PERGAMON

Available online at www.sciencedirect.com
SCIENCE DIRECT®

Chaos, Solitons and Fractals 17 (2003) 283–288

CHAOS
SOLITONS & FRACTALS

www.elsevier.com/locate/chaos

Wave dynamics in two-dimensional samples of n-GaAs with point contacts

R. Escobedo *, L.L. Bonilla

Department of Mathematics, Universidad Carlos III de Madrid, Av. de la Universidad, 30, Leganés, 28911 Madrid, Spain

Accepted 19 August 2002

Abstract

We present results of numerical simulations of a two-dimensional Kroemer model for the Gunn effect in samples with different number of point contacts. These results can be interpreted with the help of asymptotic theories for particular geometries: a one-dimensional sample and a Corbino disk (a circular sample with a point contact at the center surrounded by a metallic contact attached to its boundary). Comments on open problems and possible experimental verification are also made.

1. Introduction

The Gunn effect [1] consists of self-sustained oscillations of the current through a rectangular sample of bulk n-GaAs with attached planar metallic contacts at different dc voltage. These oscillations are due to repeated creation and destruction of charge dipole waves at the contacts and their one-dimensional motion inside the sample from one contact to the other. While the physical origin of the effect is well understood (electron transfer between different valleys of the conduction band), there are many interesting nonlinear problems related to the Gunn effect. If point contacts are attached appropriately to a planar GaAs sample, a rich variety of novel nonlinear behaviors can be observed, mostly due to wavefront curvature and interaction between different wavetrains.

2. Two-dimensional Kroemers model

The Kroemer model of the Gunn effect in n-GaAs [2] consists of the following equations and boundary conditions (in dimensionless units [3]):

$$\Delta\varphi = n - 1 \quad \text{in } \Omega \times (0, T], \tag{1}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v} - \delta\nabla n) = 0 \quad \text{in } \Omega \times (0, T], \tag{2}$$

$$n = n_0 \quad \text{in } \Omega \text{ at } t = 0, \tag{3}$$

$$\vec{E} \cdot \vec{N} = \rho(n\vec{v} - \delta\nabla n) \cdot \vec{N} \quad \text{on } \Sigma_{c,a} \times (0, T], \tag{4}$$

* Corresponding author.
E-mail address: escobedo@math.uc3m.es (R. Escobedo).

PII: S0960-0779(02)00353-3

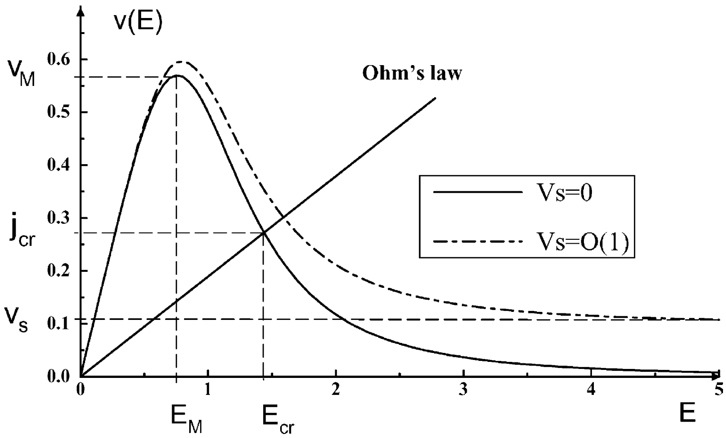

Fig. 1. Electron drift velocity curve as a function of electric field for two different saturation values v_s. Contact resistivity is chosen so that the boundary current density $j = E/\rho$ intersects $v(E)$ on its second branch, past the maximum (E_M, v_M), as shown in the figure.

$$\varphi = 0 \quad \text{on } \Sigma_c \times (0, T], \tag{5}$$

$$\varphi = \phi \quad \text{on } \Sigma_a \times (0, T], \tag{6}$$

$$\nabla n \cdot \vec{N} = \nabla\varphi \cdot \vec{N} = 0 \quad \text{on } \partial\Omega \times (0, T]. \tag{7}$$

Here the unknowns φ, $\vec{E} = \nabla\varphi$ and n are the electric potential, minus the electric field and electron density, respectively. The sample is a rectangle Ω with attached point contacts connected to an external circuit. Σ_c (cathodes) and Σ_a (anodes) are the interfaces between semiconductor and contacts. $\vec{N}$ is the unit normal to $\Sigma_{c,a}$ directed towards Ω. Eqs. (1) and (2) are the Poisson and charge continuity equations, respectively. In the latter, δ is the diffusion coefficient (taken as a constant) and $\vec{v}(\vec{E}) = \vec{E}(1 + v_s E^3)/(1 + E^4)$ is the electron drift velocity, where $E = |\vec{E}|$. See Fig. 1. Lastly, ρ is the contact resistivity and ϕ the applied voltage.

3. Numerical simulation

We have solved the Kroemer's model in different geometries, characterized by the number and the location of point contacts. The result consists in sequences of images representing the electron density $n(x, y, t)$ in a grey color scale which goes from black ($n = 0$) to white (maximum positive value of n). See Fig. 2, which corresponds to the electron density distribution depicted in Fig. 3.

For the numerical resolution we consider the problem as an evolution problem for $n(x, y, t)$, governed by a parabolic equation (the continuity equation) whose coefficients are given at each time-step by the Poisson equation. The continuity equation is discretized in a system of ODEs in time which stiffness is treated with an explicit Euler method. For Poisson equation we have used a GMRES method with incomplete LU factorization preconditioning [4]. This choice is due to the fact that the matrix which comes from the discretization of the Poisson equation is nonsymmetric because the domain has holes inside.

Fig. 2. Electron density image corresponding to the distribution depicted in Fig. 3.

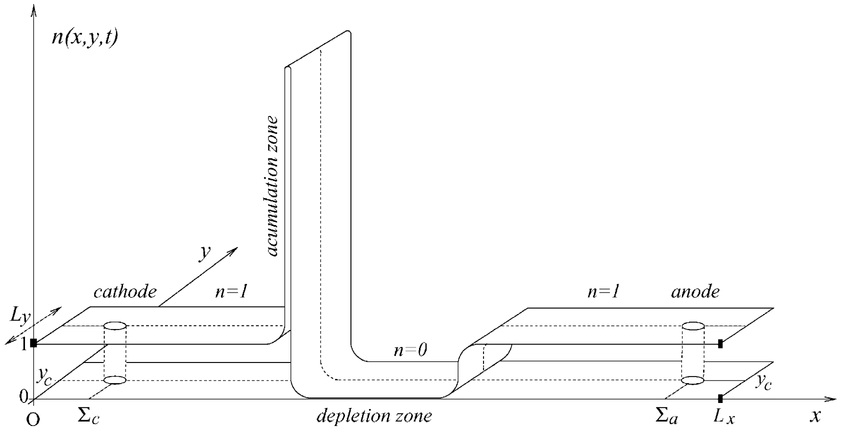

Fig. 3. Electron density dipole wave in a rectangular sample of size $L_x \times L_y = 36 \times 6$.

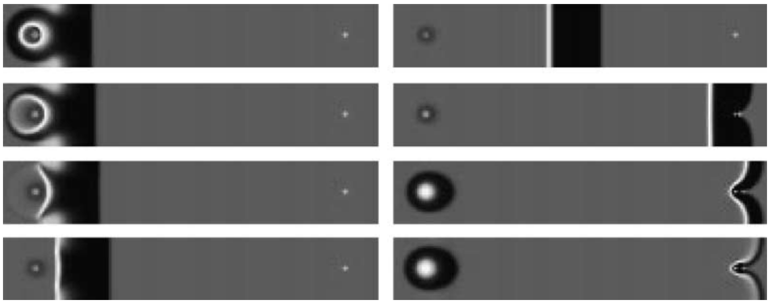

Fig. 4. One-dimensional pattern in a sample in which $L_x \gg L_y$.

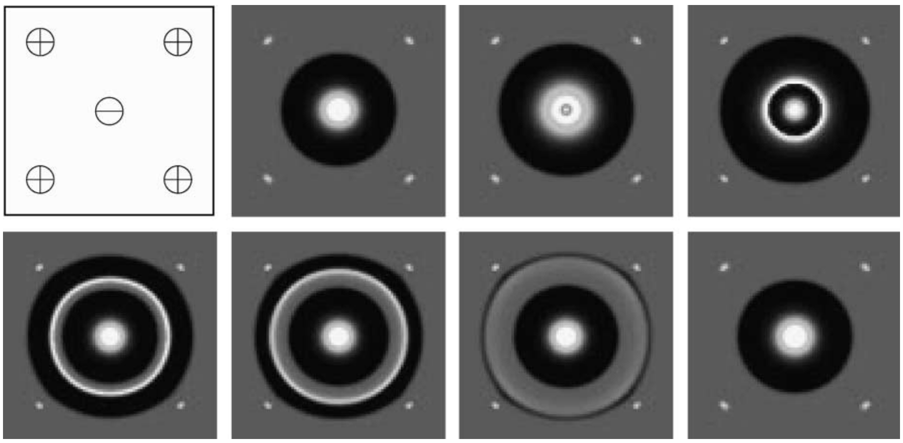

Fig. 5. Ring pattern like in Corbino geometry and in experimental results [6,7].

4. Results

In Fig. 4 the sample is the one depicted in Figs. 2 and 3. Dimensions are such that $L_x \gg L_y$. The resulting patterns are very similar to the one-dimensional case, in which the electron density is a plane dipole wave which moves from cathode to anode periodically. The geometry used in Fig. 5 reproduces the ring patterns obtained in Corbino disks [9,10] and in experimental results [6,7]. In this case the wave disappears inside the sample before reaching the anode. These two geometries are explained with the help of asymptotic theories [5,8,10].

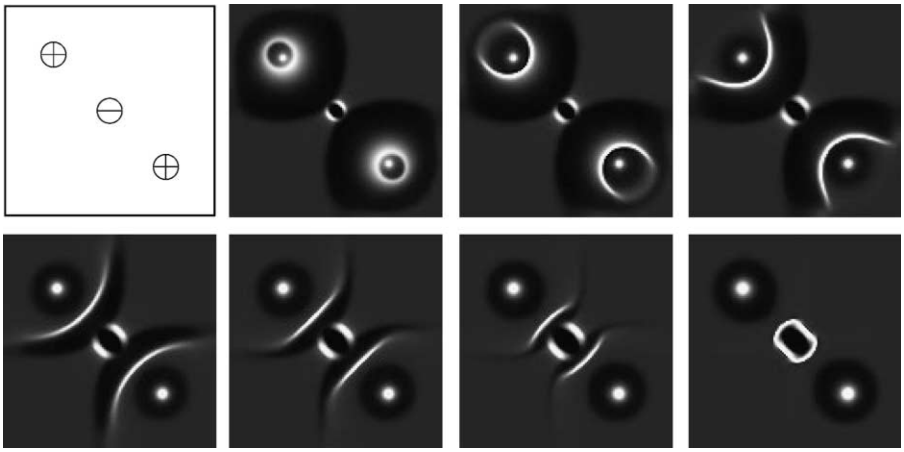

Fig. 6. Two symmetric one-dimensional patterns with respect to the midpoint of the sample.

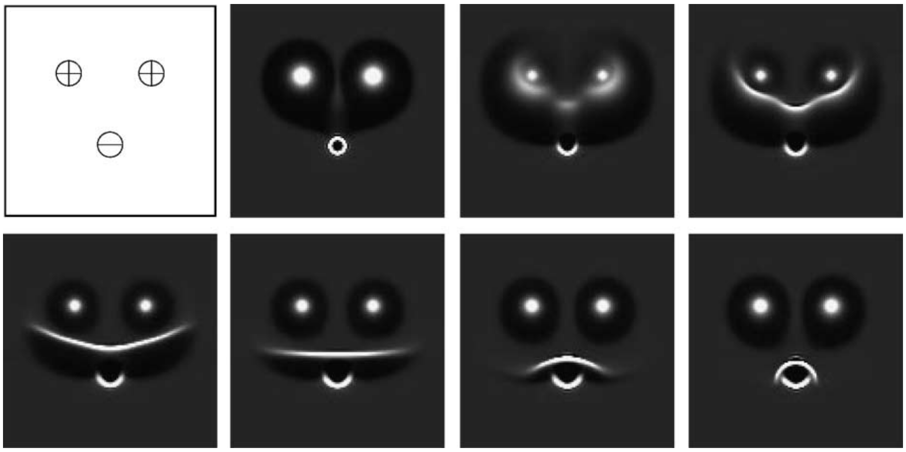

Fig. 7. One-dimensional pattern, in which the two upper cathodes behave as a simple cathode located at the midpoint of them.

Figs. 6 and 7 show that patterns can be frequently interpreted as combinations of one-dimensional and ring patterns. The first one looks like two symmetric one-dimensional patterns with respect to the midpoint of the sample. In the second one we can say that the two upper contacts (cathodes) produce the same behavior than a simple cathode located in the middle of them.

The variety of novel nonlinear behaviors which can be obtained is however very rich, just by attaching the contacts appropriately. In Fig. 8 we have chosen a position for two cathodes and two anodes in order to produce a collision between two dipole waves. The last two figures are an attempt to obtain spiral waves. In Fig. 9 we have put five cathodes around two anodes, and the result is a symmetric behavior. In Fig. 10 we have introduced a perturbation of the pentagon, joining two cathodes in the upper part. The result is that the wave appears before in the region of the two contacts and then it is joined by the other waves coming from the other three cathodes. It is noticeable that all the patterns presented here are periodic in time.

A detailed asymptotic study requires using larger samples. This in turn requires improving the numerical algorithm. Alternatively, the multidimensional Gunn effect can be reformulated in terms of a free boundary problem, provided that the width of the dipole wave is very small compared to the sample size. Then, the wave can be identified with a free surface (white *bands* in Figs. 4–10) moving over a uniform domain in which $\Delta\varphi = 0$. This formulation is much more economical computationally and it could be used advantageously to study three-dimensional problems. Details will be presented elsewhere.

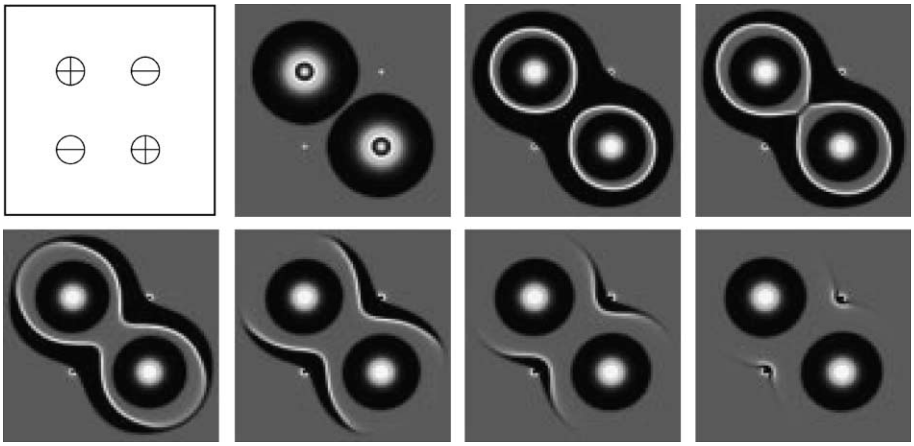

Fig. 8. Collision between two dipole waves.

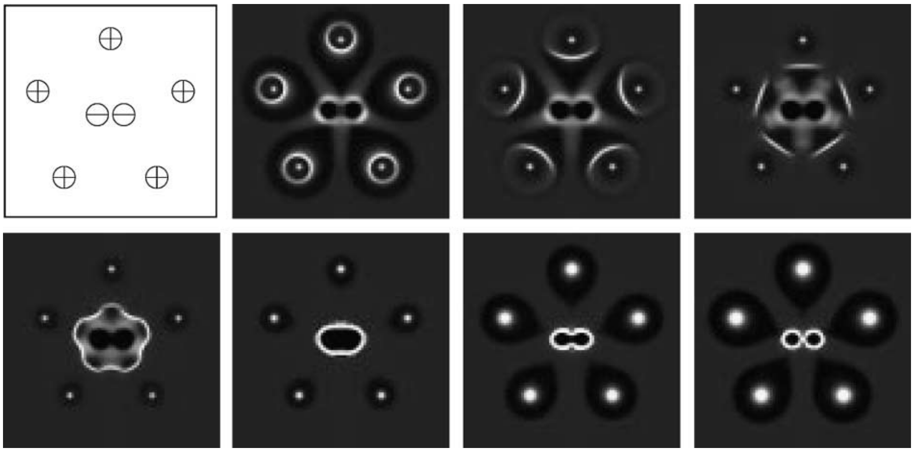

Fig. 9. Symmetric pattern in a pentagonal distribution.

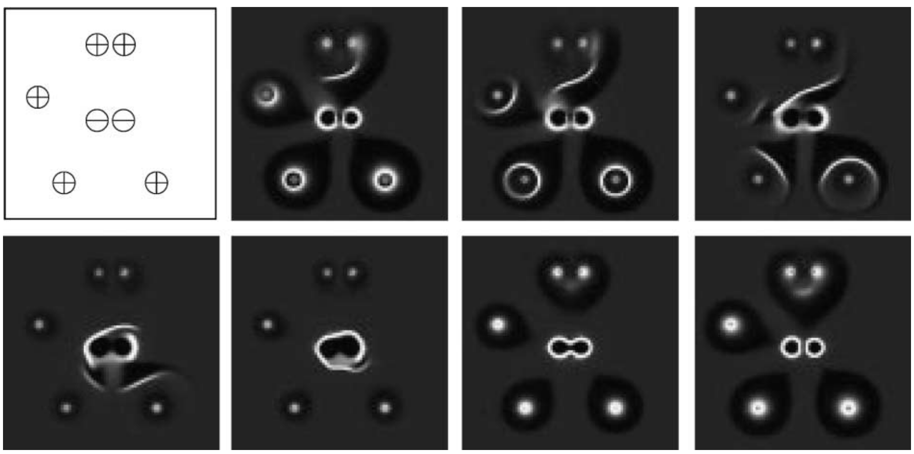

Fig. 10. Perturbation of the pentagonal distribution, in order to obtain spiral waves.

References

[1] Gunn JB. Solid State Commun 1963;1:88–91.
[2] Kroemer H. IEEE Trans Electron Dev 1966;ED-13:27–40.
[3] Bonilla LL, Higuera FJ. Phys D 1991;52:458–76.
[4] Saad Y, Schultz MH. SIAM J Sci Statist Comput 1986;7(3):856–69.
[5] Higuera FJ, Bonilla LL. Phys D 1992;57:161–84.
[6] Willing B, Maan JC. Phys Rev B 1994;49:13995.
[7] Willing B. PhD Thesis, University of Nijmegen, 1994.
[8] Bonilla LL, Escobedo R. Phys Rev E 2001;64:036203.
[9] Bonilla LL, Escobedo R, Higuera FJ. In Proc 25th ICPS, Osaka, Japan. Springer; 2001. p. 134–5.
[10] Bonilla LL, Escobedo R, Higuera FJ. Phys Rev E 2001;65:016607.