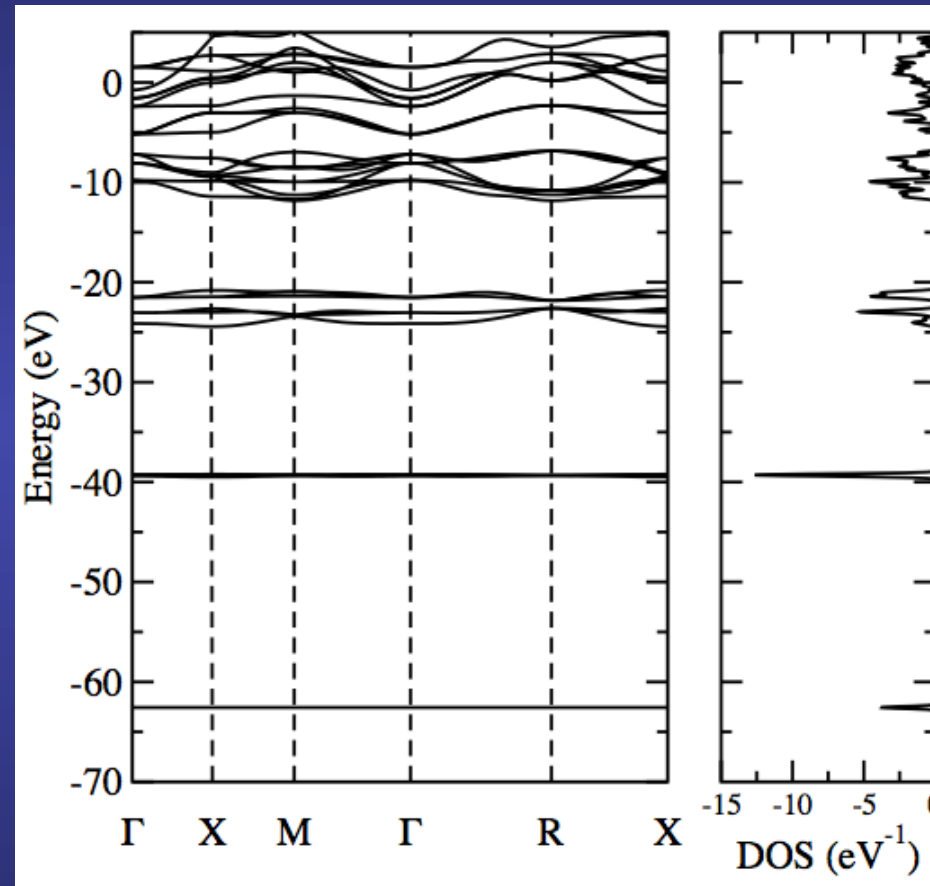


How to compute the projected density of states (PDOS)

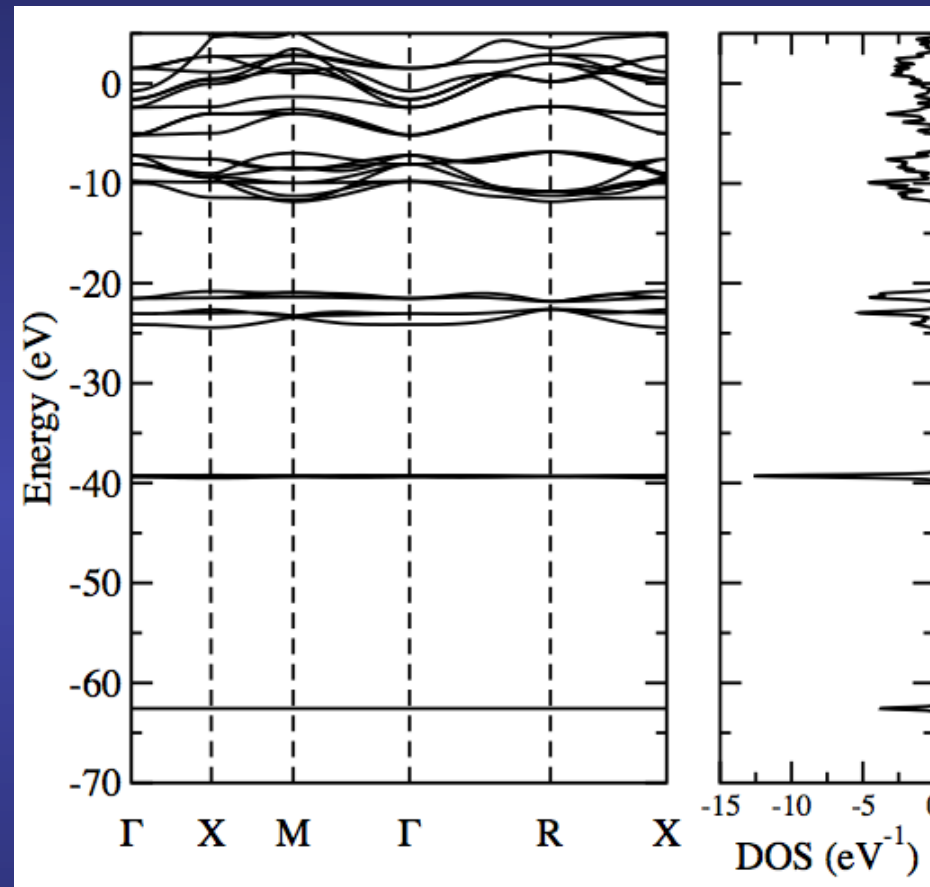


Javier Junquera

Density Of States (DOS)

$g(E)dE$ the number of one-electron levels between E and $E + dE$

SrTiO₃ bulk



$$g(E) = \frac{1}{N_{\vec{k}}} \sum_i^{bands} \sum_{\vec{k}} \delta(E - E_i(\vec{k}))$$

Units: (Energy)⁻¹

Projected Density Of States (PDOS)

$g_\mu(E)dE$ the number of one-electron levels with weight on orbital μ between E and $E + dE$

$$g_\mu(E) = \frac{1}{N_{\vec{k}}} \sum_i^{bands} \sum_{\vec{k}} \sum_{\nu} c_{\nu i}^*(\vec{k}) c_{\mu i}(\vec{k}) S_{\nu\mu}(\vec{k}) \delta(E - E_i(\vec{k}))$$

Coefficients of the eigenvector $\psi_i(\vec{k})$
with eigenvalue $E_i(\vec{k})$

Overlap matrix of the atomic basis

Units: (Energy)⁻¹

Relation between the DOS and PDOS:

$$g(E) = \sum_{\mu} g_\mu(E)$$

Normalization of the DOS and PDOS

$$\int_{-\infty}^{+\infty} g(E) dE = \text{Number of bands per k-point} = \text{Number of atomic orbitals in the unit cell}$$

$$\int_{-\infty}^{+\infty} g(E) \underset{\substack{\uparrow \\ \text{Occupation factor at energy } E}}{n(E)} dE = \text{Number of electrons in the unit cell}$$

Important labels to compute the density of states and the projected density of states

```
%block ProjectedDensityOfStates
  -70.00  5.00  0.150 3000  eV
%endblock ProjectedDensityOfStates

%PDOS.kgrid_Monkhorst_Pack
  60  0  0  0.5
  0 60  0  0.5
  0  0 60  0.5
%end PDOS.kgrid_Monkhorst_Pack
```

A separate set of k-points, usually on a finer grid than the one used to achieve self-consistency.
Same format as the Monkhorst-Pack grid.

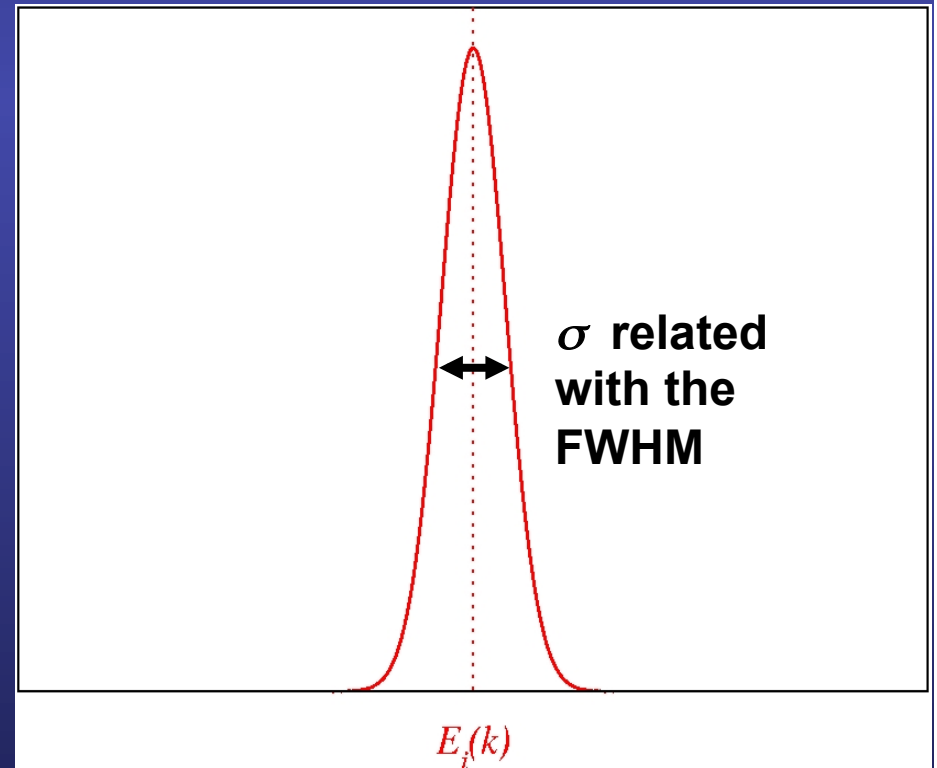
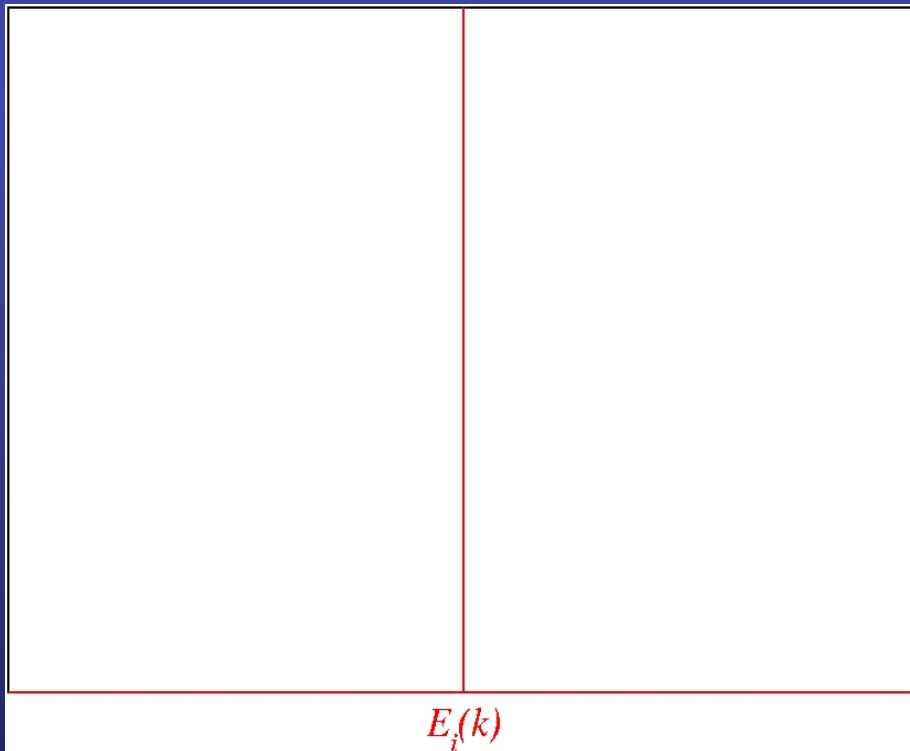
How to compute the DOS and PDOS

```
%block ProjectedDensityOfStates  
-70.0 5.0 0.150 3000 eV  
%endblock ProjectedDensityOfStates
```

-70.0 5.0 : Energy window where the DOS and PDOS will be computed
(relative to the program's zero, i.e. the same as the
eigenvalues printed by the program)

The eigenvalues are broadening by a gaussian to smooth the shape of the DOS and PDOS

$$\delta \left(E - E_i(\vec{k}) \right) \rightarrow \frac{1}{\sigma \sqrt{\pi}} e^{-\frac{(E - E_i(\vec{k}))^2}{\sigma^2}}$$



How to compute the DOS and PDOS

```
%block ProjectedDensityOfStates  
-70.0 5.0 0.150 3000 eV  
%endblock ProjectedDensityOfStates
```

-70.0 5.0 : Energy window where the DOS and PDOS will be computed
(relative to the program's zero, i.e. the same as the
eigenvalues printed by the program)

0.150 : Peak width of the gaussian used to broad the eigenvalues (energy)

**It should be twice as large as the fictitious electronic
temperature used during self-consistency.**

In this example:

ElectronicTemperature 0.075 eV

(see Appendix B of M. Stengel et al. Phys. Rev. B 83, 235112 (2011))

How to compute the DOS and PDOS

```
%block ProjectedDensityOfStates  
-70.0 5.0 0.150 3000 eV  
%endblock ProjectedDensityOfStates
```

-70.0 5.0 : Energy window where the DOS and PDOS will be computed
(relative to the program's zero, i.e. the same as the
eigenvalues printed by the program)

0.150 : Peak width of the gaussian used to broad the eigenvalues (energy)

**It should be twice as large as the fictitious electronic
temperature used during self-consistency**

(see Appendix B of M. Stengel et al. Phys. Rev. B 83, 235112 (2011))

3000 : Number of points in the histogram

How to compute the DOS and PDOS

```
%block ProjectedDensityOfStates  
-70.0 5.0 0.150 3000 eV  
%endblock ProjectedDensityOfStates
```

-70.0 5.0 : Energy window where the DOS and PDOS will be computed
(relative to the program's zero, i.e. the same as the
eigenvalues printed by the program)

0.150 : Peak width of the gaussian used to broad the eigenvalues (energy)

**It should be twice as large as the fictitious electronic
temperature used during self-consistency**

(see Appendix B of M. Stengel et al. Phys. Rev. B 83, 235112 (2011))

3000 : Number of points in the histogram

eV : Units in which the previous energies are introduced

Output for the Density Of States

SystemLabel.DOS

Format

Energy (eV) DOS Spin Up (eV⁻¹) DOS Spin Down (eV⁻¹)

-69.99993	0.00000	0.00000
-69.97492	0.00000	0.00000
-69.94991	0.00000	0.00000
.	.	.
.	.	.
.	.	.

Output for the Projected Density Of States

SystemLabel.PDOS

Written in XML

```
<pdos>
<nspin>1</nspin>
<norbitals> 72</norbitals>
<energy_values units="eV">
    -69.99993
    -69.97492
    -69.94991
    .
    .
    .
</energy_values>
<orbital
    index="                1"
    atom_index="            1"
    species="Sr"
    position="  -0.000000  -0.000000  0.000000"
    n="                4"
    l="                0"
    m="                0"
    z="                1"
>
<data>
    0.00000
    0.00000
    0.00000
    .
    .
    .
</data>
</orbital>
</pdos>
```

Energy Window

One element <orbital> for
every atomic orbital in the basis
set

How to digest the SystemLabel.PDOS file

fmpdos (by Andrei Postnikov)

Go to the directory Util/Contrib/Apostnikov, or download from

<http://www.home.uni-osnabrueck.de/apostnik/download.html>

Compile the code (in the Util directory, simply type \$ make)

Execute fmpdos and follow the instructions at run-time

```
$ <your_siesta_directory_path>/Util/Contrib/APostnikov/fmpdos
  Input file name (PDOS):
SrTiO3.PDOS
  Output file name :
Sr.PDOS.dat
  Extract data for atom index (enter atom NUMBER, or 0 to select all),
  or for all atoms of given species (enter its chemical LABEL):
1
  Extract data for n= ... (0 for all n ):
0
```

Repeat this for all the atoms you might be interested in

How to digest the SystemLabel.PDOS file

Plot the layer by layer Projected Density of States

```
$ gnuplot
```

```
G N U P L O T
```

```
Version 4.2 patchlevel 5
```

```
last modified Mar 2009
```

```
System: Darwin 11.4.2
```

```
Copyright (C) 1986 - 1993, 1998, 2004, 2007 - 2009
```

```
Thomas Williams, Colin Kelley and many others
```

```
Type 'help' to access the on-line reference manual.
```

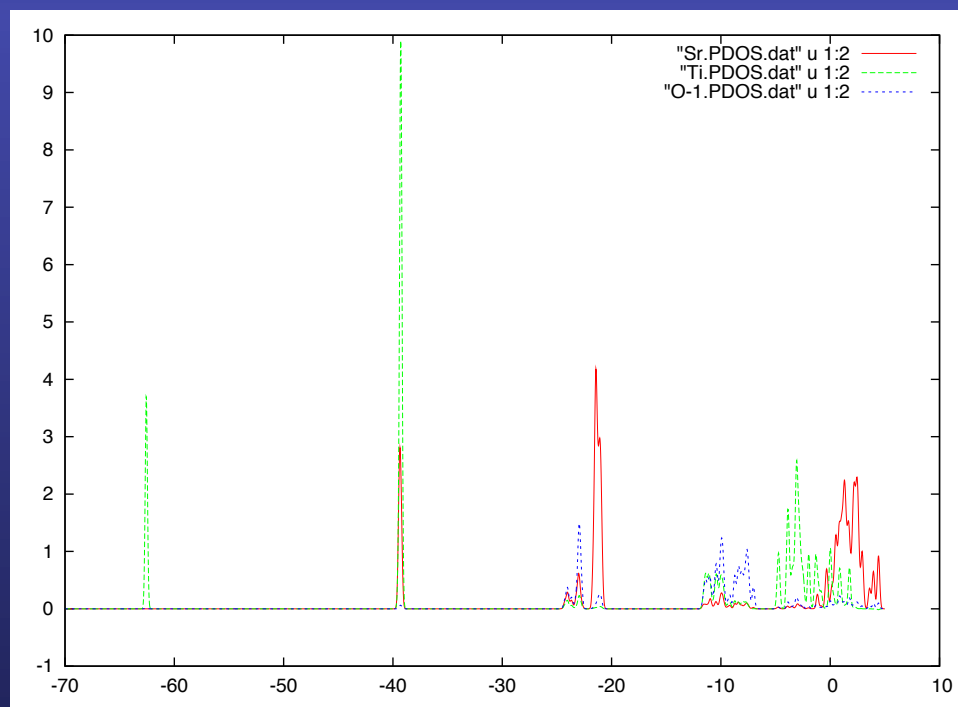
```
The gnuplot FAQ is available from http://www.gnuplot.info/faq/
```

```
Send bug reports and suggestions to <http://sourceforge.net/projects/gnuplot>
```

```
Terminal type set to 'aqua'
```

```
gnuplot> plot "Sr.PDOS.dat" u 1:2 w l, "Ti.PDOS.dat" u 1:2 w l, "O-1.PDOS.dat" u 1:2 w l
```

The PDOS for the three O atoms
are equivalent by symmetry



How to digest the SystemLabel.PDOS file

Plot the layer by layer Projected Density of States

```
$ gnuplot
```

```
G N U P L O T
```

```
Version 4.2 patchlevel 5
```

```
last modified Mar 2009
```

```
System: Darwin 11.4.2
```

```
Copyright (C) 1986 - 1993, 1998, 2004, 2007 - 2009
```

```
Thomas Williams, Colin Kelley and many others
```

```
Type 'help' to access the on-line reference manual.
```

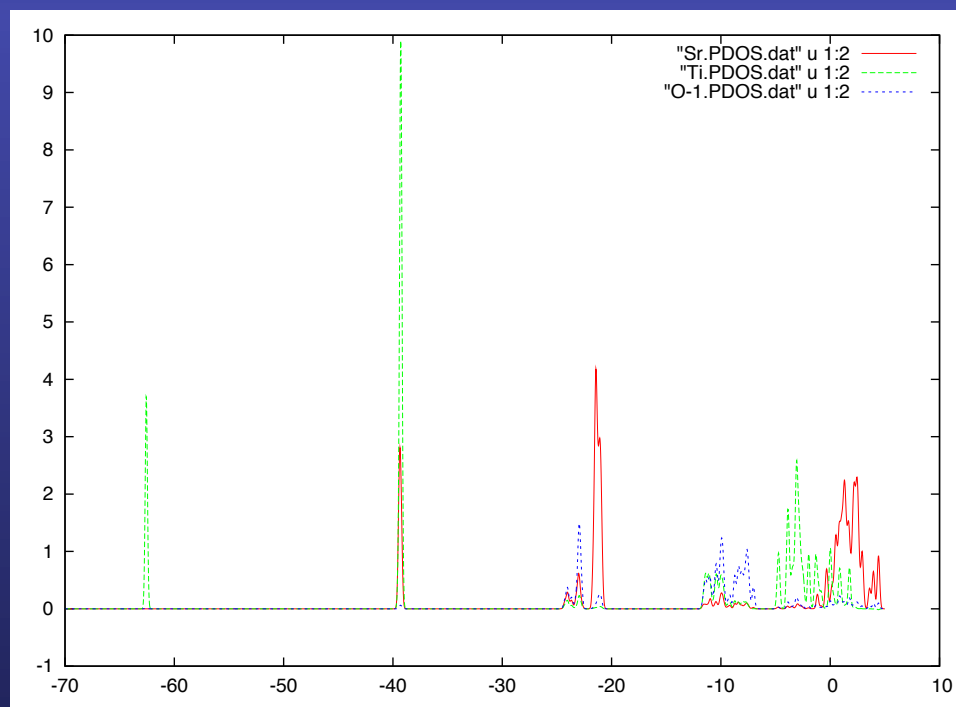
```
The gnuplot FAQ is available from http://www.gnuplot.info/faq/
```

```
Send bug reports and suggestions to <http://sourceforge.net/projects/gnuplot>
```

```
Terminal type set to 'aqua'
```

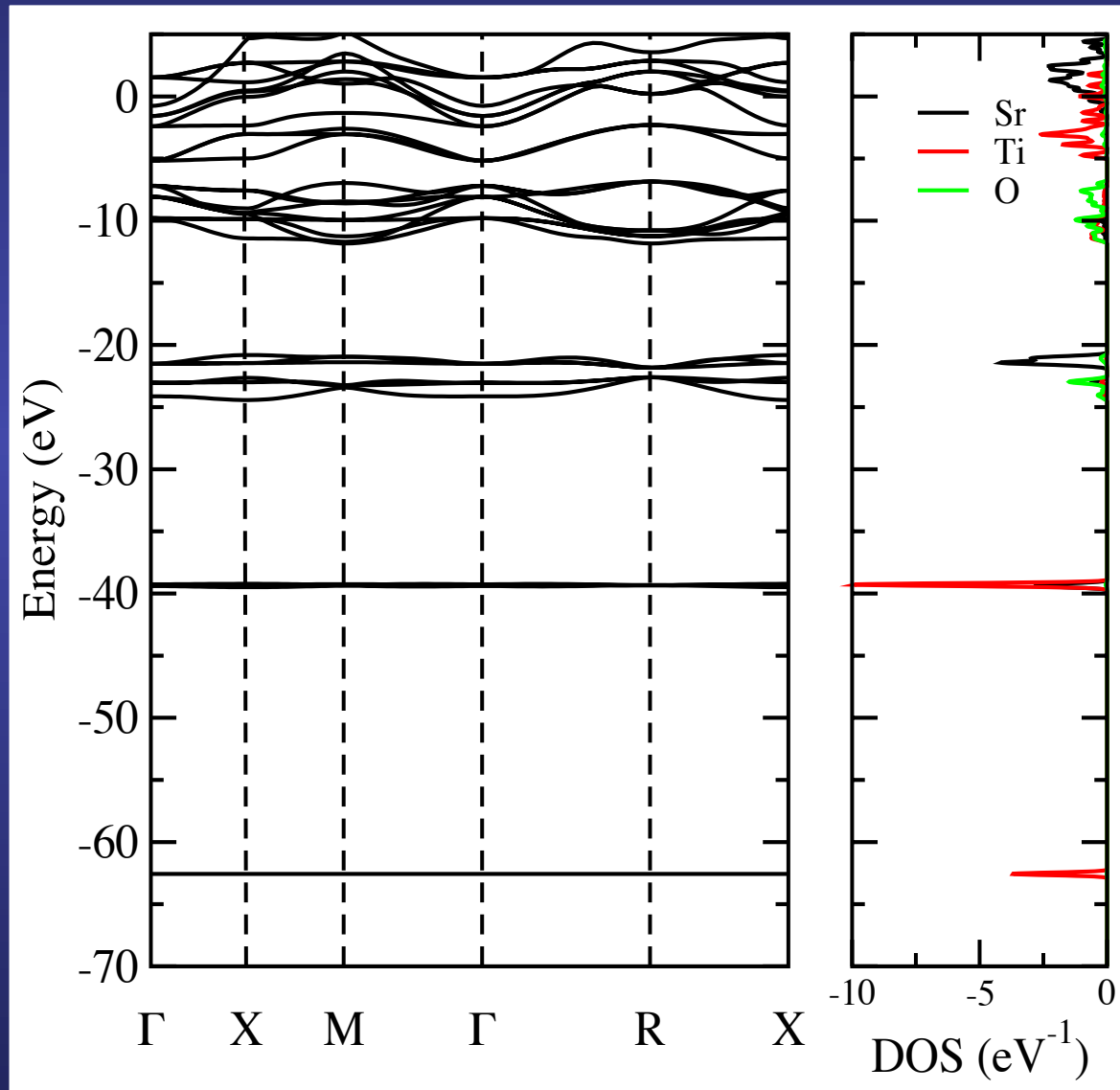
```
gnuplot> plot "Sr.PDOS.dat" u 1:2 w l, "Ti.PDOS.dat" u 1:2 w l, "O-1.PDOS.dat" u 1:2 w l
```

The PDOS for the three O atoms
are equivalent by symmetry



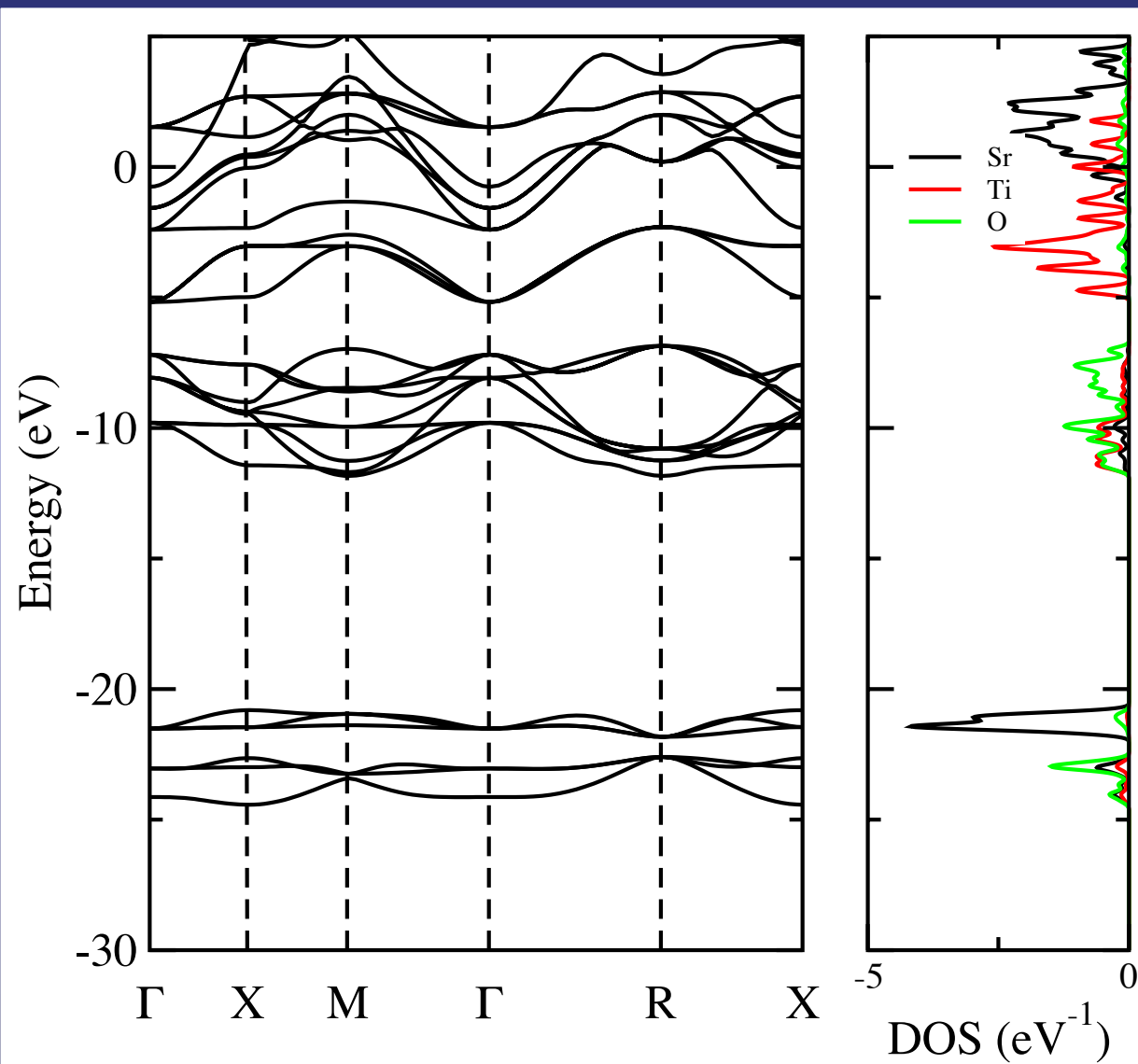
We can analyze the character of the different bands

Which atoms contribute more to the bands at a particular energy window



We can analyze the character of the different bands

Which atoms contribute more to the bands at a particular energy window
Zoom around the top of the valence bands and bottom of conduction bands



Bottom of conduction bands: mostly Ti character

Top of valence bands: mostly O character

We can project on particular atomic orbitals within an atom to further define the character.

To know the order of the orbitals, explore the SystemLabel.ORB_INDX file

```
$ more SrTiO3.ORB_INDX
72 15552 = orbitals in unit cell and supercell. See end of file.
```

io	ia	is	spec	iao	n	l	m	z	p	sym	rc	isc	iuo
1	1	1	Sr	1	4	0	0	1	F	s	6.602	0 0 0	1
2	1	1	Sr	2	5	0	0	1	F	s	7.116	0 0 0	2
3	1	1	Sr	3	5	0	0	2	F	s	5.542	0 0 0	3
4	1	1	Sr	4	4	1	-1	1	F	py	6.769	0 0 0	4
5	1	1	Sr	5	4	1	0	1	F	pz	6.769	0 0 0	5
6	1	1	Sr	6	4	1	1	1	F	px	6.769	0 0 0	6
7	1	1	Sr	7	5	1	-1	1	F	py	4.004	0 0 0	7
8	1	1	Sr	8	5	1	0	1	F	pz	4.004	0 0 0	8
9	1	1	Sr	9	5	1	1	1	F	px	4.004	0 0 0	9
10	1	1	Sr	10	4	2	-2	1	F	dxy	6.602	0 0 0	10
11	1	1	Sr	11	4	2	-1	1	F	dyz	6.602	0 0 0	11
12	1	1	Sr	12	4	2	0	1	F	dz2	6.602	0 0 0	12
13	1	1	Sr	13	4	2	1	1	F	dxz	6.602	0 0 0	13
14	1	1	Sr	14	4	2	2	1	F	dx2-y2	6.602	0 0 0	14
15	2	2	Ti	1	3	0	0	1	F	s	5.806	0 0 0	15
16	2	2	Ti	2	4	0	0	1	F	s	6.104	0 0 0	16
17	2	2	Ti	3	4	0	0	2	F	s	5.124	0 0 0	17
18	2	2	Ti	4	3	1	-1	1	F	py	5.806	0 0 0	18
19	2	2	Ti	5	3	1	0	1	F	pz	5.806	0 0 0	19
20	2	2	Ti	6	3	1	1	1	F	px	5.806	0 0 0	20
21	2	2	Ti	7	4	1	-1	1	F	py	3.108	0 0 0	21
22	2	2	Ti	8	4	1	0	1	F	pz	3.108	0 0 0	22
23	2	2	Ti	9	4	1	1	1	F	px	3.108	0 0 0	23
24	2	2	Ti	10	3	2	-2	1	F	dxy	5.953	0 0 0	24
25	2	2	Ti	11	3	2	-1	1	F	dyz	5.953	0 0 0	25
26	2	2	Ti	12	3	2	0	1	F	dz2	5.953	0 0 0	26
27	2	2	Ti	13	3	2	1	1	F	dxz	5.953	0 0 0	27
28	2	2	Ti	14	3	2	2	1	F	dx2-y2	5.953	0 0 0	28
29	2	2	Ti	15	3	2	-2	2	F	dxy	4.754	0 0 0	29
30	2	2	Ti	16	3	2	-1	2	F	dyz	4.754	0 0 0	30
31	2	2	Ti	17	3	2	0	2	F	dz2	4.754	0 0 0	31
32	2	2	Ti	18	3	2	1	2	F	dxz	4.754	0 0 0	32
33	2	2	Ti	19	3	2	2	2	F	dx2-y2	4.754	0 0 0	33
34	3	3	O	1	2	0	0	1	F	s	4.931	0 0 0	34
35	3	3	O	2	2	0	0	2	F	s	3.653	0 0 0	35
36	3	3	O	3	2	1	-1	1	F	py	5.056	0 0 0	36
37	3	3	O	4	2	1	0	1	F	pz	5.056	0 0 0	37
38	3	3	O	5	2	1	1	1	F	px	5.056	0 0 0	38

.

.

.

Ti 3d

O 2p

Projections on the Ti 3d t_{2g} orbitals

```
$ <your_siesta_path>/Util/Contrib/APostnikov/fmpdos
Input file name (PDOS):
SrTiO3.PDOS
Output file name :
Ti-dxy.PDOS.dat
Extract data for atom index (enter atom NUMBER, or 0 to select all),
or for all atoms of given species (enter its chemical LABEL):
2
Extract data for n= ... (0 for all n ):
3
Extract data for l= ... (-1 for all l ):
2
Extract data for m= ... (9 for all m ):
-2

$ <your_siesta_path>/Util/Contrib/APostnikov/fmpdos
Input file name (PDOS):
SrTiO3.PDOS
Output file name :
Ti-dyz.PDOS.dat
Extract data for atom index (enter atom NUMBER, or 0 to select all),
or for all atoms of given species (enter its chemical LABEL):
2
Extract data for n= ... (0 for all n ):
3
Extract data for l= ... (-1 for all l ):
2
Extract data for m= ... (9 for all m ):
-1

$ <your_siesta_path>/Util/Contrib/APostnikov/fmpdos
Input file name (PDOS):
SrTiO3.PDOS
Output file name :
Ti-dxz.PDOS.dat
Extract data for atom index (enter atom NUMBER, or 0 to select all),
or for all atoms of given species (enter its chemical LABEL):
2
Extract data for n= ... (0 for all n ):
3
Extract data for l= ... (-1 for all l ):
2
Extract data for m= ... (9 for all m ):
1
```

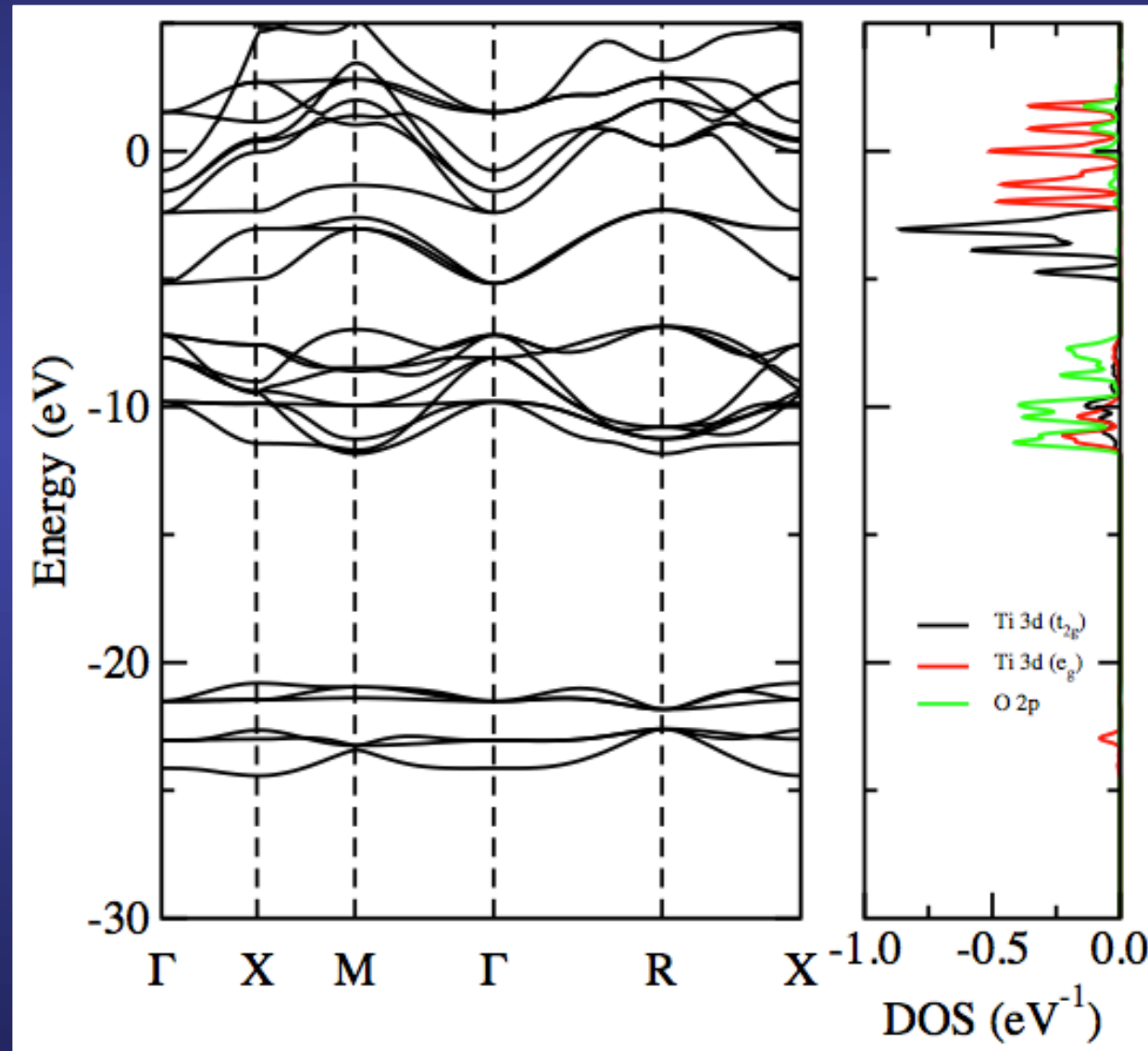
Projections on the Ti 3d e_g orbitals

```
$ <your_siesta_path>/Util/Contrib/APostnikov/fmpdos
Input file name (PDOS):
SrTiO3.PDOS
Output file name :
Ti-dz2.PDOS.dat
Extract data for atom index (enter atom NUMBER, or 0 to select all),
or for all atoms of given species (enter its chemical LABEL):
2
Extract data for n= ... (0 for all n ):
3
Extract data for l= ... (-1 for all l ):
2
Extract data for m= ... (9 for all m ):
0

$ <your_siesta_path>/Util/Contrib/APostnikov/fmpdos
Input file name (PDOS):
SrTiO3.PDOS
Output file name :
Ti-dx2-y2.PDOS.dat
Extract data for atom index (enter atom NUMBER, or 0 to select all),
or for all atoms of given species (enter its chemical LABEL):
2
Extract data for n= ... (0 for all n ):
3
Extract data for l= ... (-1 for all l ):
2
Extract data for m= ... (9 for all m ):
2
```

We can analyze the character of the different bands

Which atoms contribute more to the bands at a particular energy window
Zoom around the top of the valence bands and bottom of conduction bands



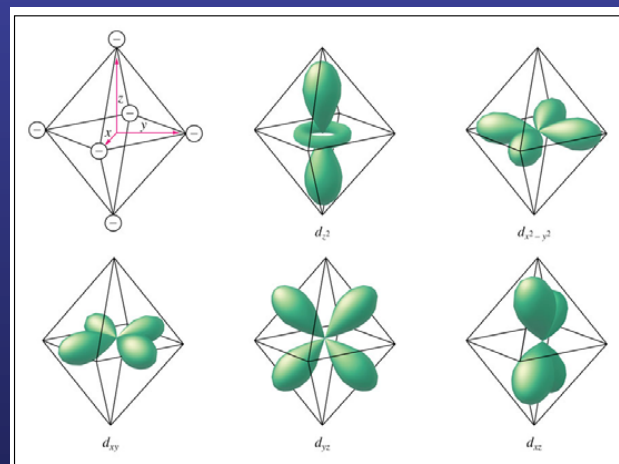
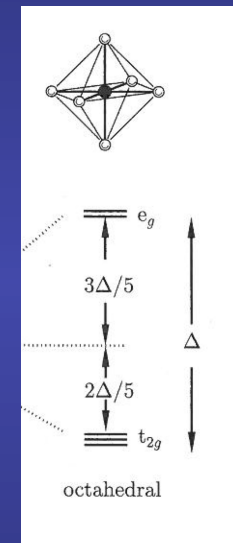
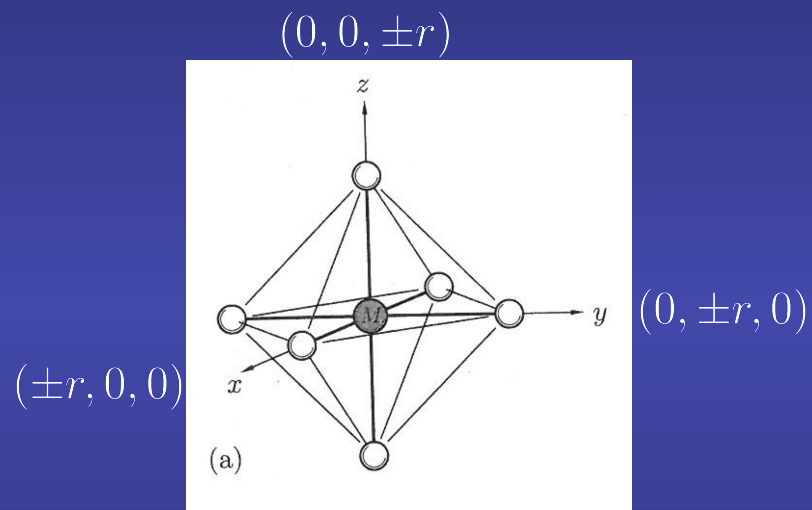
Bottom of conduction bands: mostly Ti character
The crystal field splits the t_{2g} and e_g bands

Top of valence bands: mostly O character (O 2p)

We can project on particular atomic orbitals within an atom to further define the character.

The environment affect the orbitals in different ways

In an octahedral environment the ligands (considered negative point charges) congregate at six points



$d_{z^2}, d_{x^2-y^2}$ point along the x, y, z axis will be raised in energy

d_{xy}, d_{yz}, d_{xz} point between the x, y, z axis will be lowered in energy