta x = a/40$, $\Delta z = b/30$ and $s = 0.8$.

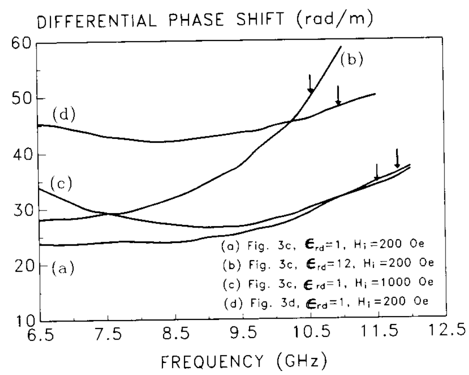

Fig. 7. Differential phase shift for the dominant mode of the structure shown in the Fig. 3(c) and (d). Arrows: cut-off of the first higher-order mode. a = 22.86 mm, b = 10.16 mm, $w = a/4, l = a/8, h = b/6, \epsilon_{rf} = 12$, $4\pi M_s = 2000$ G, $\alpha = 0$, $\Delta x = a/40$, $\Delta z = b/30$ and $s = 0.8$. Curve a: Fig. 3(c), $\epsilon_{rd} = 1$, $H_i = 200$ Oe; Curve b: Fig. 3(c), $\epsilon_{rd} = 12$, $H_i = 200$ Oe; Curve c: Fig. 3(c), $\epsilon_{rd} = 1$, $H_i = 1000$ Oe; Curve d: Fig. 3(d), $\epsilon_{rd} = 1$, $H_i = 200$ Oe.

structures are depicted in Fig. 7 . For the case of two ferrite slabs, dielectric loading is also considered with similar results to those obtained in the case of the single ferrite slab. When the intensity of the internal dc magnetic field is augmented, the differential phase shift only increases in the lower part of the band. This is because the resonance frequency is far below the operating frequency for these examples. For the case of four ferrite slabs, the bandwidth is reduced because of the onset of the first higher order mode. A way of increasing the bandwidth for wideband applications may be to use reduced guides or wide-band structures such as ridge or T-septa waveguides.

As an example of a longitudinally magnetized waveguide, Fig. 8 shows results for the quasi-TE_{10} and quasi-TE_{01} modes of a ferrite-filled square waveguide calculated by FDTD and by Schelkunof's method. There is good agreement between the results obtained with the two methods, although there is a small discrepancy in the attenuation constants. This is affected by errors in the determination of the group velocity, which can

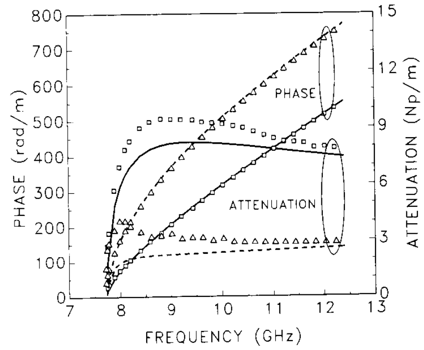

Fig. 8. Phase and attenuation constants of the Quasi-TE_{10} and Quasi-TE_{01} modes of the square ferrite-filled waveguide shown in the Fig. 3(d). a = 22.86 mm, ϵ_{rf}= 12, $4\pi M_s$= 1500 G, H_i= 1000 Oe, α= 0.03, $\Delta x = \Delta y = a/10$ and s = 1/4. Quasi-TE_{10}: ____ Schelkunoff's method; ■ FDTD. Quasi-TE_{01}: _ _ _ Schelkunoff's method; △ FDTD.

be important near cut-off, and also by the errors associated with the computation of Q-factors by Prony's method, which are sensitive to the presence of numerical noise in the time-domain data. Furthermore, for a lossy waveguide, the fields are still assumed to be of the form given in (10) (or (16)), which strictly speaking, is only valid for lossless structures and approximately valid for $\beta >> \alpha'$.

V. Conclusion

The FDTD method has been extended to include magnetized ferrites. The treatment of the ferrite material is based on the equation of motion of the magnetization vector (differential approach). The discretization scheme is based on central finite-differences and linear interpolation. This scheme maintains the fully explicit nature of the FDTD methods and shares its advantages for isotropic materials: it is flexible, conceptually simple, and easy to implement. Other alternative schemes, such as the rotated Richtmyer finite-difference scheme, can also be used to discretize Maxwell's equation together with the equation of motion [30]. The extended FDTD method for ferrite treatment provides a powerful tool for analyzing complex structures such as junction circulators or ferrite substrate patch antennas.

The extended FDTD method for magnetized ferrites has been applied to the full-wave analysis of ferrite-loaded waveguides. The dispersion curves have been calculated by a 2D- FDTD formulation. A number of numerical results for both propagation and attenuation constants of various transversely and longitudinally magnetized ferrite-loaded waveguides have been obtained and compared with the exact values or with those obtained by Schelkunoff's method, with good agreement being obtained.

Appendix

The coefficients of the difference form of the equation of motion ((6) and (7)) are given by

$$f_i = \frac{N_i}{D} \; i = 0, ..., 5 \tag{18}$$

where

$$D = \gamma^2 \Delta t^2 \mu_0^2 (H_i + M_s)^2 + 4\gamma\alpha\Delta t \mu_0 (H_i + M_s) + 4(\alpha^2 + 1) \tag{19a}$$

$$N_0 = 4(\alpha^2 + 1) - \gamma^2 \Delta t^2 \mu_0^2 (H_i + M_s)^2 \tag{19b}$$

$$N_1 = \frac{1}{\mu_0}\left[\gamma^2 H_i \Delta t^2 \mu_0^2 (H_i + M_s) + 2\gamma\alpha\Delta t \mu_0 (2H_i + M_s) + 4(\alpha^2 + 1)\right] \tag{19c}$$

$$N_2 = \frac{1}{\mu_0}\left[\gamma^2 H_i \Delta t^2 \mu_0^2 (H_i + M_s) - 2\gamma M_s \alpha \Delta t \mu_0 - 4(\alpha^2 + 1)\right] \tag{19d}$$

$$N_3 = -2\gamma \Delta t M_s \tag{19e}$$

$$N_4 = 2\gamma \Delta t (2H_i + M_s) \tag{19f}$$

$$N_5 = -4\gamma \Delta t \mu_0 (H_i + M_s) \tag{19g}$$

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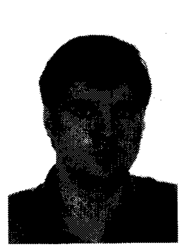

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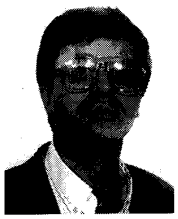

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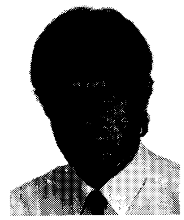

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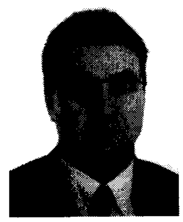

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