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COMPUTER-AIDED GEOMETRIC DESIGN AND COMPUTER GRAPHICS:

ILLUMINATION MODELS

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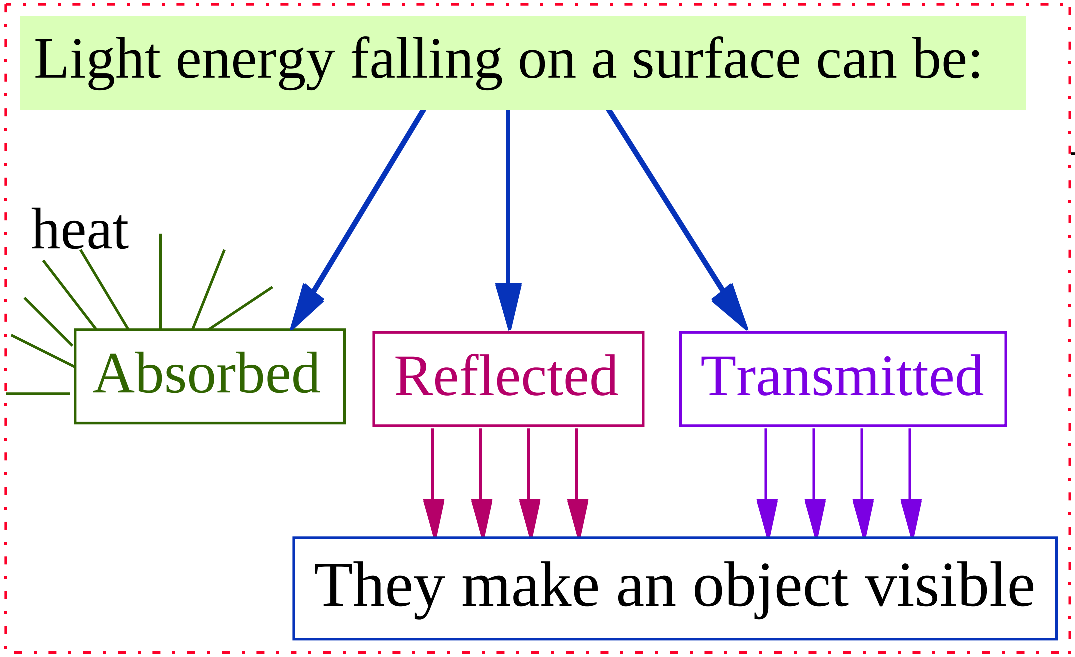
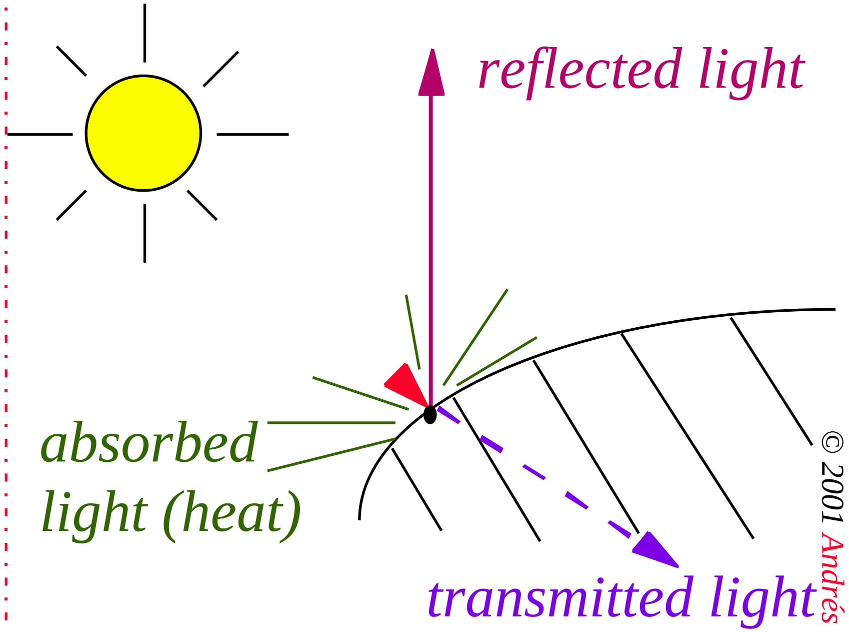
Illumination Models





The amount of energy absorbed, transmitted or deflected depends on the *wavelength* (w.l.) of the light.

If	then, the object
all the incident light energy is absorbed..	is invisible
nearly all the incident light energy is absorbed................................	appears black
only a small fraction is absorbed	appears white
the incident light energy is nearly equally reduced for all w.l.	appears gray
the incident light energy is selectively reduced for all w.l.	appears colored

Illumination Models

Rogers, D.F. *Procedural Elements for Computer Graphics*, McGraw-Hill, 2nd. Edition, 1998

The character of the light reflected or transmitted...

depends on:

- Composition of the light source
- Direction of the light source
- Geometry of the light source
- The surface orientation
- The surface properties of the object

and can be:

Diffuse: light that has penetrated below the surface of the object, been absorbed and then reemitted

→ Scattered equally in all directions

→ Observer's position is unimportant

Especular: light does not penetrate below the surface

→ It is not scattered

→ Character of light reamins unchanged

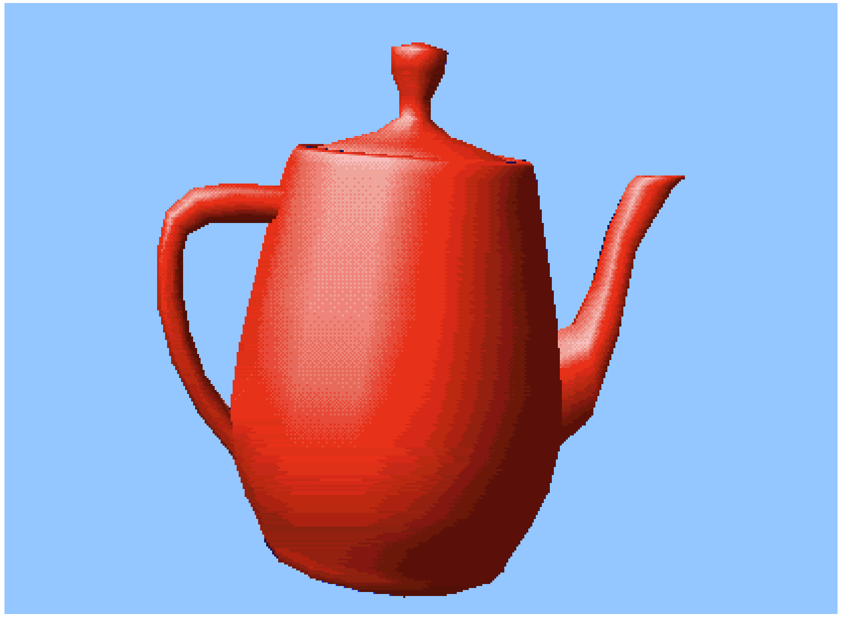
→ Light is independent of surface's color

Illumination Models

Each point may have several sources of illumination:

direct illumination
light arrives straight from the light sources

indirect illumination
light arrives after interacting with the rest of the scene



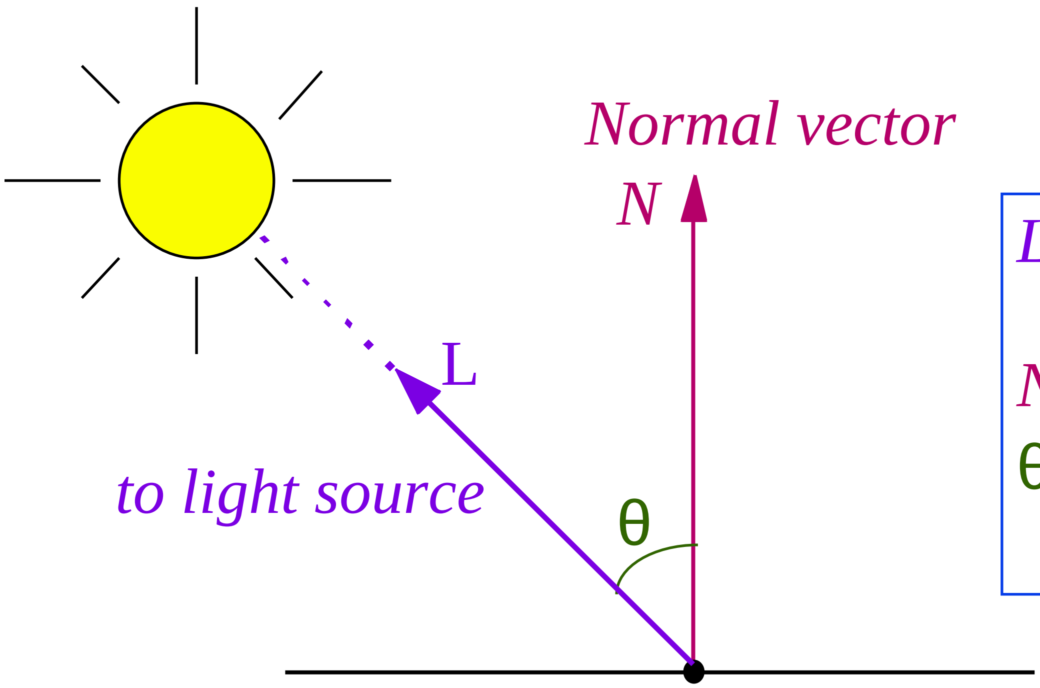
According to how they handle these sources, algorithms can be grouped into:

global illumination algorithms
Both kinds of sources are considered

local illumination algorithms
Only direct lights are taken into account

Illumination Models

Some useful definitions:



L - light source direction
N - Normal vector
θ − angle of incidence

- **N** is the **surface normal**
- **L** is the **direction to light source**
- Vectors **N** and **L** are *unit* vectors
- θ is the **angle of incidence**

Illumination Models

Illumination model 1: Ambient light

Ambient light

- Uniform from all directions
- K_α measures reflectivity of surface for diffuse light (values in the range: 0-1)

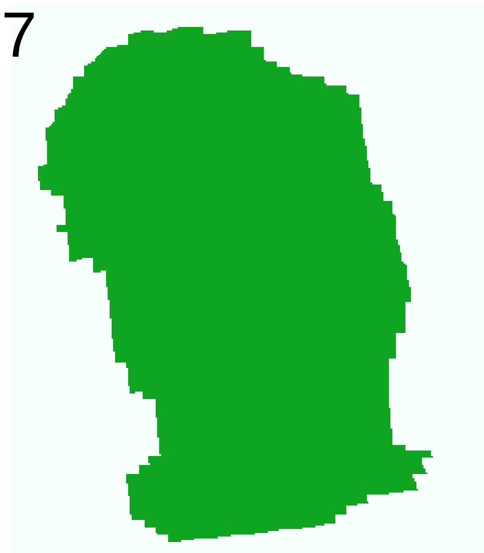
$$\boxed{I = K_\alpha \; I_\alpha}$$

I_α: Intensity of ambient light

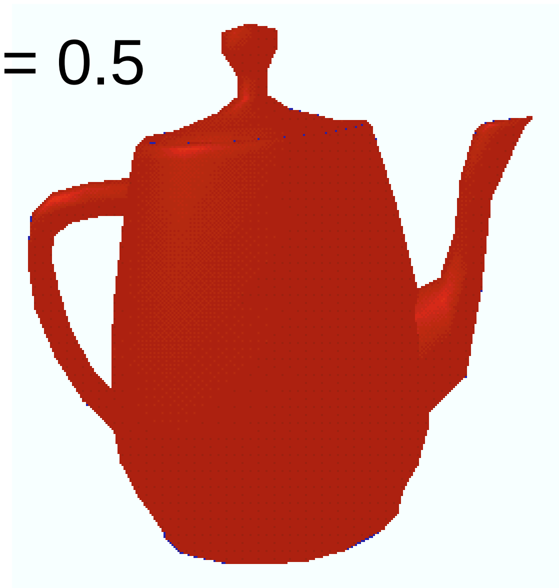
K_α: Ambient reflection coefficient

Problem: an object is illuminated uniformly

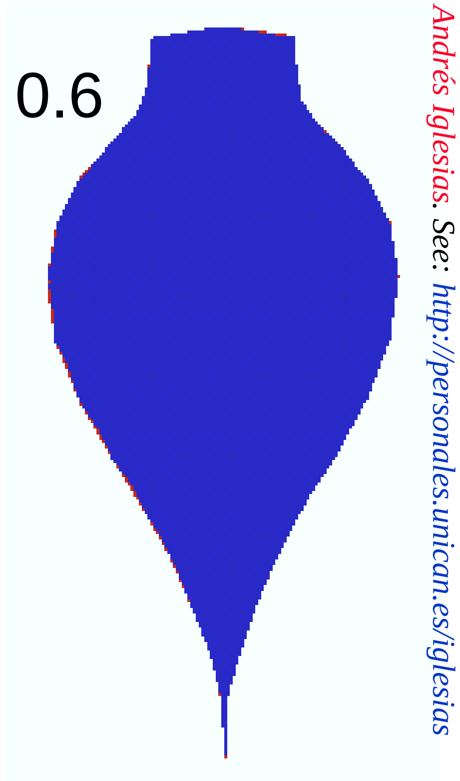
$K_\alpha = 0.7$



$K_\alpha = 0.5$



$K_\alpha = 0.6$



Illumination Models

Illumination model 2: Ambient + diffuse light

Lambert's Cosine Law

incident intensity from a point light source

wavelenght

diffuse reflection function $\quad (0 \leq K_d(\lambda) \leq 1)$

$$\boxed{I_d(\lambda) = I_l(\lambda)\, K_d(\lambda)\, \cos(\theta)} \qquad 0 \leq \theta \leq \frac{\pi}{2}$$

intensity of reflected diffuse light

Therefore, the Lambertian illumination model becomes:

$$I(\lambda) = \underbrace{I_l(\lambda)\, K_d(\lambda)\, \cos(\theta)}_{\text{diffuse light}} + \underbrace{K_\alpha(\lambda)\, I_\alpha(\lambda)}_{\text{ambient light}}$$

Illumination Models

Illumination model 2: Ambient + diffuse light

In practice, dependence on the wavelength λ is usually omitted:

$$\boldsymbol{I} = \underbrace{\boldsymbol{I_l}\ \boldsymbol{K_d} \cos(\theta)}_{\text{diffuse light}} + \underbrace{\boldsymbol{K_\alpha}\ \boldsymbol{I_\alpha}}_{\text{ambient light}} \qquad 0 \le \theta \le \frac{\pi}{2} \qquad \boldsymbol{K_\alpha} + \boldsymbol{K_d} < 1$$

Since N and L are unit vectors, it holds that: $\cos(\theta) = \boldsymbol{N} \bullet \boldsymbol{L}$

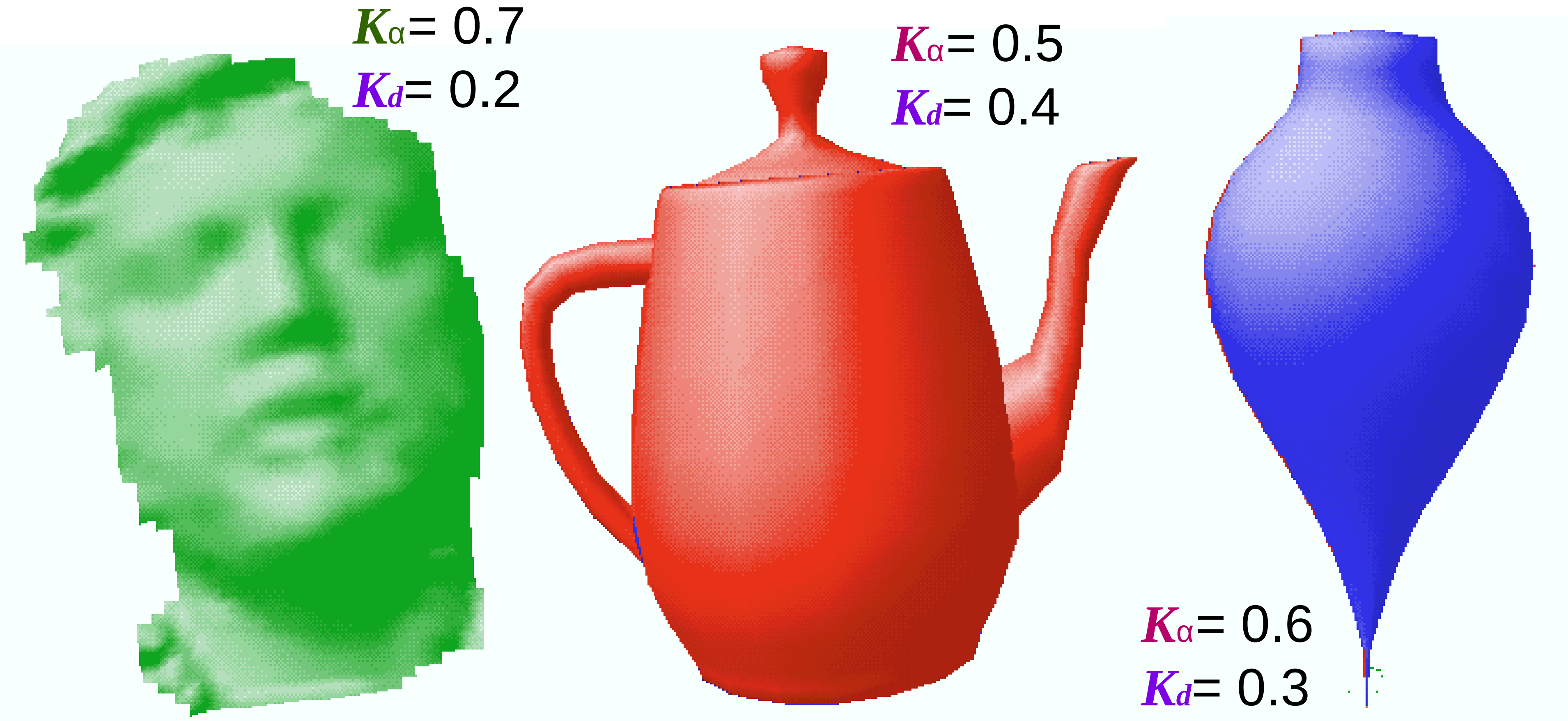
dot product

$$\boldsymbol{I} = \underbrace{\boldsymbol{I_l}\ \boldsymbol{K_d}\ (\boldsymbol{N} \bullet \boldsymbol{L})}_{\text{diffuse light}} + \underbrace{\boldsymbol{K_\alpha}\ \boldsymbol{I_\alpha}}_{\text{ambient light}} \qquad 0 \le \theta \le \frac{\pi}{2} \qquad \boldsymbol{K_\alpha} + \boldsymbol{K_d} < 1$$

Illumination Models

Illumination model 2: Ambient + diffuse light

Surfaces with a simple Lambertian diffuse reflection appear to have a *dull matte* surface:



Illumination Models

Illumination model 3: Ambient + diffuse + specular light

Many surfaces are *shiny*

At certain viewing angles, *shiny* surfaces produces *highlights*

Highlights are due to *specular reflection*

Specular reflection is *directional*

Highlights occur over a narrow range of angles

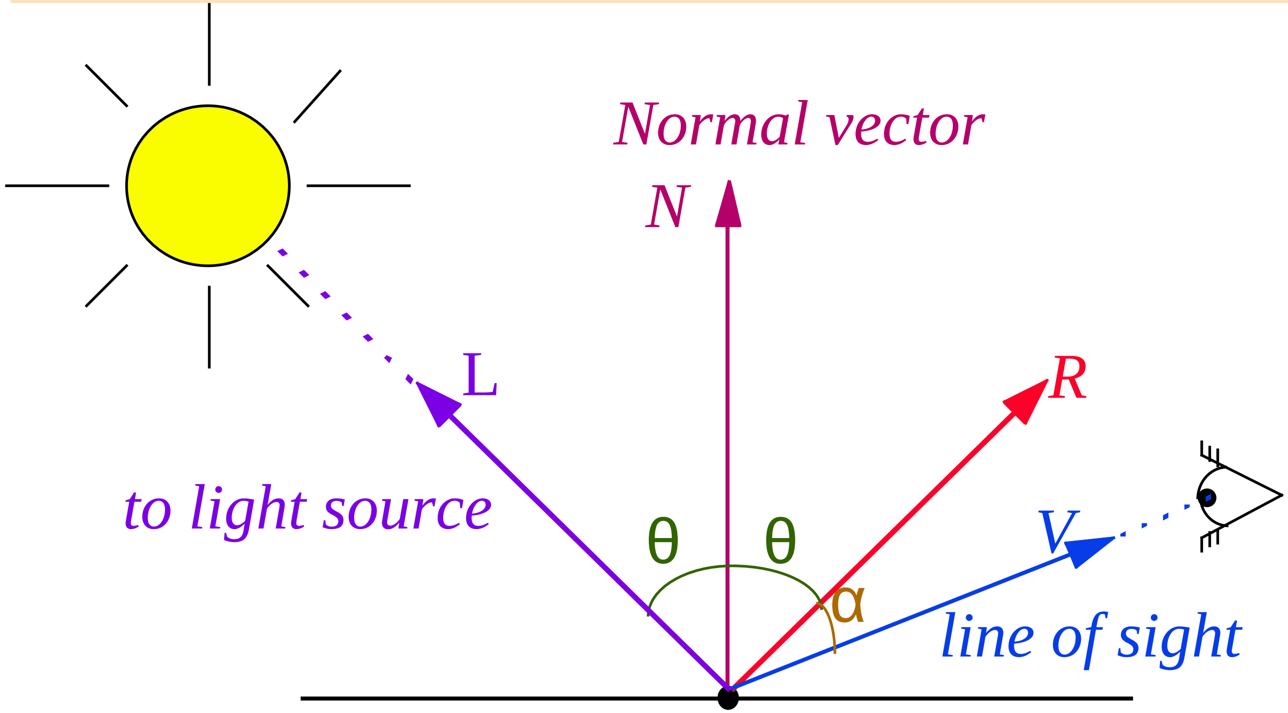
Highlights color same as the illuminating light rather than the color of the surface

Highlights move with the observer

- For a perfect reflecting surface (a mirror) the angle of reflection is equal to the angle of incidence
- For smooth surfaces, the spatial distribution of specular light is narrow.
- For rough surfaces, it is spread out.

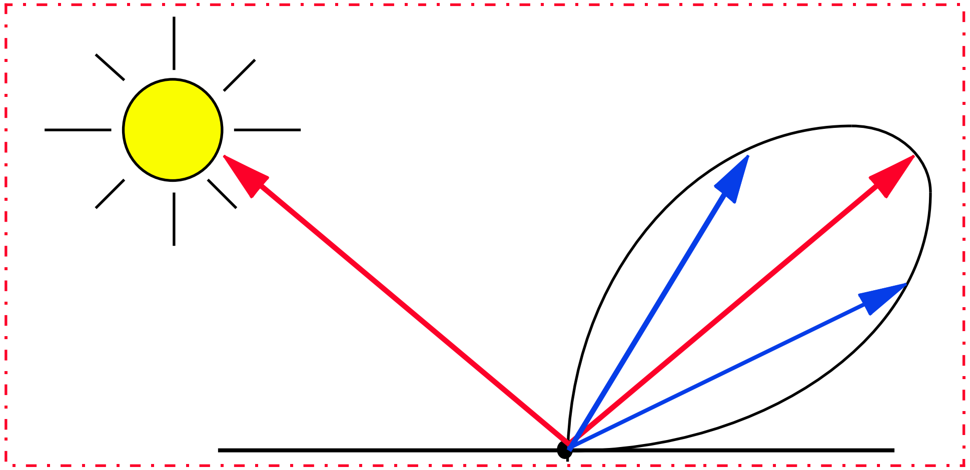
Illumination Models

Illumination model 3: Ambient + diffuse + specular light



L - light source direction
N - Normal vector
θ – angle of incidence
V - line of sight
R - direction of ideal specular reflection
α – angle between *R* and *V*

If α =0, we have a perfect reflecting surface (a mirror). An observer located here sees any specularly reflected light. Otherwise, we have a spatial distribution like:



Illumination Models

Illumination model 3: Ambient + diffuse + specular light

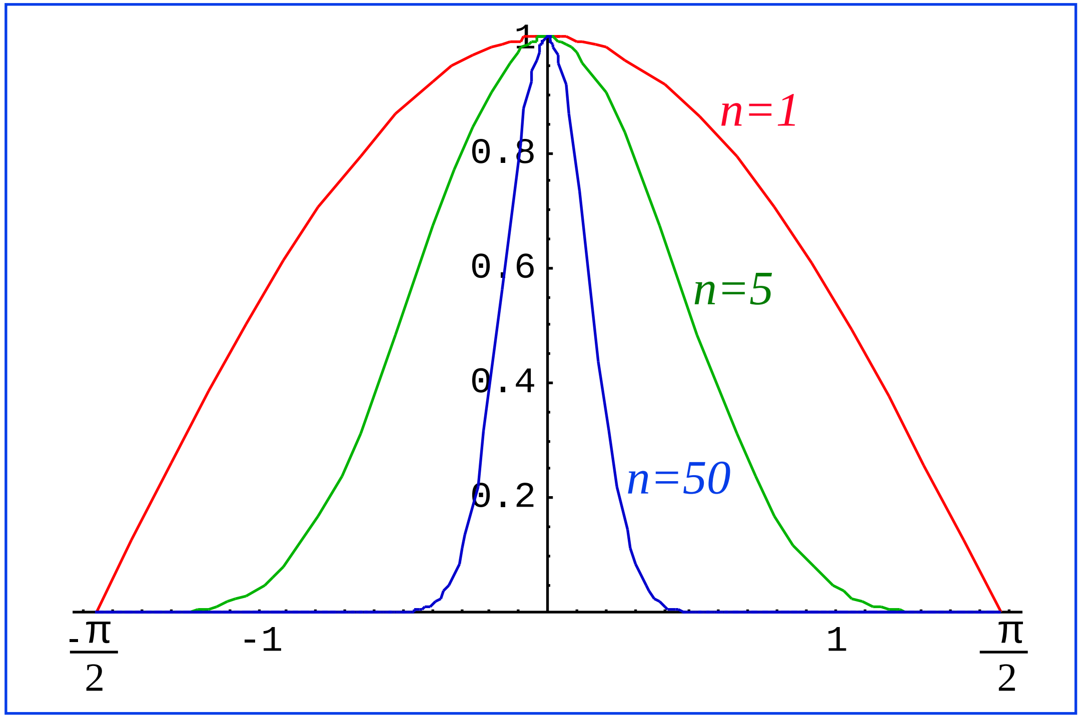
Phong Model

Because of the complex physical characteristics of the specular light, an *empirical model* based on taking the function:

$$f(\alpha)=\cos^n(\alpha)$$

where n depends on surface properties. For:

- a perfect reflector, $n=\infty$
- very poor reflector $n=1$
- in practice use $1 \leq n \leq 200$



In general, we use:
Larger values of n for metals and other shiny surfaces
Small values of n for nonmetallic surfaces (e.g., paper)

Illumination Models

Illumination model 3: Ambient + diffuse + specular light

Phong's empirical model controls the size of the specular highlight

incident intensity from a point light source

wavelenght

reflectance curve

$$\boxed{I_s(\lambda) = I_l(\lambda)\, w(i,\lambda)\, \cos^n(\alpha)}$$

intensity of reflected specular light

$w(i,\lambda)$: ratio of the specularly reflected light to the incident light, as a function of the incidence angle, i, and the wavelength λ

Combining this term with model 2:

Total Intensity light

$$I(\lambda) = K_\alpha(\lambda)\, I_\alpha(\lambda) + I_l(\lambda)\, K_d(\lambda)\, \cos(\theta) + I_l(\lambda)\, w(i,\lambda)\, \cos^n(\alpha)$$

Ambient light +

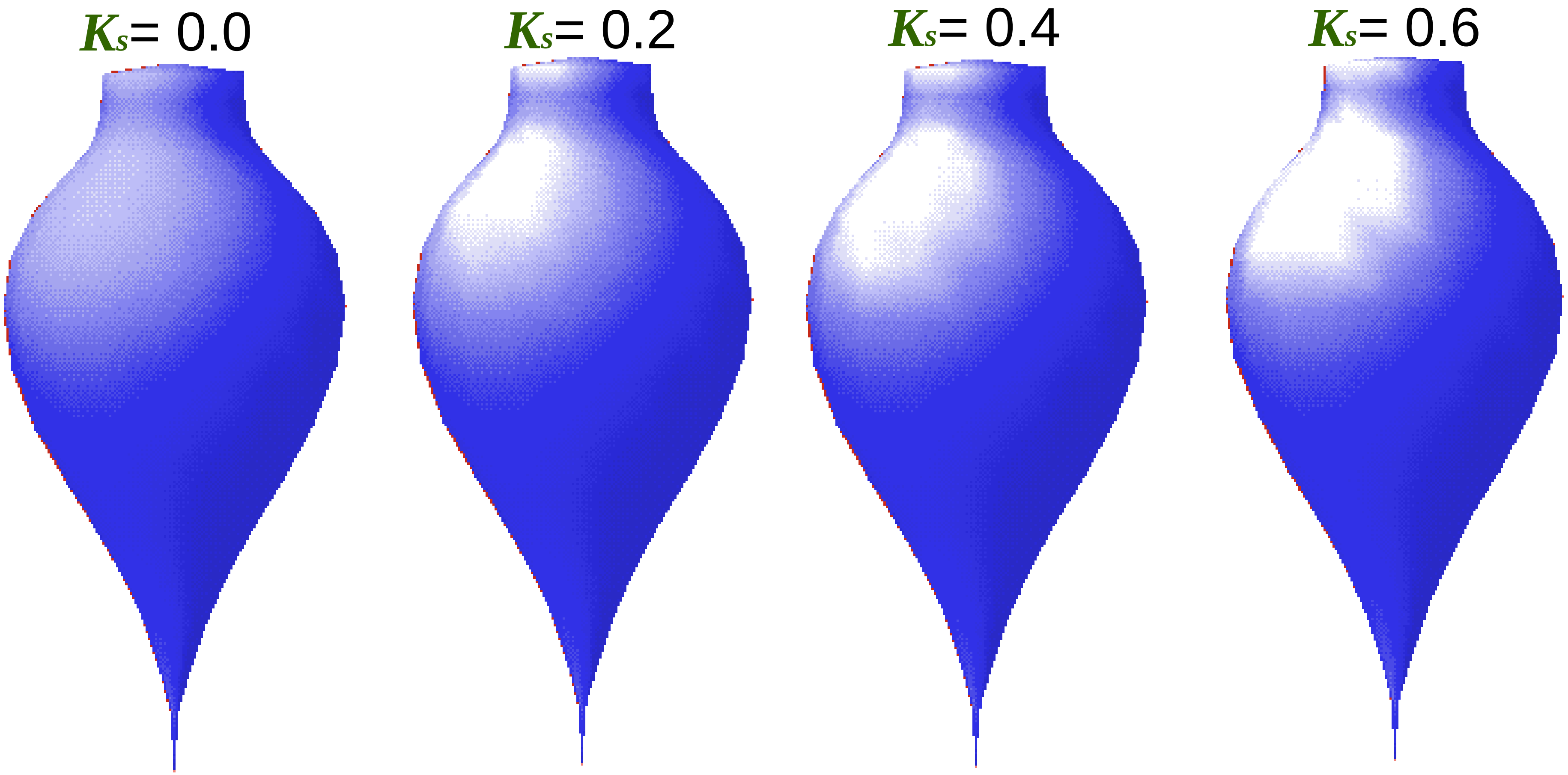
Diffuse light +

Specular light

Illumination Models

Illumination model 3: Ambient + diffuse + specular light

In practice, dependence on the wavelength λ is usually omitted. In addition, $w(i,\lambda)$ is a very complex function, so it is replaced by an aesthetically or experimentally determined constant k_s



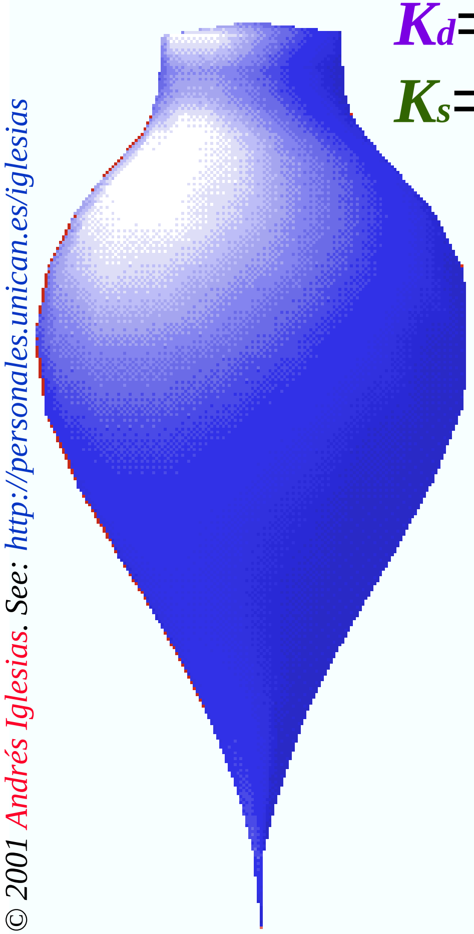
Illumination Models

Illumination model 3: Ambient + diffuse + specular light

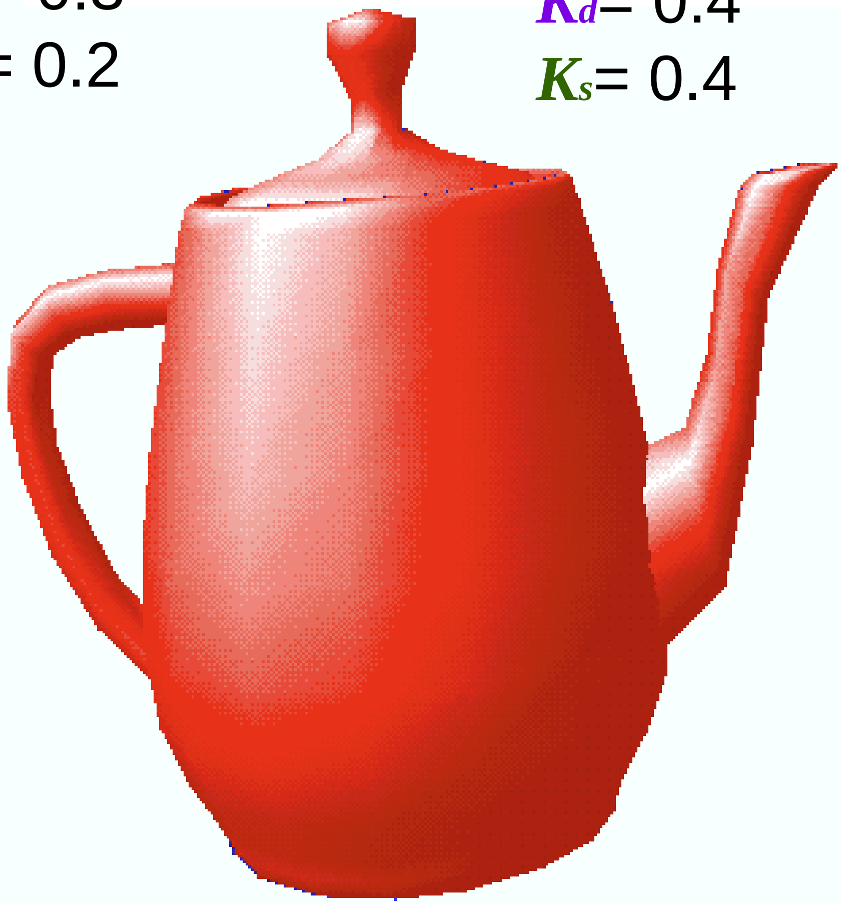
$$I = K_\alpha I_\alpha + I_l K_d \cos(\theta) + I_l K_s \cos^n(\alpha)$$

K_α *Ambient reflection*
K_d *Diffuse reflection*
K_s *Specular reflection*

$K_\alpha = 0.6$
$K_d = 0.3$
$K_s = 0.2$



$K_\alpha = 0.5$
$K_d = 0.4$
$K_s = 0.4$



$K_\alpha = 0.7$
$K_d = 0.2$
$K_s = 0.8$

Illumination Models

Illumination model 3: Ambient + diffuse + specular light

Noting that:

$\cos(\theta) = N.L$

$\cos(\alpha) = R.V$

the model becomes:

$$I = K_\alpha I_\alpha + I_l K_d (N.L) + I_l K_s (R.V)^n$$

However, two objects *at different distances* but with *the same orientation* to the light source exhibit *the same intensity*.

The intensity of light decreases inversely as the square of the distance.

Objects farther away appear dimmer !!!!!

$$I = \frac{I_{source}}{4 \pi d^2}$$

d = source distance

In practice, this model produces unrealistic variations in intensity.

Experimental model:

$$I = \frac{I_{source}}{d^p + K}$$

$0 \le p \le 2$

K is an arbitrary constant used to prevent infinite intensity when d=0

Illumination Models

Illumination model 4: *Ambient* + *diffuse* + *specular* + *attenuation* *light*

Phong, B. T. *Illumination for Computer Generated Images*, Communications of the ACM, Vol. 18, pp. 311-317, 1975.

Phong Model:

$$I = K_\alpha I_\alpha + \frac{I_l}{d^p + K}\left(K_d \cos(\theta) + K_s \cos^n(\alpha)\right)$$

$$I = Ambient + attenuation\ (\ diffuse + specular\ )$$

$$I = K_\alpha I_\alpha + \frac{I_l}{d^p + K}\left(K_d\ (N.L) + K_s\ (R.V)^{\,n}\right)$$

Individual shading functions are used for each of the three primary colors

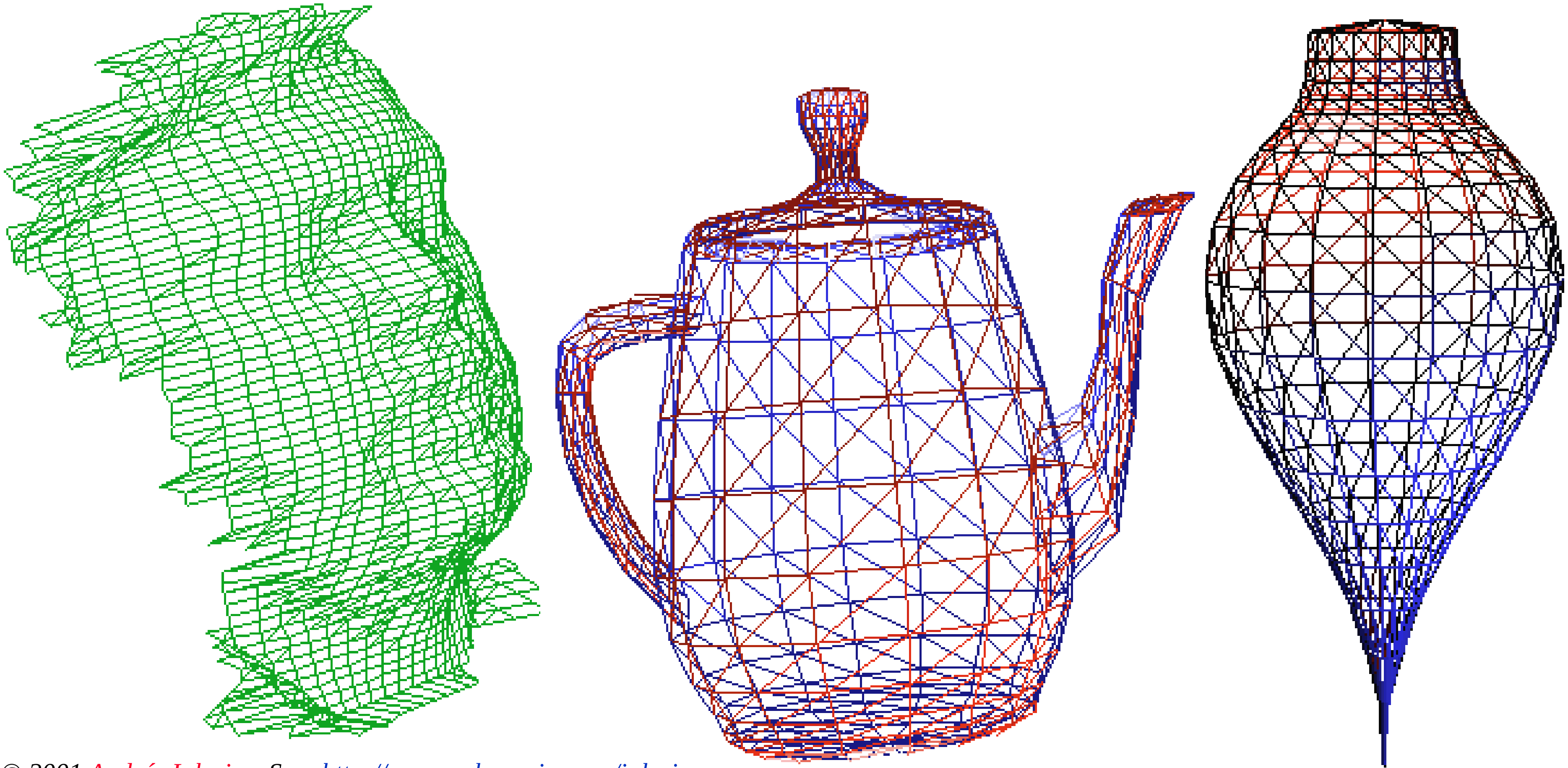
Multiple light sources: their effects are to be *linearly added*.

Illumination Models

Polygon shading

Until now, we compute the intensity light at a single point on a surface

But, many objects are given by meshes of polygons !!!!!



Illumination Models

Polygon shading

How to compute the intensity across the polygon ?

1. ~~Compute the shade at all points: ***Unnecessary and impractical***~~

2. Compute the shade at the centre and use this to represent the whole polygon: ***Flat shading***

3. Compute shade at key points and interpolate for the rest: ***Gouraud*** and ***Phong shading***

Flat

Gouraud

Phong

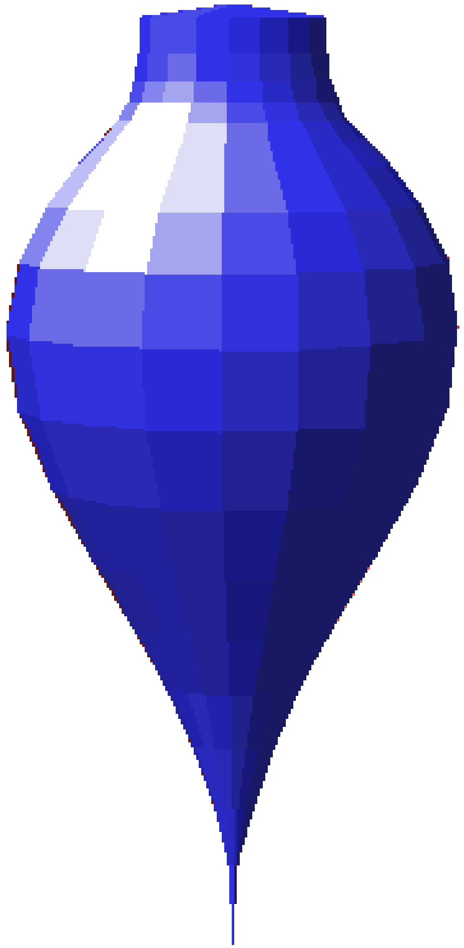
Illumination Models

Polygon shading: Flat shading

Is the simplest method and the most computationally efficient.
But it is also:

1. *Not realistic*: it exhibits polygon structure.

Faceted Appearance

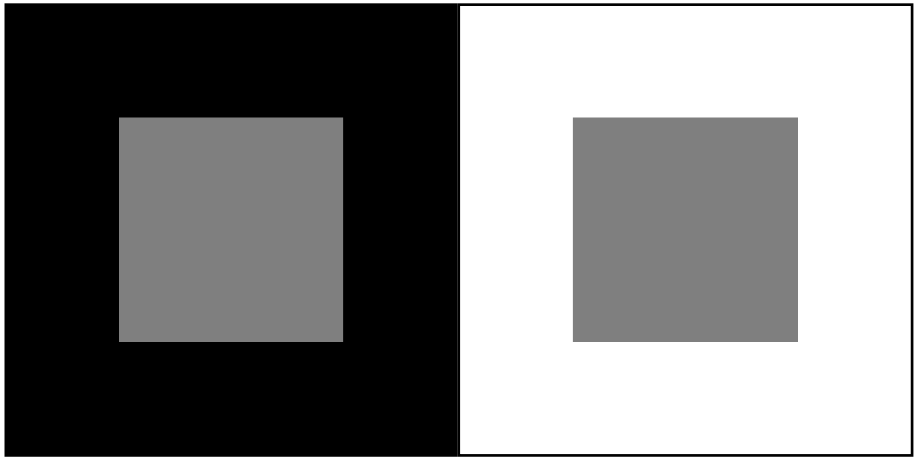


Illumination Models

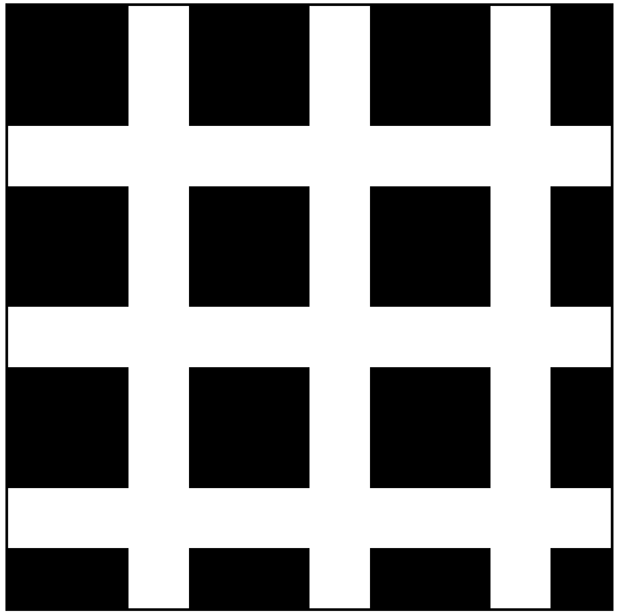
Polygon shading: Flat shading

2. *Simultaneous contrast*: an area of constant brightness surrounded by a dark area is perceived to be brighter than the same area surrounded by a light area

3. *Mach banding*: brightness perceived by the eye tends to overshoot at the boundaries of regions of constant intensity



$I_{background} = 1$	$I_{background} = 0$
$I_{center} = 0.5$	$I_{center} = 0.5$



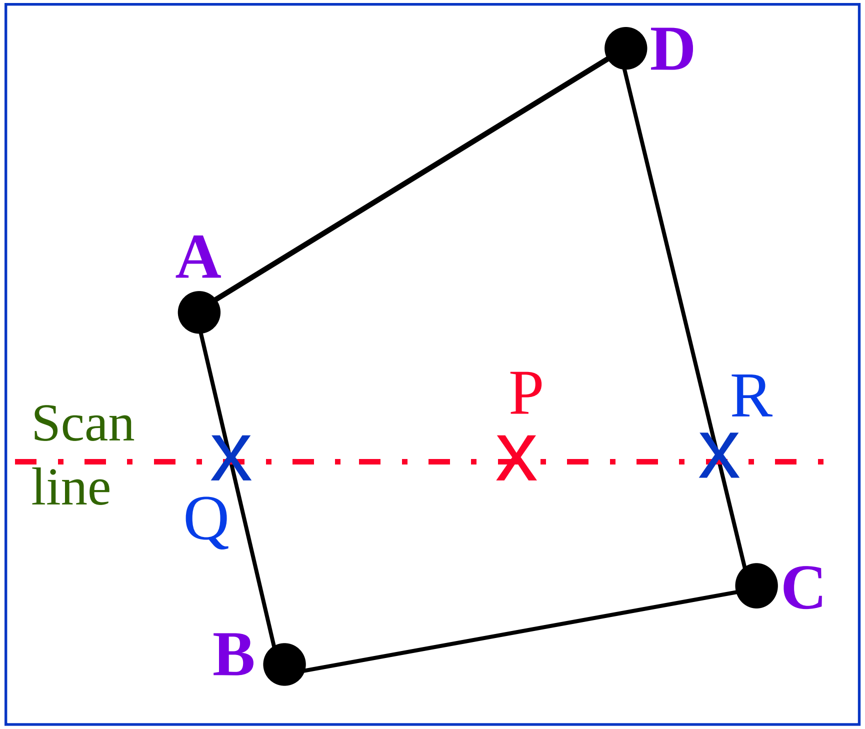
Abrupt changes in the shading of two adjacent polygons are perceived to be even greater

Illumination Models

Polygon shading: Gouraud shading

Gouraud, H. *Computer Display of Curved Surfaces*, IEEE Transactions on Comput., C-20, pp. 623-628, 1971.

Given a polygon and a scan-line, the problem is to determine the intensity at an interior point, such as P



1. First compute the intensity values at each polygon vertex

   Output: I_A, I_B, I_C, I_D

2. Next compute the intensity at points Q and R using linear interpolation

   Output: I_Q, I_R

3. Finally, linearly interpolate between I_Q and I_R to get intensity at point P

   Output: I_P

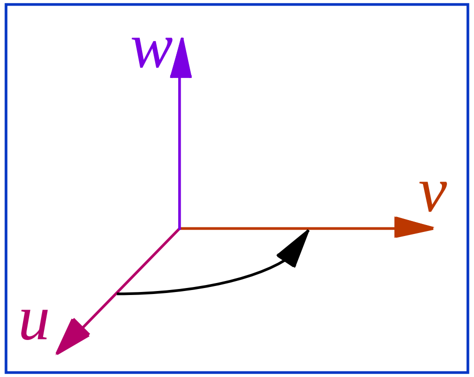
Illumination Models

Polygon shading: Gouraud shading

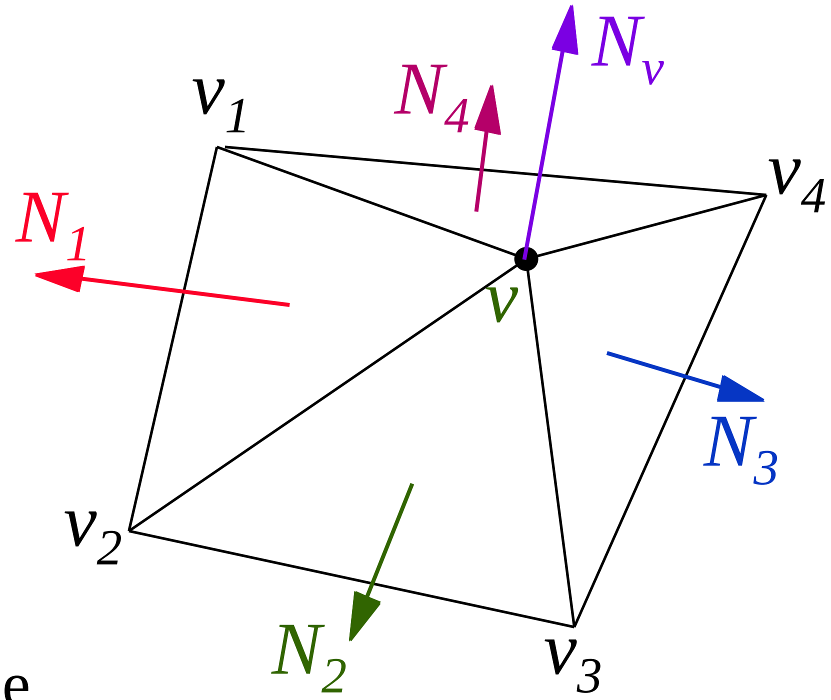
1. First compute the intensity values at each polygon vertex

We need the vertex normal vector N_v

Cross product



$$w = u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$



avering the cross products x of all the edges that terminate at the vertex

$$\boxed{N_v = \underbrace{v.v_1 \times v.v_2}_{N_1} + \underbrace{v.v_2 \times v.v_3}_{N_2} + \underbrace{v.v_3 \times v.v_4}_{N_3} + \underbrace{v.v_4 \times v.v_1}_{N_4}}$$

Polygon shading: Gouraud shading

2. Next compute the intensity at points Q and R using linear interpolation

Intensity at a point Q:

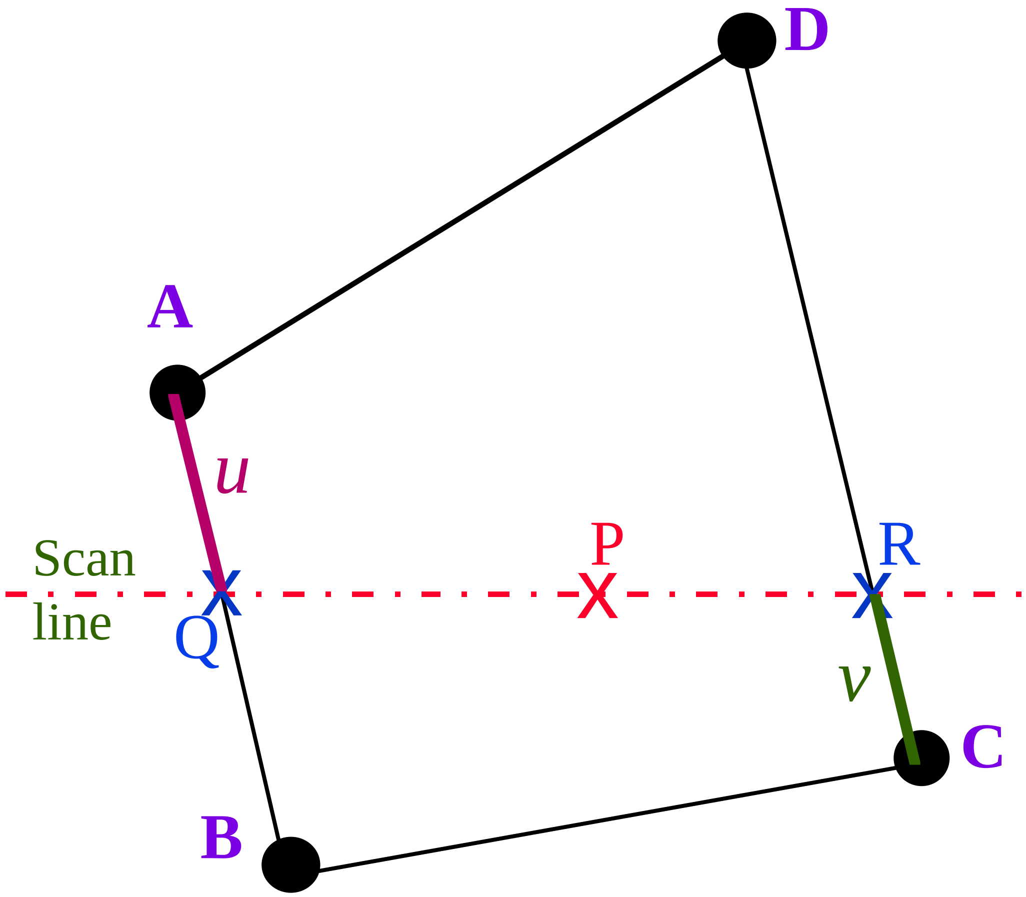
$$I_Q = u\, I_A + (1-u)\, I_B$$

where $u = \dfrac{AQ}{AB}$

Intensity at a point R:

$$I_R = v\, I_C + (1-v)\, I_D$$

where $v = \dfrac{CR}{CD}$



Illumination Models

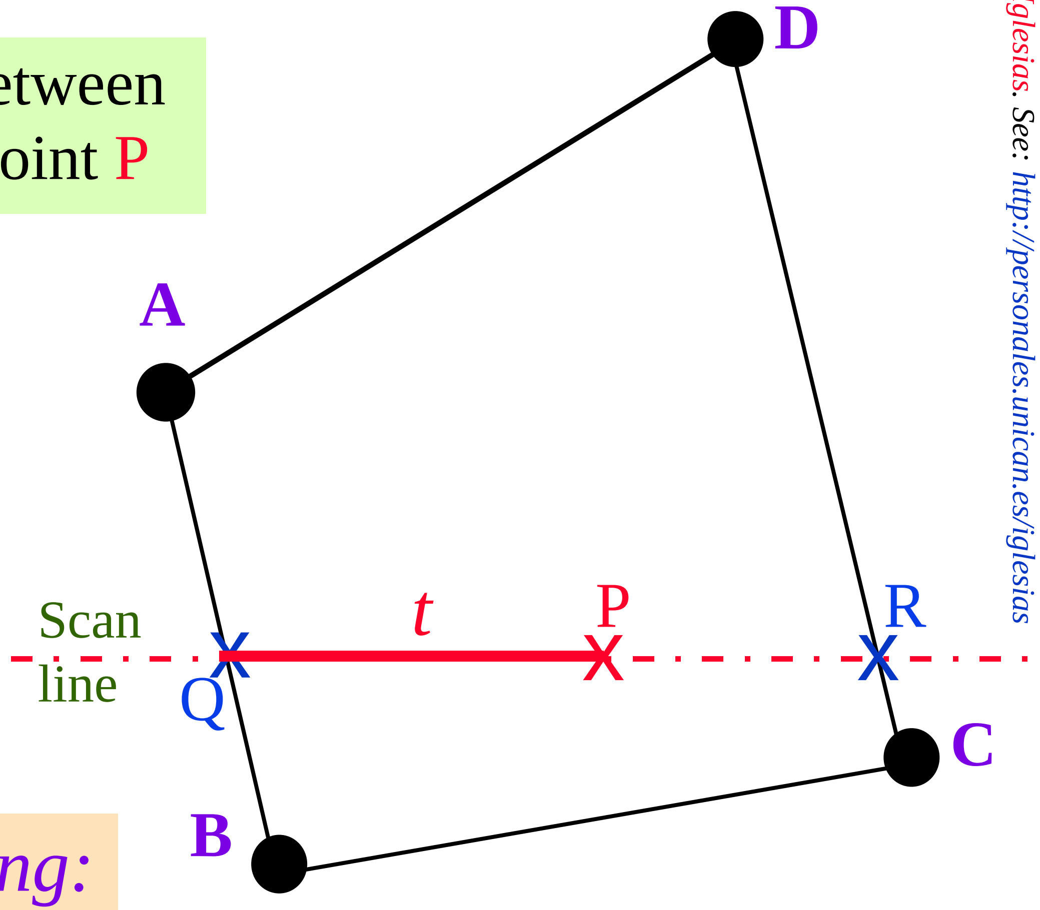
Polygon shading: Gouraud shading

3. Finally, linearly interpolate between I_Q and I_R to get intensity at the point P

Intensity at a point P:

$$I_R = t\, I_Q + (1-t)\, I_R$$

where $t = \dfrac{QP}{QR}$



Problems of Gouraud shading:

- This method yields only to continuity of intensity, but not continuity of changes of intensity => *Mach banding*
- Silhouette edges are still polygonal

Illumination Models

Polygon shading: Phong shading

It is similar to Gouraud shading, except we linearly interpolate the *surface normal vector* across the polygon

Determine: N_A, N_B, N_C, N_D

$$N_Q = u\,N_A + (1-u)\,N_B$$
$$N_R = v\,N_C + (1-v)\,N_D$$

$$N_R = t\,N_Q + (1-t)\,N_R$$

It is computationally more expensive.

Gouraud shading loses specular reflection if the hightlight lies inside a single polygon