PROBLEM SET NUMBER 2
Due Wednesday, October 29, 2008 (in lecture)

Reading Assignment for week of October 20.

Review Power-Point notes on methods for solving linear systems.

Learning Objectives for Week of October 20

1. Know the fundamental steps for Gaussian elimination and LU decomposition and be able to solve small problems by hand.
2. Be able to explain the importance and value of partial pivoting, and of scaling, with Gaussian elimination.
3. Know the definition of a FLOP and be able to explain its importance, as well as use it in the comparison of alternative numerical algorithms, and estimate variations in run times with problem size.
4. Be able to determine the number of FLOPS necessary to perform matrix computations.
5. Know the properties of vector and matrix norms.
6. Be able to compute the condition number of a matrix A (either 2x2 or diagonal 3x3) using the infinity and spectral norms; and know how the condition number can be used to predict the loss of significance in equation solving.
7. Know the steps in the Jacobi algorithm for the iterative solution of systems of linear equations.

Assignment

1.- Consider the following system of linear equations:

$$\begin{aligned} 10^{-n}x + 2y &= 8 \\ x + y &= 2 \end{aligned}$$

For $n = 2, 3$ solve the system by using gaussian elimination without pivoting and floating point arithmetic with 3 significant digits (and rounding). Analyse how different the result obtained is in comparison to the exact solution of the problem.

2.- Obtain the LU factorization of the matrix

$$A = \begin{pmatrix} 2 & 4 & -1 & 0 \\ 4 & 10 & -1 & -1 \\ 6 & 10 & -7 & 1 \\ 0 & 2 & 1 & -2 \end{pmatrix}$$

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3.- Show that the nonsingular matrix

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

has no LU factorization, while the singular matrix $A - I$ has it.

4.- Obtain the Choleski factorization of the matrix

$$A = \begin{pmatrix} 4 & 2 & -2 \\ 2 & 5 & 5 \\ -2 & 5 & 11 \end{pmatrix}$$