Even and odd mode cut-off parameters of pairs of ridges in rectangular waveguide using a mode matching solver

A. Mediavilla, J.A. Pereda, A. Casanueva, M. McKay and J. Helszajn

Abstract: A waveguide geometry of some interest consists of a symmetrical pair of ridges in a rectangular waveguide. The paper gives the even and odd mode cut-off spaces of such a structure in addition to its corresponding standing wave patterns. The classic power-voltage, power-current and voltage-current definitions of impedance of the two solutions are also evaluated. The calculations are based on a mode matching procedure and are verified separately using a finite element solver. A knowledge of these parameters is sufficient for the design of a proximity, reverse, directional coupler.

1 Introduction

The cut-off space and impedance levels of rectangular waveguides with single and double ridge inserts have, by now, been investigated thoroughly in the open literature [1–9]. The parameters of rectangular waveguides with triple ridges have been described also [10]. Circular waveguides with triple, quadruple and multiple ridges have been dealt with separately [11, 12]. Another ridge waveguide is one consisting of either a pair of single or double ridges symmetrically displaced from the symmetry plane of a regular rectangular waveguide. A feature of this waveguide is that it has, with two of the ridges suitably terminated, the properties of a 4-port directional coupler between the other two ridges. The purpose of this paper is to evaluate the dominant and first order even and odd mode cut-off spaces of the double ridge arrangement. The corresponding voltage-current, power-voltage and power-current definitions of impedance at infinite frequency are calculated separately and the standing wave patterns of the two types of solution are also drawn. The impedance levels of single, double, triple and quadruple ridge waveguides are evaluated separately and compared. The procedure adopted in this paper is based on the mode matching (MM) method and the results are verified separately by having recourse to a finite element (FE) solver. The structure under consideration is illustrated in Fig. 1. Other papers of note on ridge waveguides are to be found in [13–17].

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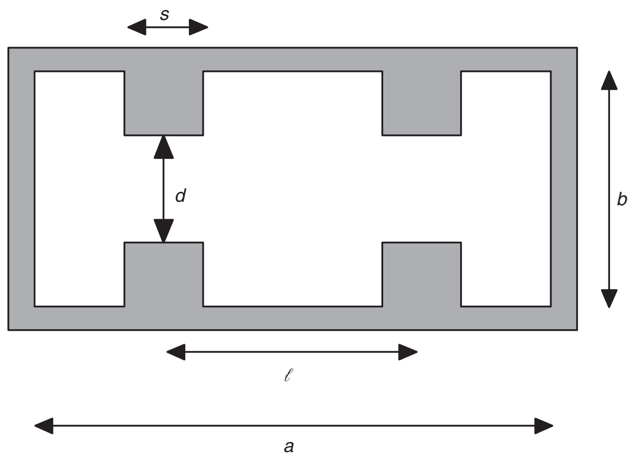

Fig. 1 *Schematic diagram of ridge waveguide loaded by doublets of ridges*

2 Even and odd mode cut-off spaces

An important quantity in the description of any waveguide is its cut-off space. The purpose of this Section is to summarise some calculations on the cut-off spaces of the even and odd mode geometries of two pairs of ridges symmetrically placed in a regular rectangular waveguide. The two circuits under consideration are illustrated in Figs. 2*a* and 2*b*. Each arrangement is defined by the spacing between the pairs of ridges (ℓ/a), the gap between the ridges (d/b) and the width of the ridges (s/a). Figures 3*a* and 3*b* depict the relationship between the dominant even mode cut-off space and the normalised spacing between the ridges for parametric values of the normalised gap and two different values of s/a. The corresponding results for the odd mode solution are indicated in Figs. 4*a* and 4*b* respectively. The results were obtained by introducing a suitable electric or magnetic wall along the H-plane symmetry of the waveguide and modelling one quarter of the original topology.

The onset of the first higher order even and odd modes are also of some interest in any design. These fix the dominant mode bandwidths in any waveguide. In

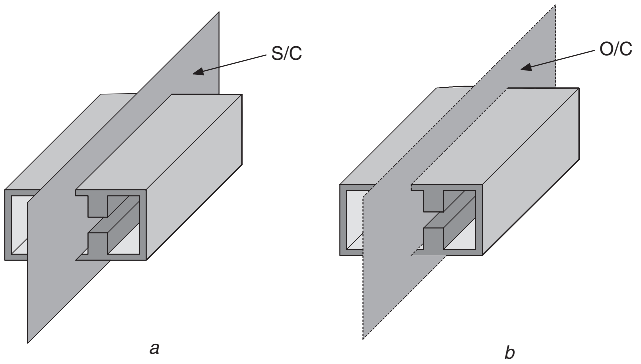

Fig. 2 *Even and odd mode geometry of waveguide with doublets of ridges*
a Even mode
b Odd mode

investigating these modes it is necessary to partition the geometry by electric and magnetic walls in all combinations. The first higher order even mode in Figs. 5*a* and 5*b* is associated with a magnetic wall along the horizontal symmetry plane. The first higher order odd mode in Figs. 6*a* and 6*b* is associated over some interval with a magnetic wall and over the remaining interval with an electric wall.

3 Even and odd mode standing wave patterns of coupled pairs of ridges

The even and odd mode electric field patterns of the arrangement under consideration are also of some interest. The purpose of this Section is to present a FE calculation on each solution. This is again done for the first two modes of the waveguide. The segmentation utilised in this work is illustrated in Fig. 7. Its details are fixed by $p = 2$, $m = 6$, $n = 159$, and $q = 362$, where p is the degree of the interpolation polynomial within each FE triangle, m is the number of nodes inside each triangle, n is the number of triangles and q is the number of nodes after assembly of the mesh. The dominant even and odd mode electric field patterns are illustrated in Fig. 8. The corresponding results for the first higher order modes are depicted in Fig. 9. These results apply to a geometry with $b/a = 0.50$, $s/a = 0.20$, $d/b = 0.375$ and $\ell/a = 0.40$.

4 Definition of impedance in waveguide

The characteristic impedance of a uniform transmission line supporting TEM propagation may be defined in one of three possible ways.

$$Z_{PV} = \frac{VV^*}{2P_t} \tag{1}$$

$$Z_{PI} = \frac{2P_t}{II^*} \tag{2}$$

$$Z_{VI} = \frac{V}{I} \tag{3}$$

$$Z_{PV} = Z_{PI} = Z_{VI} \tag{4}$$

In rectangular or ridge waveguides, the definitions of voltage and current are not unique so that

$$Z_{PV} \neq Z_{PI} \neq Z_{VI} \tag{5}$$

It is readily observed that

$$Z_{VI} = \sqrt{Z_{PV} Z_{PI}} \tag{6}$$

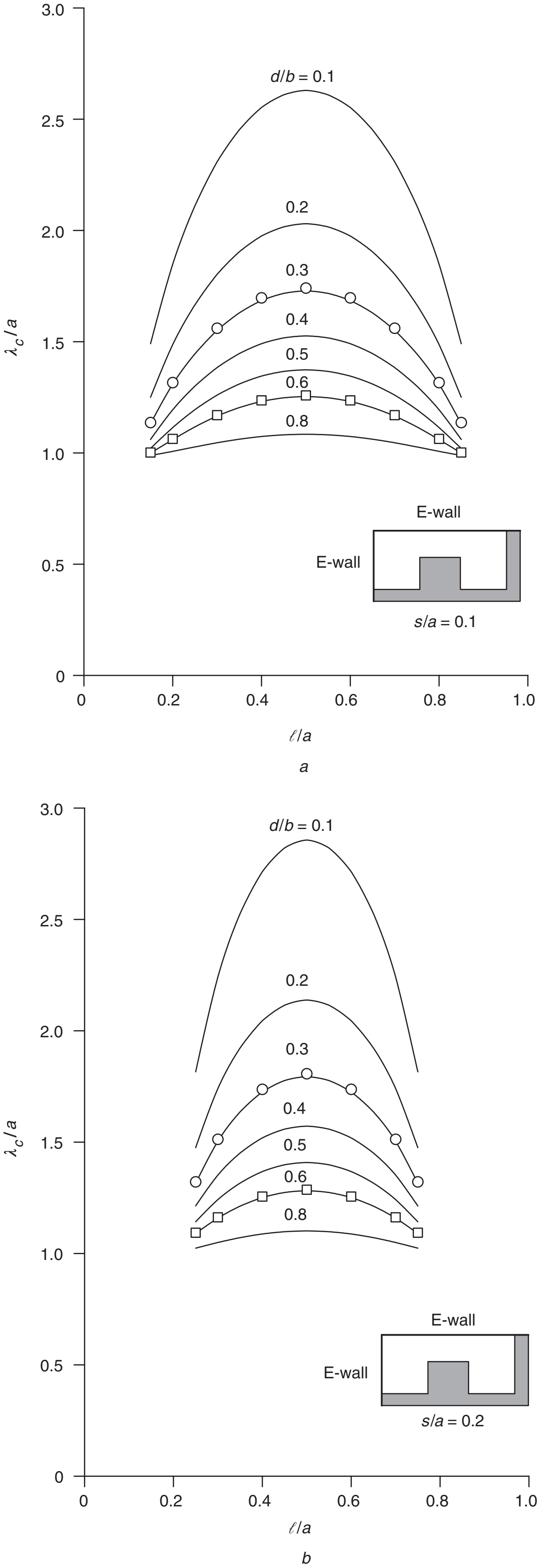

Fig. 3 *Dominant even mode cut-off space of doublets of ridges against ℓ/a for parametric values of d/b*
a $s/a = 0.10$
b $s/a = 0.20$
MM —; FE: ○ ($d/b = 0.30$), □ ($d/b = 0.60$)

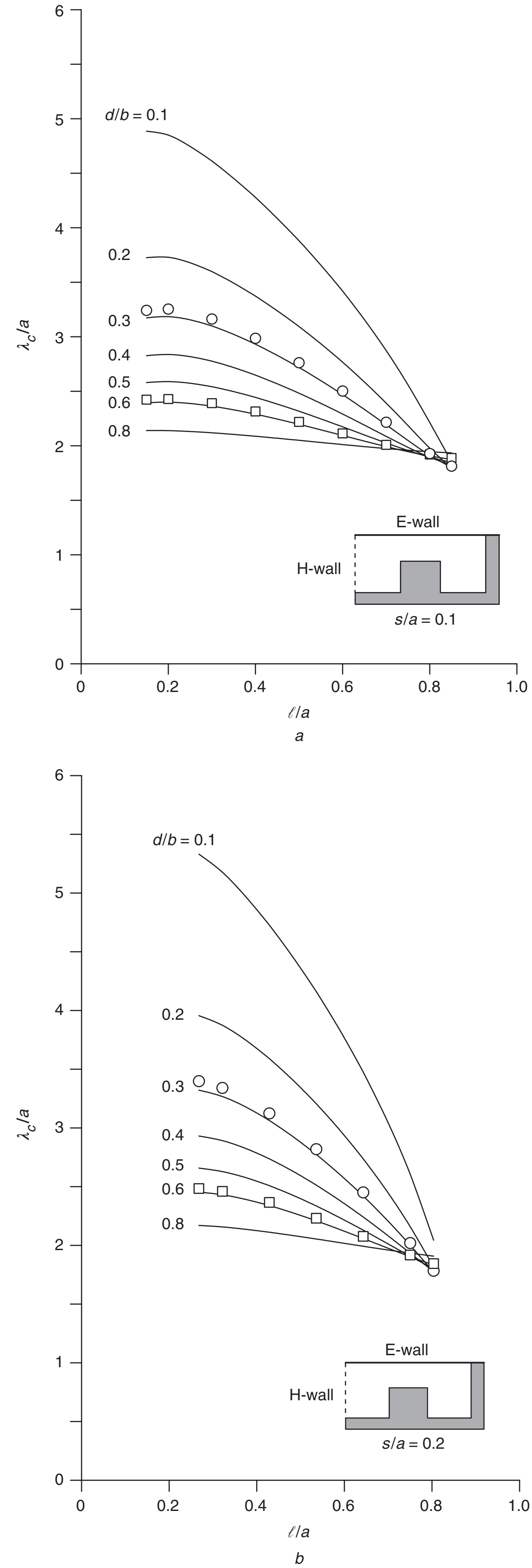

Fig. 4 *Dominant odd mode cut-off space of doublets of ridges against* ℓ/a *for parametric values of* d/b
a $s/a = 0.10$
b $s/a = 0.20$
MM —; FE: ○ ($d/b = 0.30$), □ ($d/b = 0.60$)

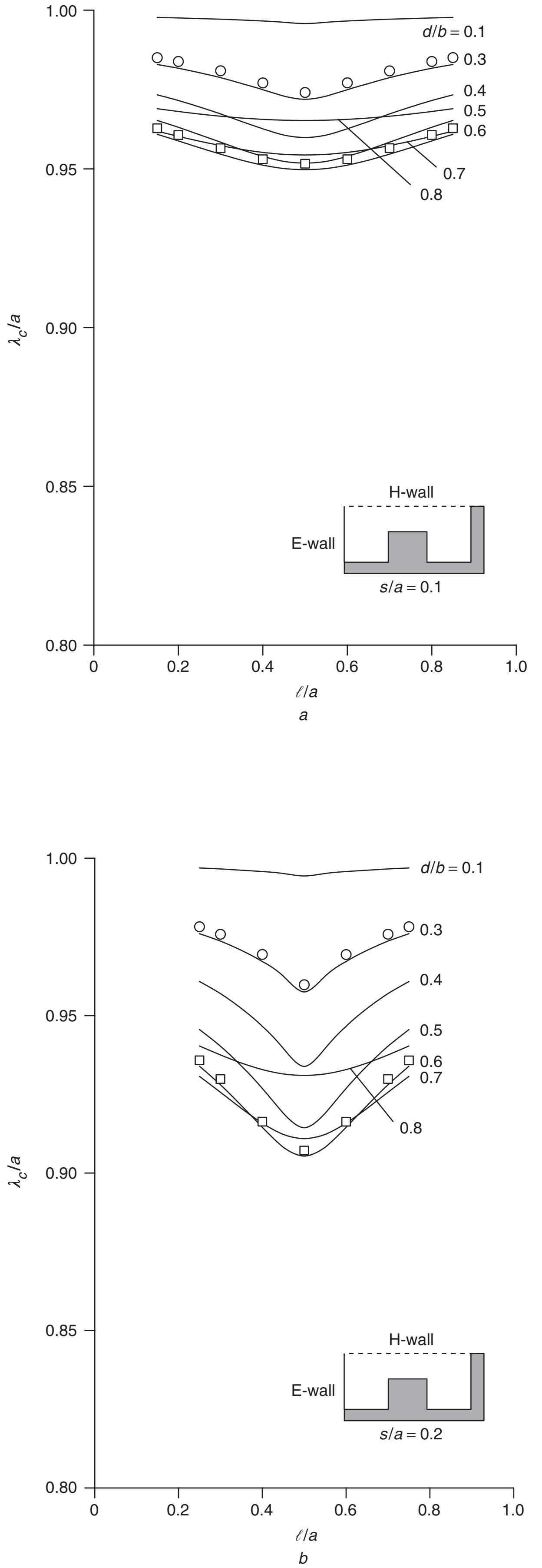

Fig. 5 *First higher order even mode cut-off space of doublets of ridges against* ℓ/a *for parametric values of* d/b
a $s/a = 0.10$
b $s/a = 0.20$
MM —; FE: ○ ($d/b = 0.30$), □ ($d/b = 0.60$)

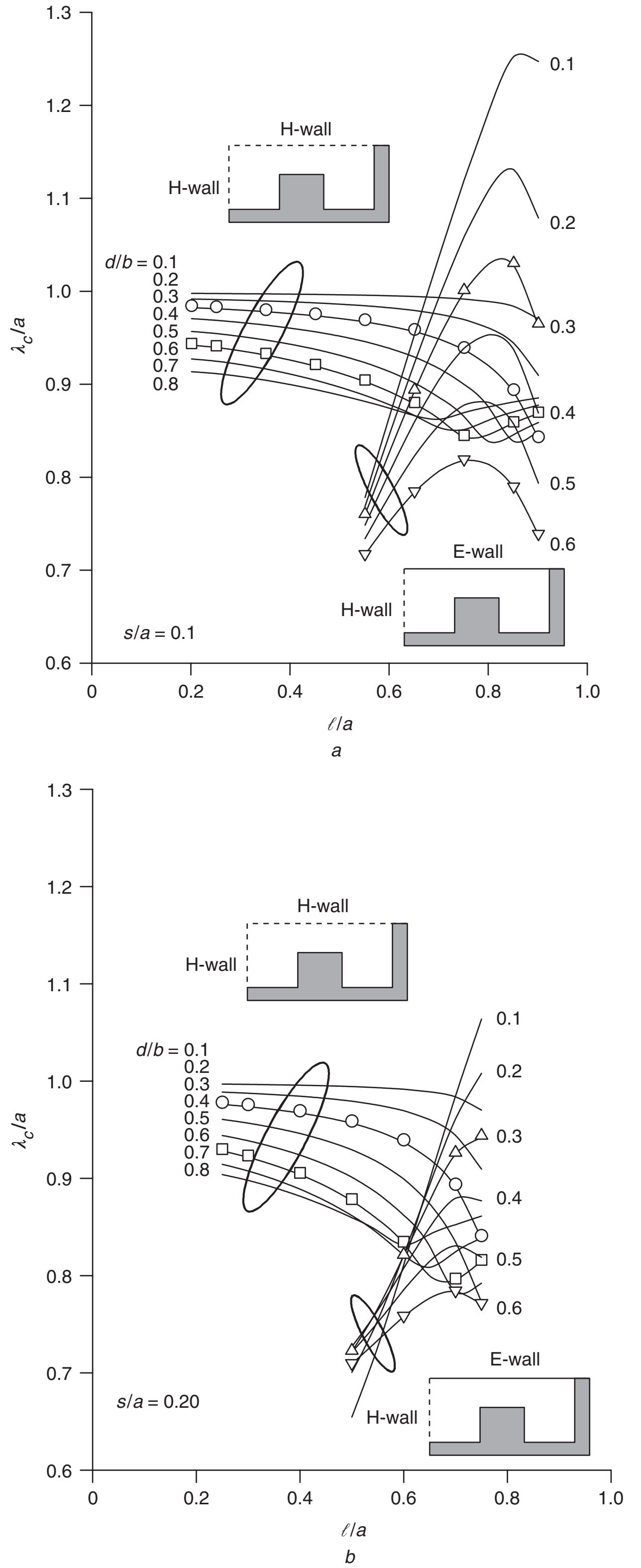

Fig. 6 *First higher order odd mode cut-off space of doublets of ridges against ℓ/a for parametric values of d/b*
a $s/a = 0.10$
b $s/a = 0.20$
MM —; FE: ○ ($d/b = 0.30$), □ ($d/b = 0.60$); △ ($d/b = 0.30$), ▽ ($d/b = 0.60$)

A knowledge of any two definitions of impedance is sufficient for the specification of the third one. It is usual in this sort of waveguide to calculate the various impedances at infinite frequency. The actual impedance is then given by

$$Z(\omega) = Z(\infty)\left(\frac{\lambda_g}{\lambda_0}\right) \quad (7)$$

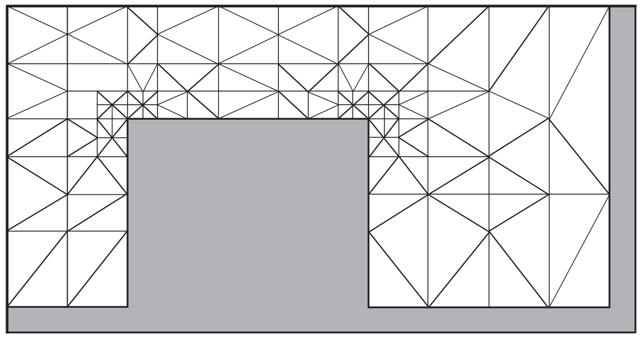

Fig. 7 *Finite element discretisation of ridge waveguide*
$b/a = 0.50$, $s/a = 0.20$, $d/b = 0.375$, $\ell/a = 0.40$

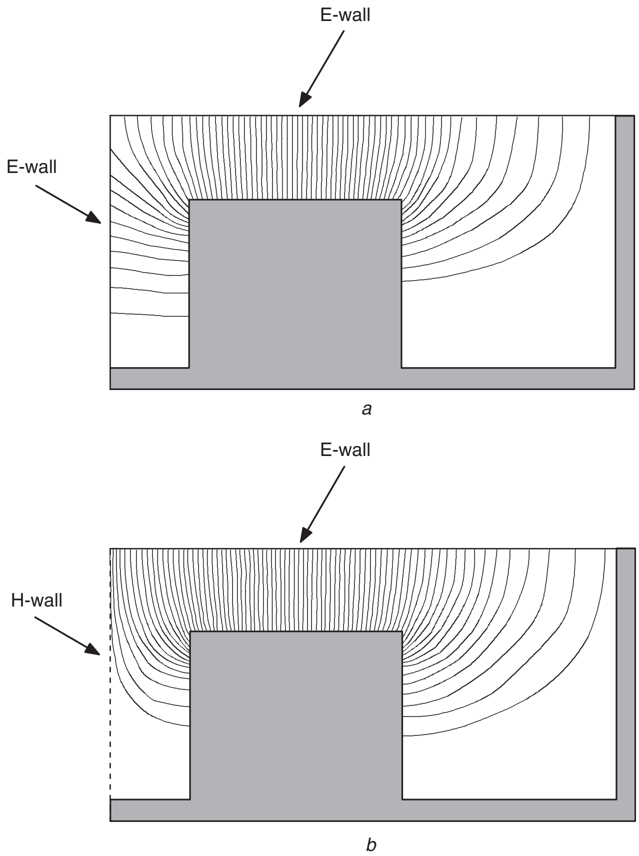

Fig. 8 *Dominant even and odd mode standing wave pattern in waveguide with doublets of ridges*
a Even mode
b Odd mode
$b/a = 0.50$, $s/a = 0.20$, $d/b = 0.375$, $\ell/a = 0.40$

This formulation permits the data to be compiled in a universal fashion and is the convention adopted here.

5 Even and odd mode impedances

The main purpose of this Section is to summarise the even and odd mode voltage-current, power-voltage and power-current definitions of impedance of the ridge waveguide under consideration. The power-voltage definition of impedance relies on a calculation of the Poynting vector in the waveguide and the voltage at the centre of a typical ridge. The even and odd mode impedance parameters are

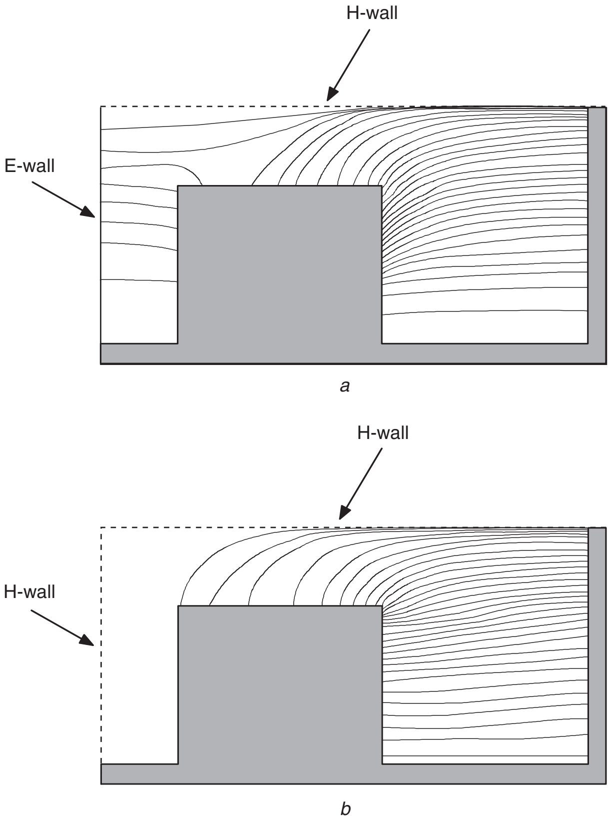

Fig. 9 *First higher order even and odd mode standing wave pattern in waveguide with doublets of ridges*
a Even mode
b Odd mode
$b/a = 0.50$, $s/a = 0.20$, $d/b = 0.375$, $\ell/a = 0.40$

again evaluated at infinite frequency.

$$Z_{PV}(\infty) = \frac{V(\infty)V^*(\infty)}{2P_t(\infty)}, \text{ even or odd mode} \quad (8)$$

The calculation proceeds by recalling that the field distributions in the waveguide are identical at cut-off and infinite frequency.

$$\frac{V(\infty)V^*(\infty)}{2P_t(\infty)} = \frac{V(f_c)V^*(f_c)}{2P_t(f_c)} \quad (9)$$

One definition of the voltage is that at the centre of one of the two pairs of ridges.

$$V(f_c) = \int_0^d E_y(f_c)dy \quad (10)$$

The Poynting vector is defined in the usual way by

$$P_t(f_c) = \frac{1}{2}\int_S \left(\overline{E}_t(f_c)x\overline{H}_t(f_c)\right)dS, \text{ W/m}^2 \quad (11)$$

$\overline{E}_t(f_c)$ and $\overline{H}_t(f_c)$ are the transverse components of the electric and magnetic field at the cut-off frequency and S is the cross-section of the waveguide. Figures 10*a* and 10*b* give some results for this definition of impedance for a typical geometry. The power-current and voltage-current definitions of impedance are separately superimposed on these illustrations.

6 Cut-off space of ridge waveguide

The regular cut-off space and the other parameters of the ridge waveguide under consideration are also of interest. The cut-off space and field patterns of the first and second modes in this waveguide coincide with the even mode

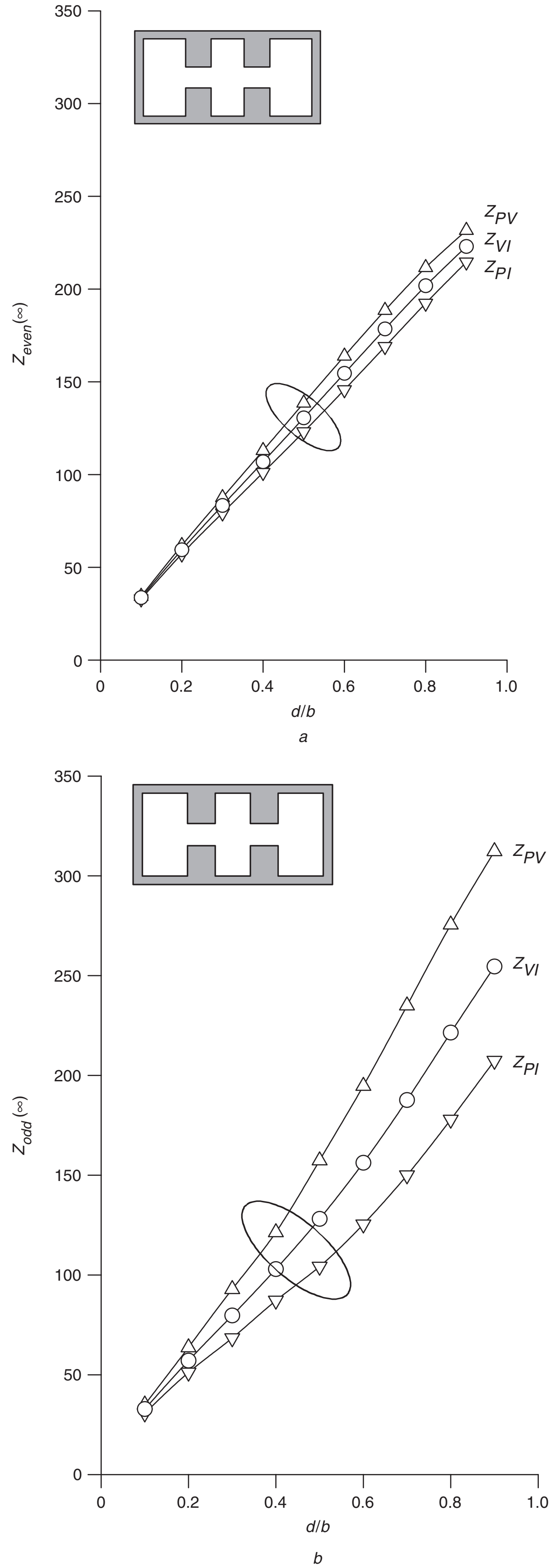

Fig. 10 *Even and odd mode definition of impedance in waveguide with doublets of ridges against d/b*
a Even mode
b Odd mode
$b/a = 0.50$, $s/a = 0.20$, $\ell/a = 0.40$

solutions. The various definitions of impedance at the centre of the waveguide, between the ridges, also coincide with these definitions.

7 Impedance levels of ridge waveguides

It is interesting to compare the impedance levels of ridge waveguides using single, double, triple ridges, and also that existing in the middle of pairs of double ridge structures. This has been done for $b/a = 0.50$, $s/a = 0.20$, $s_1/a = 0.10$, $\ell/a = 0.50$, $d_1/b = 0.50$ and parametric values of d/b. Figure 11*a* depicts the result and Fig. 11*b* gives the details employed in the calculations. It is apparent from these calculations that the triple ridge structure is compatible with very low impedance waveguides.

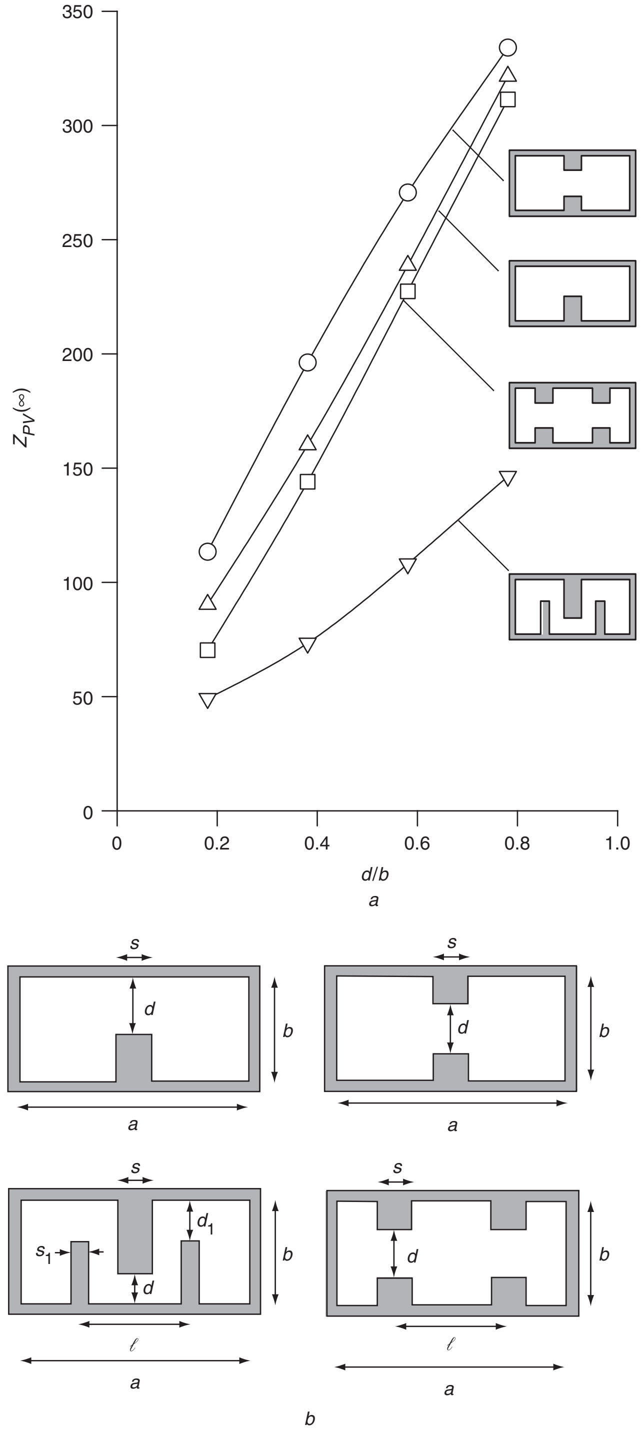

Fig. 11 *Comparison of $Z_{PV}(\infty)$ at centre of waveguide for single, double, triple and quadruple ridge waveguide against d/b and schematic diagram of single, double, triple and quadruple ridge waveguides*
a $Z_{PV}(\infty)$ against d/b
b Schematic diagrams
$b/a = 0.50$, $s/a = 0.20$, $s_1/a = 0.10$, $\ell/a = 0.50$, $d_1/b = 0.50$

8 The directional coupler

One application of a pair of coupled lines is in the design of a parallel line proximity directional coupler [18]. The geometry of one typical arrangement is indicated in Fig. 12. In this sort of circuit, ports 3 and 4 are terminated in matched loads, and ports 1 and 2 are taken as the input and output ports of the structure.

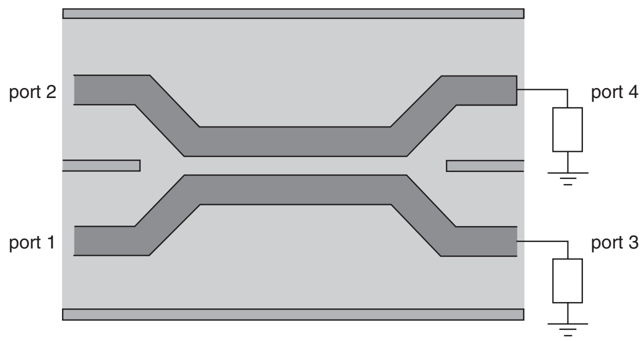

Fig. 12 *Parallel line ridge directional coupler*

The odd and even mode impedances are related to the coupling coefficient of a single 90° section by

$$Z_{odd} = Z_0\sqrt{\frac{1+k}{1-k}} \tag{12}$$

$$Z_{even} = Z_0\sqrt{\frac{1-k}{1+k}} \tag{13}$$

where

$$Z_{odd}Z_{even} = Z_0^2 \tag{14}$$

The voltage-coupling coefficient (k) between ports 1 and 2 of a 90° section is

$$k = \frac{Z_{odd} - Z_{even}}{Z_{odd} + Z_{even}} \tag{15}$$

The voltage transfer ratio at port 3 is

$$\sqrt{1-k^2} \tag{16}$$

The coupling coefficient is in practice fixed by the length of the coupler. In an ideal reverse coupler

$$\beta_{odd} = \beta_{even} = \beta_0 \tag{17}$$

and

$$\beta_0 L = \frac{\pi}{2} \tag{18}$$

The former condition is not naturally satisfied in the present situation. One classic means of doing so is to capacitively load the odd mode circuit.

Z_{odd} and Z_{even} are deduced from (12) and (13) once the coupling coefficient is stipulated.

9 Conclusions

One waveguide geometry that is of interest in the design of a proximity directional coupler is the rectangular waveguide loaded with a symmetrical pair of ridges. The main purpose of this paper has been to evaluate the even and odd mode cut-off space and the three classic definitions of impedance by having recourse to both a mode matching and finite element solver. The standing wave pattern of the dominant and first higher order even and odd modes have also been included.

10 Acknowledgment

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