Nonlinear electronic transport in semiconductor superlattices

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Nonlinear charge transport in strongly coupled semiconductor superlattices is described by single or two-miniband Wigner-Poisson kinetic equations with BGK collision terms. Balance equations for miniband populations and electric field are derived using the Chapman-Enskog method. Numerical solutions show stable self-oscillations of the current through a voltage biased superlattice.

Keywords: Quantum drift-diffusion equations, Chapman-Enskog method, Rashba spin-orbit interaction, modified Kane model.

1. Introduction

Semiconductor superlattices are essential ingredients in fast nanoscale oscillators, quantum cascade lasers and infrared detectors. A superlattice (SL) is a quasi-one-dimensional crystal originally proposed by Esaki and Tsu to observe Bloch oscillations, i.e., the periodic coherent motion of electrons in a miniband when an electric field is applied. Once the materials were grown, many interesting nonlinear phenomena were observed, such as self-oscillations of the current through the SL due to charge dipole motion, multistability of stationary charge and field profiles, etc. See the review 1.

Nonlinear charge transport in SLs has been widely studied in the last decade using balance equations for electron densities and electric field.

These equations are either proposed using phenomenological arguments or derived ad hoc from kinetic theories.[1,2] Systematic derivations are scarce. For a single-miniband SL, the Chapman-Enskog (CE) method applied to a semiclassical Boltzmann-Poisson system whose collision term is of Bhatnagar-Gross-Krook (BGK) type yields a generalized drift-diffusion equation (GDDE),[3] and a quantum drift-diffusion equation (QDDE) when applied to a Wigner-Poisson-BGK (WPBGK) system.[4] For a semiclassical parabolic-band BGK-Poisson semiconductor system, the CE method had earlier been used to obtain balance equations.[5] The quantum WPBGK system contains two pseudo-differential operators, involving the band dispersion relation and the electric potential. The leading order approximation in the hyperbolic limit balances collisions and electric potential, and its solution is not obvious because the potential is an a priori unknown solution of the Poisson equation. SLs are simpler because their Wigner functions are periodic in the reciprocal lattice, the potential terms become multiplication operators in Fourier space, and the leading order approximation is straightforward to solve.[4]

For sufficiently high applied electric fields, electrons may populate higher minibands, then be scattered to the lowest, etc. Moreover, SLs with diluted magnetic impurities subject to a magnetic field may present spin polarization effects whose understanding is crucial to develop spintronic devices.[6] Even without magnetic impurities, spin polarization could appear due to Rashba spin-orbit interaction.[7] Once we consider electron spin, each miniband is split in two and single-miniband SLs become two-miniband SLs. We shall systematically derive quantum balance equations by the CE method.

2. Single miniband superlatice

The Wigner-Poisson-Bhatnagar-Gross-Krook (WPBGK) system for 1D electron transport in the lowest miniband of a strongly coupled SL is:

$$
\begin{aligned}
&\frac{\partial f}{\partial t} + \frac{i}{\hbar}\left[\mathcal{E}\left(k + \frac{1}{2i}\frac{\partial}{\partial x}\right) - \mathcal{E}\left(k - \frac{1}{2i}\frac{\partial}{\partial x}\right)\right] f \\
&\quad + \frac{ie}{\hbar}\left[W\left(x + \frac{1}{2i}\frac{\partial}{\partial k}, t\right) - W\left(x - \frac{1}{2i}\frac{\partial}{\partial k}, t\right)\right] f \\
&\quad = Q[f] \equiv -\nu_e\left(f - f^{FD}\right) - \nu_i\,\frac{f(x,k,t) - f(x,-k,t)}{2}, \qquad (2.1)
\end{aligned}
$$

$$
\varepsilon\,\frac{\partial^2 W}{\partial x^2} = \frac{e}{l}\,(n - N_D), \qquad (2.2)
$$

with

$$n = \frac{l}{2\pi}\int_{-\pi/l}^{\pi/l} f(x,k,t)dk = \frac{l}{2\pi}\int_{-\pi/l}^{\pi/l} f^{FD}(k;n)dk, \tag{2.3}$$

$$f^{FD}(k;n) = \frac{m^* k_B T}{\pi^2\hbar^2}\,\ln\left[1+\exp\left(\frac{\mu-\mathcal{E}(k)}{k_B T}\right)\right]. \tag{2.4}$$

Here f, n, N_D, $\mathcal{E}(k)$, d_B, d_W, $l = d_B + d_W$, W, ε, m^*, k_B, T, ν_e, ν_i and $-e < 0$ are the one-particle Wigner function, the 2D electron density, the 2D doping density, the miniband dispersion relation, the barrier width, the well width, the SL period, the electric potential, the SL permittivity, the effective mass of the electron, the Boltzmann constant, the lattice temperature, the frequency of the inelastic collisions responsible for energy relaxation, the frequency of the elastic impurity collisions and the electron charge, respectively.

The left-hand side of Eq. (2.1) can be straightforwardly derived from the Schrödinger-Poisson equation for the wave function in the miniband using the definition of the 1D Wigner function:

$$f(x,k,t) = \frac{2l}{S}\sum_{j=-\infty}^{\infty}\int \langle\psi^\dagger(x+\frac{jl}{2},y,z,t)\psi(x-\frac{jl}{2},y,z,t)\rangle e^{ijkl}dxdy. \tag{2.5}$$

The second quantized wave function ψ is a superposition of the Bloch states corresponding to the miniband and S is the SL cross section.[4] The right hand side in Eq. (2.1) is the sum of $-\nu_e\left(f - f^{FD}\right)$, which represents energy relaxation towards a 1D effective Fermi-Dirac (FD) distribution $f^{FD}(k;n)$ (local equilibrium, which is the 3D Fermi-Dirac distribution integrated over the lateral components of the wave vector (k, k_y, k_z)), and $-\nu_i[f(x,k,t) - f(x,-k,t)]/2$, which accounts for impurity elastic collisions.[3] For simplicity, the collision frequencies ν_e and ν_i are fixed constants. Exact and FD distribution functions have the same electron density, thereby preserving charge continuity. $\mu = \mu(n)$ results from solving (2.3) with (2.4).

The WPBGK system (2.1) to (2.4) should have a $2\pi/l$-periodic (in k) solution satisfying appropriate initial and boundary conditions:

$$f(x,k,t) = \sum_{j=-\infty}^{\infty} f_j(x,t)\, e^{ijkl}. \tag{2.6}$$

Defining $F = \partial W/\partial x$ (*minus* the electric field) and the average

$$\langle F\rangle_j(x,t) = \frac{1}{jl}\int_{-jl/2}^{jl/2} F(x+s,t)\,ds, \tag{2.7}$$

it is possible to obtain the following equivalent form of the Wigner equation[4]

$$\frac{\partial f}{\partial t} + \sum_{j=-\infty}^{\infty} \frac{ijl}{\hbar}\, e^{ijkl} \left(\mathcal{E}_j \, \frac{\partial}{\partial x} \langle f \rangle_j + e\, \langle F \rangle_j \, f_j \right) = Q[f]. \tag{2.8}$$

Integrating (2.8) over k yields the charge continuity equation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} \sum_{j=1}^{\infty} \frac{2jl}{\hbar} \left\langle \mathrm{Im}(\mathcal{E}_{-j} f_j) \right\rangle_j = 0. \tag{2.9}$$

Here we can eliminate the electron density by using the Poisson equation and then integrate over x, thereby obtaining the nonlocal Ampère's law for the total current density $J(t)$:

$$\varepsilon \, \frac{\partial F}{\partial t} + \frac{2e}{\hbar} \sum_{j=1}^{\infty} j \langle \mathrm{Im}(\mathcal{E}_{-j} f_j) \rangle_j = J(t). \tag{2.10}$$

To derive the QDDE, we shall assume that the electric field contribution in (2.8) is comparable to the collision terms and that they dominate the other terms (*the hyperbolic limit*).[3] Let v_M and F_M be the electron velocity and field positive values at which the (zeroth order) drift velocity reaches its maximum. In this limit, the time t_0 it takes an electron with speed v_M to traverse a distance $x_0 = \varepsilon F_M l/(eN_D)$, over which the field variation is of order F_M, is much longer than the mean free time between collisions, $\nu_e^{-1} \sim \hbar/(eF_M l) = t_1$. We therefore define the *small parameter* $\lambda = t_1/t_0 = \hbar v_M N_D/(\varepsilon F_M^2 l^2)$ and formally multiply the first two terms on the left side of (2.1) or (2.8) by λ.[3,4] The result is

$$\lambda \left(\frac{\partial f}{\partial t} + \sum_{j=-\infty}^{\infty} \frac{ijl}{\hbar} e^{ijkl} \mathcal{E}_j \frac{\partial}{\partial x} \langle f \rangle_j \right) = Q[f] - \sum_{j=-\infty}^{\infty} \frac{iejl}{\hbar} e^{ijkl} \langle F \rangle_j f_j \tag{2.11}$$

The solution of Eq. (2.11) for $\lambda = 0$ is calculated in terms of its Fourier coefficients as

$$f^{(0)}(k;F) = \sum_{j=-\infty}^{\infty} \frac{(1 - ij\mathcal{F}_j/\tau_e)\, f_j^{FD}}{1 + j^2 \mathcal{F}_j^2}\, e^{ijkl}, \tag{2.12}$$

where $\mathcal{F}_j = \langle F \rangle_j / F_M$, $F_M = \hbar \sqrt{\nu_e(\nu_e + \nu_i)}/(el)$ and $\tau_e = \sqrt{(\nu_e + \nu_i)/\nu_e}$.

The CE ansatz for the Wigner function is:[4]

$$f(x,k,t;\lambda) = f^{(0)}(k;F,n) + \sum_{m=1}^{\infty} f^{(m)}(k;F,n)\, \lambda^m, \tag{2.13}$$

$$\varepsilon \frac{\partial F}{\partial t} + \sum_{m=0}^{\infty} J^{(m)}(F,n)\, \lambda^m = J(t). \tag{2.14}$$

The coefficients $f^{(m)}(k;F,n)$ depend on the 'slow variables' x and t only through their dependence on F and n, which obey (2.2) and (2.14). The functionals $J^{(m)}(F,n)$ are chosen so that the $f^{(m)}(k;F,n)$ are bounded and $2\pi/l$-periodic in k. Keeping the desired number of terms and setting $\lambda=1$ in (2.14) yields the sought QDDE. Inserting (2.13) - (2.14) in (2.11), we find the hierarchy:

$$\mathcal{L}f^{(1)} = -\left.\frac{\partial f^{(0)}}{\partial t}\right|_0 + \sum_{j=-\infty}^{\infty} \frac{ijl\mathcal{E}_j e^{ijkl}}{\hbar}\frac{\partial}{\partial x}\langle f^{(0)}\rangle_j \tag{2.15}$$

$$\mathcal{L}f^{(2)} = -\left.\frac{\partial f^{(1)}}{\partial t}\right|_0 + \sum_{j=-\infty}^{\infty} \frac{ijl\mathcal{E}_j e^{ijkl}}{\hbar}\frac{\partial}{\partial x}\langle f^{(1)}\rangle_j - \left.\frac{\partial}{\partial t}f^{(0)}\right|_1, \tag{2.16}$$

and so on. Here

$$\mathcal{L}u(k) \equiv \frac{ie}{\hbar}\sum_{-\infty}^{\infty} jl\langle F\rangle_j u_j e^{ijkl} + \left(\nu_e + \frac{\nu_i}{2}\right)u(k) - \frac{\nu_i}{2}\,u(-k), \tag{2.17}$$

and the subscripts 0 and 1 in the right hand side of these equations mean that $\varepsilon\,\partial F/\partial t$ is replaced by $J - J^{(0)}(F)$ and by $-J^{(1)}(F)$, respectively.

Inserting the expansion (2.13) into (2.3), we obtain the *compatibility condition* $f_0^{(m)} = 0$ (for $m>0$), which implies that $(\mathcal{L}f^{(m)}) = 0$, for $m > 0$. These solvability conditions yield $J^{(m)} = 2e\sum_{j=1}^{\infty} j\langle \mathrm{Im}(\mathcal{E}_{-j}f_j^{(m)})\rangle_j/\hbar$, which can also be obtained by insertion of Eq. (2.12) in (2.10).

We shall particularize our results to the tight-binding dispersion relation $\mathcal{E}(k) = \Delta\,(1-\cos kl)/2$ (Δ is the miniband width and $v(k) = (\Delta l\,\sin kl)/(2\hbar)$ is the group velocity), having nonzero Fourier coefficients $\mathcal{E}_0 = \Delta/2$, $\mathcal{E}_{\pm1} = -\Delta/4$. The leading order of Ampère's law (2.14) is

$$\varepsilon\frac{\partial F}{\partial t} + \frac{ev_M}{l}\left\langle n\mathcal{M}\frac{2\mathcal{F}_1}{1+\mathcal{F}_1^2}\right\rangle_1 = J(t), \tag{2.18}$$

where $v_M = \Delta l\,\mathcal{I}_1(M)/[4\hbar\tau_e\mathcal{I}_0(M)]$, $\mathcal{M} = \mathcal{I}_1(\tilde\mu)\,\mathcal{I}_0(M)/[\mathcal{I}_1(M)\,\mathcal{I}_0(\tilde\mu)]$, $\mathcal{I}_m(s) = \int_{-\pi}^{\pi}\cos(mk)\,\ln\left(1+e^{s-\delta+\delta\cos k}\right)dk$, $\delta = \Delta/(2k_BT)$, $\tilde\mu = \mu/(k_BT)$. $\mu = Mk_BT$ (calculated graphically in Fig. 1 of Ref. 3) solves (2.3) with $n = N_D$.

The solution of (2.15) yields $J^{(1)}$ in (2.14), which is the first correction to the QDDE (2.18). The details can be found in References 4 and 8 (for the numerical procedure). An important point is that the nonlocal terms in the QDDE require that boundary conditions be imposed on the intervals $[-2l,0]$ and $[Nl, Nl+2l]$ for a N-period SL. Fig. 1 shows the current self-oscillations that appear when the QDDE is solved with boundary conditions $\varepsilon\partial F/\partial t + \sigma F = J$ at each point of the intervals $[-2l,0]$ and $[Nl, Nl+2l]$

and appropriate σ and dc voltage bias. Parameter values correspond to a 157-period 3.64 nm GaAs/0.93 nm AlAs SL at 5K, with $N_D = 4.57 \times 10^{10}$ cm^{-2}, $\nu_i = 2\nu_e = 18\times10^{12}$ Hz under a dc voltage bias of 1.62 V, which yield $x_0 = 16$ nm, $t_0 = 0.24$ ps, $J_0 = ev_M N_D/l = 1.10 \times 10^5$ A/cm^2. Cathode and anode contact conductivities σ are 2.5 and 0.62 Ω^{-1}cm^{-1}, respectively.

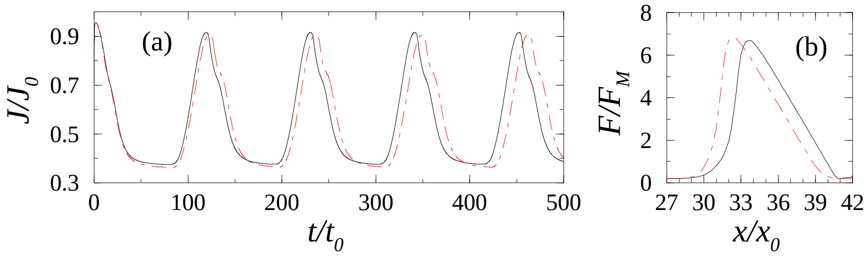

Fig. 1. (a) Current vs. time during self-oscillations, and (b) fully developed dipole wave. Solid line: QDDE, dashed line: GDDE.

3. Wigner description of a two-miniband superlattice

We shall consider a 2×2 Hamiltonian $\mathbf{H}(x, -i\partial/\partial x)$, in which

$$\mathbf{H}(x,k) = [h_0(k) - eW(x)]\boldsymbol{\sigma}_0 + \vec{h}(k)\cdot\vec{\boldsymbol{\sigma}}] \equiv \begin{pmatrix} (\alpha+\gamma)(1-\cos kl) - eW(x) + g & -i\beta\sin kl \\ i\beta\sin kl & (\alpha-\gamma)(1-\cos kl) - eW(x) - g \end{pmatrix}. \quad (3.1)$$

Then $h_0 = \alpha(1 - \cos kl)$, $h_1 = 0$, $h_2 = \beta \sin kl$, $h_3 = \gamma(1 - \cos kl) + g$, and

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \boldsymbol{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (3.2)$$

The Hamiltonian (3.1) corresponds to the simplest 2×2 Kane model in which the quadratic and linear terms $(kl)^2/2$ and kl are replaced by $(1 - \cos kl)$ and $\sin kl$, respectively. For a SL with two minibands, $2g$ is the miniband gap and $\alpha = (\Delta_1 + \Delta_2)/4$ and $\gamma = (\Delta_1 - \Delta_2)/4$, provided Δ_1 and Δ_2 are the miniband widths. In the case of a lateral SL, $g = \gamma = 0$, and $h_2\boldsymbol{\sigma}_2$ corresponds to the precession term in the Rashba spin-orbit interaction.[7] The other term, the intersubband coupling, depends on the momentum in the y direction and we have not included it here. Small modifications of (3.1) represent a single miniband SL with dilute magnetic impurities in the presence of a magnetic field B: $g = \gamma = h_2 = 0$, and $h_1 = \beta(B)$.[6] As in the case of a single miniband SL, $W(x)$ is the electric potential.

The energy minibands $\mathcal{E}^{\pm}(k)$ are the eigenvalues of the free Hamiltonian $\mathbf{H}_0(k) = h_0(k)\boldsymbol{\sigma}_0 + \vec{h}(k)\cdot\vec{\boldsymbol{\sigma}}$ and are given by

$$\mathcal{E}^{\pm}(k) = h_0(k) \pm |\vec{h}(k)|. \tag{3.3}$$

The corresponding spectral projections are $\mathbf{P}^{\pm}(k) = (\boldsymbol{\sigma}_0 \pm \vec{\nu}(k)\cdot\vec{\boldsymbol{\sigma}})/2$, with $\vec{\nu} = \vec{h}/|\vec{h}(k)|$, so that we can write $\mathbf{H}_0(k) = \mathcal{E}^{+}(k)\mathbf{P}^{+}(k) + \mathcal{E}^{-}(k)\mathbf{P}^{-}(k)$.

We shall now write the WPBGK equations for the Wigner matrix written in terms of the Pauli matrices $\boldsymbol{\sigma}_i$:

$$\mathbf{f}(x,k,t) = \sum_{i=1}^{3} f^i(x,k,t)\boldsymbol{\sigma}_i = f^0(x,k,t)\boldsymbol{\sigma}_0 + \vec{f}(x,k,t)\cdot\vec{\boldsymbol{\sigma}}. \tag{3.4}$$

The Wigner components are real and can be related to the coefficients of the Hermitian Wigner matrix by $f_{11} = f^0 + f^3$, $f_{12} = f^1 - if^2$, $f_{21} = f^1 + if^2$, $f_{22} = f^0 - f^3$. Hereinafter we shall use the equivalent notations

$$f = \begin{pmatrix} f^0 \\ \vec{f} \end{pmatrix} = \begin{pmatrix} f^0 \\ f^1 \\ f^2 \\ f^3 \end{pmatrix}. \tag{3.5}$$

The populations of the minibands with energies $\mathcal{E}^{\pm}$ are the moments:

$$n^{\pm}(x,t) = \frac{l}{2\pi}\int_{-\pi/l}^{\pi/l}\left[f^0(x,k,t) \pm \vec{\nu}\cdot\vec{f}(x,k,t)\right] dk, \tag{3.6}$$

and the total electron density is $n^+ + n^-$.

We shall restrict ourselves to the Rashba case, $g = \gamma = h_3 = 0$, from now on. Then $\vec{\nu} = (0,1,0)$ and $n^{\pm}$ are the densities of electrons having spin $\pm$. After some algebra, we can obtain the following WPBGK equations for the Wigner components

$$\frac{\partial f^0}{\partial t} + \frac{\alpha}{\hbar}\sin kl\,\Delta^- f^0 + \frac{\beta\cos kl}{\hbar}\,\Delta^- f^2 - \Theta f^0 = Q^0[f], \tag{3.7}$$

$$\frac{\partial \vec{f}}{\partial t} + \frac{\alpha\sin kl}{\hbar}\Delta^-\vec{f} + \frac{\beta}{\hbar}[\vec{\nu}\Delta^- f^0\cos kl + \Delta^+(\vec{\nu}\times\vec{f})\sin kl] \\ -\Theta\vec{f} = \vec{Q}[f], \tag{3.8}$$

$$\varepsilon\,\frac{\partial^2 W}{\partial x^2} = \frac{e}{l}\,(n^+ + n^- - N_D), \tag{3.9}$$

$$\Theta f^i(x,k,t) = \sum_{j=-\infty}^{\infty}\frac{ejl}{i\hbar}\langle F(x,t)\rangle_j e^{ijkl} f^i_j(x,t). \tag{3.10}$$

Our collision model contains two terms: a BGK term which tries to send $f^0 \pm f^2$ to its local equilibrium (approximated by Boltzmann statistics at $T = 300$K) and a scattering term which tries to equalize n^+ and n^-:[6]

$$Q^0[f] = -\frac{f^0 - \Omega^0}{\tau}, \quad \vec{Q}[f] = -\frac{\vec{f} - \vec{\Omega}}{\tau} - \frac{\vec{f}}{\tau_{\rm sc}}, \tag{3.11}$$

$$\Omega^0 = \frac{\phi^+ + \phi^-}{2}, \quad \vec{\Omega} = \frac{\phi^+ - \phi^-}{2}\,\vec{\nu}, \tag{3.12}$$

$$\phi^\pm(k; n^\pm) = \frac{m^* k_B T}{\pi^2 \hbar^2}\, \exp\left(\frac{\mu^\pm - \mathcal{E}^\pm(k)}{k_B T}\right), \tag{3.13}$$

$$\frac{l}{2\pi}\int_{-\pi/l}^{\pi/l} \phi^\pm(k; n^\pm)\, dk = n^\pm. \tag{3.14}$$

In (3.13), $\mu^\pm = \mu^\pm(n^\pm)$ solve (3.14). Our collision model satisfies charge continuity. In fact, (3.7) to (3.9) yield:

$$\frac{\partial n^\pm}{\partial t} + \frac{l\Delta^-}{2\pi\hbar}\int_{-\pi/l}^{\pi/l}[\alpha \sin kl\,(f^0 \pm f^2) + \beta \cos kl\,(f^2 \pm f^0)]\, dk$$
$$= \frac{l\Delta^-}{2\pi}\int_{-\pi/l}^{\pi/l}(Q^0[f] \pm Q^2[f])\, dk = \mp\frac{n^+ - n^-}{\tau_{\rm sc}}, \tag{3.15}$$

where we have employed $\int \Theta f^{0,2} dk = 0$. Then we obtain:

$$\frac{\partial}{\partial t}(n^+ + n^-) + \Delta^-\left[\frac{l}{\pi\hbar}\int_{-\pi/l}^{\pi/l}(\alpha\,\sin kl\, f^0 + \beta\,\cos kl\, f^2)\, dk\right] = 0. \tag{3.16}$$

Since $\Delta^- u(x) = l\,\partial\langle u(x)\rangle_1/\partial x$, (3.16) provides charge continuity. From (3.9) and (3.16), we get Ampère's law ($J(t)$ is the total current density):

$$\varepsilon\frac{\partial F}{\partial t} + \left\langle \frac{el}{\pi\hbar}\int_{-\pi/l}^{\pi/l}(\alpha\,\sin kl\, f^0 + \beta\,\cos kl\, f^2)\, dk\right\rangle_1 = J(t). \tag{3.17}$$

4. Quantum drift-diffusion equations

In the simpler case of a lateral SL with the precession term of Rashba spin-orbit interaction (but no intersubband coupling), we can obtain explicit rate equations for $n^\pm$ by means of the CE method. The general case (3.1) will be treated elsewhere. First of all, we should decide the order of magnitude of the terms in the WPBGK equations (3.7) and (3.8) in the hyperbolic limit. Recall that in this limit, the collision frequency $1/\tau$ and the Bloch frequency $eF_M l/\hbar$ are of the same order, about 10 THz for the SL of Section 2. The scattering time $\tau_{\rm sc}$ is much longer than the collision time τ, and we shall consider $\tau/\tau_{\rm sc} = O(\lambda) \ll 1$. From (3.7) and (3.8), we can write the scaled WPBGK equations as follows:

$$\mathbb{L}f \equiv f - \tau\,\Theta f = \Omega - \lambda\,\left(\tau\,\frac{\partial f}{\partial t} + \Lambda f\right), \tag{4.1}$$

$$\Lambda f = \frac{\tau}{\tau_{\rm sc}}\,\begin{pmatrix} 0 \\ \vec{f} \end{pmatrix} + \frac{\alpha\tau}{\hbar}\sin kl\,\Delta^- f + \frac{\beta\tau}{\hbar}\,\cos kl\,\Delta^- \begin{pmatrix} f^2 \\ 0 \\ f^0 \\ 0 \end{pmatrix} \tag{4.2}$$

$$+\,\frac{\beta\tau}{\hbar}\,\sin kl\,\Delta^+ \begin{pmatrix} 0 \\ f^3 \\ 0 \\ -f^1 \end{pmatrix}.$$

To derive the reduced balance equations, we use the following CE ansatz:

$$f(x,k,t;\epsilon) = f^{(0)}(k;n^+,n^-,F) + \sum_{m=1}^{\infty} f^{(m)}(k;n^+,n^-,F)\,\lambda^m, \tag{4.3}$$

$$\varepsilon\frac{\partial F}{\partial t} + \sum_{m=0}^{\infty} J_m(n^+,n^-,F)\,\lambda^m = J(t), \tag{4.4}$$

$$\frac{\partial n^\pm}{\partial t} = \sum_{m=0}^{\infty} A_m^\pm(n^+,n^-,F)\,\lambda^m. \tag{4.5}$$

$A_m^\pm$ and J_m are related through the Poisson equation (3.9), so that

$$A_m^+ + A_m^- = -\frac{l}{e}\,\frac{\partial J_m}{\partial x}. \tag{4.6}$$

Inserting (4.3) to (4.5) into (4.2), we get

$$\mathbb{L}f^{(0)} = \Omega, \tag{4.7}$$

$$\mathbb{L}f^{(1)} = -\,\tau\,\frac{\partial f^{(0)}}{\partial t}\bigg|_0 - \Lambda f^{(0)}, \tag{4.8}$$

$$\mathbb{L}f^{(2)} = -\,\tau\,\frac{\partial f^{(1)}}{\partial t}\bigg|_0 - \Lambda f^{(1)} - \tau\,\frac{\partial f^{(0)}}{\partial t}\bigg|_1, \tag{4.9}$$

and so on. The subscripts 0 and 1 in the right hand side of these equations mean that we replace $\varepsilon\,\partial F/\partial t|_m = J\delta_{0m} - J_m$, $\partial n^\pm/\partial t|_m = A_m^\pm$. Moreover, inserting (4.3) into (3.6) yields the following compatibility and solvability conditions:

$$f_0^{(m)\,0} = f_0^{(m)\,2} = 0 \Longrightarrow (\mathbb{L}f^{(m)\,0})_0 = (\mathbb{L}f^{(m)\,2})_0 = 0, \quad m \geq 1. \tag{4.10}$$

To solve (4.7) for $f^{(0)} \equiv \varphi$, we first note that

$$-\tau\,\Theta\varphi = \sum_{j=-\infty}^{\infty} i\vartheta_j \varphi_j e^{ijkl}, \quad \vartheta_j \equiv \frac{\tau e j l}{\hbar}\,\langle F\rangle_j. \tag{4.11}$$

Then (4.7) and (3.12) yield

$$\varphi_j^0 = \frac{\phi_j^+ + \phi_j^-}{2}\,\frac{1-i\vartheta_j}{1+\vartheta_j^2}, \quad \varphi_j^1 = \varphi_j^3 = 0, \quad \varphi_j^2 = \frac{\phi_j^+ - \phi_j^-}{2}\,\frac{1-i\vartheta_j}{1+\vartheta_j^2}, \tag{4.12}$$

where we have used that the Fourier coefficients $\phi_j^\pm$ are real because $\phi^\pm$ are even functions of k. Similarly, the solution of (4.8) is $f^{(1)} \equiv \psi$ with

$$\psi_j^m = r_j^m\,\frac{1-i\vartheta_j}{1+\vartheta_j^2} \quad (m=0,2), \quad \psi_j^1 = \psi_j^3 = 0. \tag{4.13}$$

Here r is the right hand side of (4.8). The balance equations can be found by calculating $A_m^\pm$ for $m = 0, 1$ from the solvability conditions for (4.8) and (4.9), or by simply inserting the solutions (4.12) and (4.13) in (3.15) and (3.17). In both cases, the result is:

$$\frac{\partial n^\pm}{\partial t} + \Delta^- D_\pm(n^+, n^-, F) = \mp R(n^+, n^-, F), \tag{4.14}$$

$$\varepsilon\,\frac{\partial F}{\partial t} + e\,\langle D_+ + D_-\rangle_1 = J, \tag{4.15}$$

$$D_\pm = -\frac{\alpha}{\hbar}\,\mathrm{Im}(\varphi_1^0 \pm \varphi_1^2 + \psi_1^0 \pm \psi_1^2) \pm \frac{\beta}{\hbar}\,\mathrm{Re}(\varphi_1^0 \pm \varphi_1^2 + \psi_1^0 \pm \psi_1^2), \quad R = \frac{n^+ - n^-}{2\tau_{sc}}. \tag{4.16}$$

A straightforward calculation of (4.16) yields

$$\begin{aligned} D_\pm = {} & \frac{(\alpha\vartheta_1 \pm \beta)\phi_1^\pm}{\hbar\,(1+\vartheta_1^2)} \mp \frac{\tau\,(\phi_1^+ - \phi_1^-)\,[2\alpha\vartheta_1 \pm \beta(1-\vartheta_1^2)]}{2\hbar\tau_{\rm sc}(1+\vartheta_1^2)^2} \\ & + \frac{[2\alpha\vartheta_1 \pm \beta(1-\vartheta_1^2)]\,\tau}{\hbar\,(1+\vartheta_1^2)^2}\,\frac{\partial\phi_1^\pm}{\partial n^\pm}\left[\Delta^-\left(\frac{\alpha\vartheta_1 \pm \beta}{\hbar\,(1+\vartheta_1^2)}\phi_1^\pm\right) \pm \frac{n^+ - n^-}{2\tau_{sc}}\right] \\ & + \frac{\alpha\,(3\vartheta_1^2 - 1) \pm \beta\vartheta_1(3-\vartheta_1^2)}{\hbar(1+\vartheta_1^2)^3}\,\frac{l\tau^2\phi_1^\pm}{\hbar\varepsilon}\left(\frac{J}{e} - \left\langle\left\langle\frac{\alpha\,(\phi_1^+ + \phi_1^-)\vartheta_1}{\hbar(1+\vartheta_1^2)}\right\rangle_1\right\rangle_1\right. \\ & \left. - \left\langle\left\langle\frac{\beta\,(\phi_1^+ - \phi_1^-)}{\hbar(1+\vartheta_1^2)}\right\rangle_1\right\rangle_1\right) - \frac{(\alpha^2+\beta^2)\tau}{2\hbar^2(1+\vartheta_1^2)}\,\Delta^- n^\pm \\ & + \frac{\tau}{2\hbar^2(1+\vartheta_1^2)}\left[(\alpha^2 - \beta^2 \mp 2\alpha\beta\vartheta_1)\,\Delta^-\left(\frac{\phi_2^\pm}{1+\vartheta_2^2}\right)\right. \\ & \left. + [(\beta^2 - \alpha^2)\vartheta_1 \mp 2\alpha\beta]\,\Delta^-\left(\frac{\vartheta_2\phi_2^\pm}{1+\vartheta_2^2}\right)\right]. \end{aligned} \tag{4.17}$$

The following values of the parameters are typical of a GaAs/AlGaAs SL: $\alpha = 10$ meV, $\beta = 2.1$ meV, $l = 5$ nm, $T = 300$ K, $\tau = 10^{-13}$ s, $\tau_{\rm sc} = 10^{-12}$ s, $N_D = 10^{10}$ cm^{-2}. Figure 2 shows the electron velocity, $v = Jl/(eN_D)$ (measured in units of $v_M = \alpha l/\hbar = 7.6 \times 10^6$ cm/s), as a function of field (measured in units of $F_M = 13.2$ kV/cm at which the electron velocity reaches its maximum) for a homogeneous solution of Eq. (4.15) with constant F and $n^+ = n^- = N_D/2$. We observe that there is a local maximum followed by a region of negative slope (negative differential velocity), which suggests a Gunn-type instability as in Section 2: self-sustained oscillations of the current through the SL due to motion of charge dipole waves under sufficiently high dc voltage bias.[1,2,4]

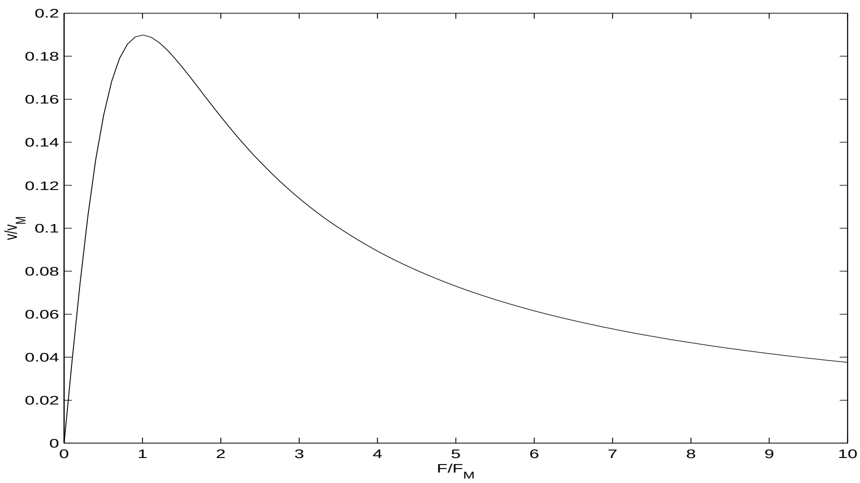

Fig. 2. Electron velocity vs. field for a homogeneous solution of (4.14) - (4.17).

5. Conclusions

For strongly coupled SLs having only one populated miniband, we have written a Wigner-Poisson-BGK system of equations and derived a quantum drift-diffusion equation for the field by using the Chapman-Enskog perturbation method. With appropriate voltage bias, a numerical solution of this equation yields self-sustained oscillations of the current due to recycling and motion of charge dipole domains. For SLs having two populated minibands coupled through a Rashba spin-orbit interaction, we have introduced a periodic version of the Kane Hamiltonian and derived the corresponding WPBGK system of equations. By using the CE method, we have derived quantum drift-diffusion equations for the miniband populations which contain generation-recombination terms. The spatially homogeneous solution of these equations provides an electron velocity which has a region of negative slope as a function of field. This hints to the possible existence of oscillatory instabilities and self-oscillations of the current due to motion of charge dipole waves under sufficiently high dc voltage bias.

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