Nonlinear electronic transport in semiconductor superlattices

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Nonlinear charge transport in strongly coupled semiconductor superlattices is described by single or two-miniband Wigner-Poisson kinetic equations with BGK collision terms. Balance equations for miniband populations and electric field are derived using the Chapman-Enskog method. Numerical solutions show stable self-oscillations of the current through a voltage biased superlattice.

e o ds: Quantum drift-diffusion equations, Chapman-Enskog method, Rashba spin-orbit interaction, modified Kane model.

1. Introduction

Semicon ctor s perlattices are essential ingre ients in ast nanoscale oscil lators ant m casca e lasers an in rare etectors. A s perlattice S is a asi one imensional cr stal originall propose b sa i an s to obser e loch oscillations i.e. the perio ic coherent motion o electrons in a miniban hen an electric el is applie . nce the materials ere gro n man interesting nonlinear phenomena ere obser e s ch as sel oscillations o the c rrent thro gh the S e to charge ipole motion m ltistabilit o stationar charge an el pro les etc. See the re ie 1.

onlinear charge transport in S s has been i el st ie in the last eca e sing balance e ations or electron ensities an electric el .

hese e ations are either propose sing phenomenological arg ments or eri e a hoc rom inetic theories.[1,2] S stematic eri ations are scarce. For a single miniban S the hapman ns og metho ap plie to a semiclassical olt mann oisson s stem hose collision term is o hatnagar ross roo t pe iel s a generali e ri t i sion e ation [3] an a ant m ri t i sion e ation hen applie to a igner oisson s stem.[4] For a semiclassi cal parabolic ban oisson semicon ctor s stem the metho ha earlier been se to obtain balance e ations.[5] he ant m s stem contains t o pse o i erential operators in ol ing the ban is persion relation an the electric potential. he lea ing or er approximation in the h perbolic limit balances collisions an electric potential an its so l tion is not ob io s beca se the potential is an a priori n no n sol tion o the oisson e ation. S s are simpler beca se their igner nctions are perio ic in the reciprocal lattice the potential terms become m ltipli cation operators in Fo rier space an the lea ing or er approximation is straight or ar to sol e.[4]

For s cientl high applie electric el s electrons ma pop late higher miniban s then be scattere to the lo est etc. Moreo er S s ith il te magnetic imp rities s b ect to a magnetic el ma present spin polari a tion e ects hose n erstan ing is cr cial to e elop spintronic e ices.[6] en itho t magnetic imp rities spin polari ation co l appear e to ashba spin orbit interaction.[7] nce e consi er electron spin each mini ban is split in t o an single miniban S s become t o miniban S s. e shall s stematicall eri e ant m balance e ations b the metho .

2. Single miniband superlatice

he igner oisson hatnagar ross roo s stem or 1 electron transport in the lo est miniban o a strongl co ple S is:

$$\frac{\partial f}{\partial t} \quad \frac{i}{\hbar} E \quad k \quad \frac{1}{2i}\frac{\partial}{\partial x} - E \quad k - \frac{1}{2i}\frac{\partial}{\partial x} \quad f$$

$$\frac{ie}{\hbar} W \quad x \quad \frac{1}{2i}\frac{\partial}{\partial k}, t - W \quad x - \frac{1}{2i}\frac{\partial}{\partial k}, t \quad f$$

$$Q\ f \equiv -\nu_e \ f - f^{FD} - \nu_i \frac{f\ x, k, t - f\ x, -k, t}{2}, \qquad 2.1$$

$$\varepsilon \frac{\partial^2 W}{\partial x^2} \quad \frac{e}{l} \ n - N_D \ , \qquad 2.2$$

ith

$$n \quad \frac{l}{2\pi} \int_{-\pi/l}^{\pi/l} f \;\; x, k, t \;\; dk \quad \frac{l}{2\pi} \int_{-\pi/l}^{\pi/l} f^{FD} \;\; k \;\; n \;\; dk, \qquad \mathbf{2.3}$$

$$f^{FD} \;\; k \;\; n \quad \frac{m^* k_B T}{\pi^2 \hbar^2} \ln \;\; 1 \quad \exp \frac{\mu - E \;\; k}{k_B T} \quad . \qquad \mathbf{2.4}$$

ere $f \;\; n \;\; N_D \;\; E \;\; k \;\; d_B \;\; d_W \;\; l \;\; d_B \;\; d_W \;\; W \;\; \varepsilon \;\; m^* \;\; k_B \;\; T \;\; \nu_e \;\; \nu_i$ an $-e < 0$ are the one particle igner nction the 2 electron en sit the 2 oping ensit the miniban ispersion relation the barrier i th the ell i th the S perio the electric potential the S per mitti it the e ecti e mass o the electron the olt mann constant the lattice temperat re the re enc o the inelastic collisions responsible or energ relaxation the re enc o the elastic imp rit collisions an the electron charge respecti el .

he le t han si e o . 2.1 can be straight or ar l eri e rom the Schro inger oisson e ation or the a e nction in the miniban sing the e nition o the 1 igner nction:

$$f \;\; x, k, t \quad \frac{2l}{S} \sum_{j=-\infty}^{\infty} \int \psi^\dagger \;\; x \quad \frac{jl}{2}, y, z, t \;\; \psi \;\; x - \frac{jl}{2}, y, z, t \;\; e^{ijkl} dx dy. \qquad \mathbf{2.5}$$

he secon anti e a e nction ψ is a s perposition o the loch states correspon ing to the miniban an S is the S cross section.[4] he right han si e in . 2.1 is the s m o $-\nu_e \;\; f - f^{FD}$ hich repre sents energ relaxation to ar s a 1 e ecti e Fermi irac F istrib tion $f^{FD} \;\; k \;\; n$ local e ilibri m hich is the 3 Fermi irac istrib tion integrate o er the lateral components o the a e ector k, k_y, k_z an $-\nu_i \;\; f \;\; x, k, t \;\; - f \;\; x, -k, t \;\; /2$ hich acco nts or imp rit elastic collisions.[3] For simplicit the collision re encies ν_e an ν_i are xe constants. x act an F istrib tion nctions ha e the same electron ensit thereb preser ing charge contin it . $\mu \quad \mu \;\; n$ res lts rom sol ing 2.3 ith 2.4 .

he s stem 2.1 to 2.4 sho l ha e a $2\pi/l$ perio ic in k sol tion satis ing appropriate initial an bo n ar con itions:

$$f \;\; x, k, t \quad \sum_{j=-\infty}^{\infty} f_j \;\; x, t \;\; e^{ijkl}. \qquad \mathbf{2.}$$

e ning $F \quad \partial W/\partial x$ *minus* the electric el an the a erage

$$\langle F \rangle_j \;\; x, t \quad \frac{1}{jl} \int_{-jl/2}^{jl/2} F \;\; x \quad s, t \;\; ds, \qquad \mathbf{2.}$$

it is possible to obtain the ollo ing e i alent orm o the igner e ation[4]

$$\frac{\partial f}{\partial t} \quad \sum_{j=-\infty}^{\infty} \frac{ijl}{\ } e^{ijkl} \quad E_j \frac{\partial}{\partial x} f_j \quad e F_j \, f_j \qquad Q f \,. \qquad (2.8)$$

ntegrating 2.8 o er k iel s the charge contin it e ation

$$\frac{\partial n}{\partial t} \quad \frac{\partial}{\partial x} \sum_{j=1}^{\infty} \frac{2jl}{\ } \ \mathrm{m}\, E_{-j} f_j \ _j \quad 0. \qquad (2.9)$$

ere e can eliminate the electron ensit b sing the oisson e ation an then integrate o er x thereb obtaining the nonlocal Ampere s la or the total c rrent ensit $J\ t$:

$$\varepsilon \frac{\partial F}{\partial t} \quad \frac{2e}{\ } \sum_{j=1}^{\infty} j \ \mathrm{m}\, E_{-j} f_j \ _j \quad J\ t \,. \qquad (2.10)$$

o eri e the e shall ass me that the electric el contrib tion in 2.8 is comparable to the collision terms an that the ominate the other terms *the hyperbolic limit* .[3] et v_M an F_M be the electron elocit an el positi e al es at hich the eroth or er ri t elocit reaches its maxim m. n this limit the time t_0 it ta es an electron ith spee v_M to tra erse a istance x_0 $\varepsilon F_M l / eN_D$ o er hich the el ariation is o or er F_M is m ch longer than the mean ree time bet een collisions ν_e^{-1} $/ eF_M l$ t_1. e there ore e ne the *small parameter* λ t_1/t_0 $v_M N_D / \varepsilon F_M^2 l^2$ an ormall m ltipl the rst t o terms on the le t si e o 2.1 or 2.8 b λ.[3,4] he res lt is

$$\lambda \ \frac{\partial f}{\partial t} \quad \sum_{j=-\infty}^{\infty} \frac{ijl}{\ } e^{ijkl} E_j \frac{\partial}{\partial x} f_j \quad Q f \ - \sum_{j=-\infty}^{\infty} \frac{iejl}{\ } e^{ijkl} F_j \, f_j . \quad (2.11)$$

he sol tion o . 2.11 or λ 0 is calc late in terms o its Fo rier coe cients as

$$f^{(0)}\ k\ F \quad \sum_{j=-\infty}^{\infty} \frac{1 - ijF_j/\tau_e \ \ f_j^{FD}}{1 \quad j^2 F_j^2} e^{ijkl}, \qquad (2.12)$$

here F_j F_j / F_M F_M $\sqrt{\nu_e\, \nu_e \quad \nu_i} / el$ an τ_e $\sqrt{\nu_e \quad \nu_i} / \nu_e$.

he ansat or the igner nction is:[4]

$$f\ x, k, t\ \lambda \quad f^{(0)}\ k\ F, n \quad \sum_{m=1}^{\infty} f^{(m)}\ k\ F, n\ \lambda^m, \qquad (2.13)$$

$$\varepsilon \frac{\partial F}{\partial t} \quad \sum_{m=0}^{\infty} J^{(m)}\ F, n\ \lambda^m \quad J\ t\,. \qquad (2.14)$$

he coe cients $f^{(m)}$ k F, n epen on the slo ariables x an t onl thro gh their epen ence on F an n hich obe 2.2 an 2.14 . he nctionals $J^{(m)}$ F, n are chosen so that the $f^{(m)}$ k F, n are bo n e an $2\pi/l$ perio ic in k. eeping the esire n mber o terms an setting λ 1 in 2.14 iel s the so ght . nserting 2.13 2.14 in 2.11 e n the hierarch :

$$Lf^{(1)} \quad -\frac{\partial f^{(0)}}{\partial t}\Big|_0 \quad \sum_{j=-\infty} \frac{ij l E_j e^{ijkl}}{} \frac{\partial}{\partial x} f^{(0)}_j \qquad 2.15$$

$$Lf^{(2)} \quad -\frac{\partial f^{(1)}}{\partial t}\Big|_0 \quad \sum_{j=-\infty} \frac{ij l E_j e^{ijkl}}{} \frac{\partial}{\partial x} f^{(1)}_j - \frac{\partial}{\partial t} f^{(0)}\Big|_1, \qquad 2.1$$

an so on. ere

$$Lu\, k \equiv \frac{ie}{} \sum_{-\infty} j l F_j u_j e^{ijkl} \quad \nu_e \quad \frac{\nu_i}{2} \; u\, k - \frac{\nu_i}{2} u\, -k, \qquad 2.1$$

an the s bscripts 0 an 1 in the right han si e o these e ations mean that $\varepsilon\,\partial F/\partial t$ is replace b $J - J^{(0)}$ F an b $-J^{(1)}$ F respecti el .

nserting the expansion 2.13 into 2.3 e obtain the *compatibility condition* $f_0^{(m)}$ 0 or $m > 0$ hich implies that $Lf^{(m)}$ 0 or $m > 0$. hese sol abilit con itions iel $J^{(m)}$ 2e $\sum_{j=1}^{\infty} j$ m $E_{-j} f_j^{(m)}$ $_j/$ hich can also be obtaine b insertion o . 2.12 in 2.10 .

e shall partic lari e o r res lts to the tight bin ing ispersion re lation E k 1 − cos kl /2 is the miniban i th an v k l sin kl / 2 is the gro p elocit ha ing non ero Fo rier coe cients E_0 /2 $E_{\pm 1}$ − /4. he lea ing or er o Ampere s la 2.14 is

$$\varepsilon \frac{\partial F}{\partial t} \quad \frac{ev_M}{l} \quad n M \frac{2F_1}{1 \; F_1^2}\Big|_1 \quad J\, t, \qquad 2.18$$

here v_M $l I_1$ M / $4\tau_e I_0$ M M I_1 μ I_0 M / I_1 M I_0 μ I_m s $\int_{-\pi}^{\pi}$ cos mk ln 1 $e^{s-\delta+\delta\cos k}$ dk δ / $2k_B T$ μ μ/ $k_B T$. μ M $k_B T$ calc late graphicall in Fig. 1 o e . 3 sol es 2.3 ith n N_D.

he sol tion o 2.15 iel s $J^{(1)}$ in 2.14 hich is the rst correction to the 2.18 . he etails can be o n in e erences 4 an 8 or the n merical proce re . An important point is that the nonlocal terms in the re ire that bo n ar con itions be impose on the inter als −2l, 0 an N l, N l 2l or a N perio S . Fig. 1 sho s the c rrent sel oscillations that appear hen the is sol e ith bo n ar con itions $\varepsilon\partial F/\partial t$ σF J at each point o the inter als −2l, 0 an N l, N l 2l an appropriate σ an c oltage bias. arameter al es correspon to a 15 perio 3. 4 nm aAs 0.93 nm AlAs S at 5 ith N_D 4.5 $\times 10^{10}$ cm^{-2} ν_i $2\nu_e$ 18×10^{12} n er a c oltage bias o 1. 2 hich iel x_0 1 nm t_0 0.24 ps J_0 $ev_M N_D / l$ 1.10×10^5 A cm^2. atho e an ano e contact con cti ities σ are 2.5 an 0. 2 $^{-1}cm^{-1}$ respecti el .

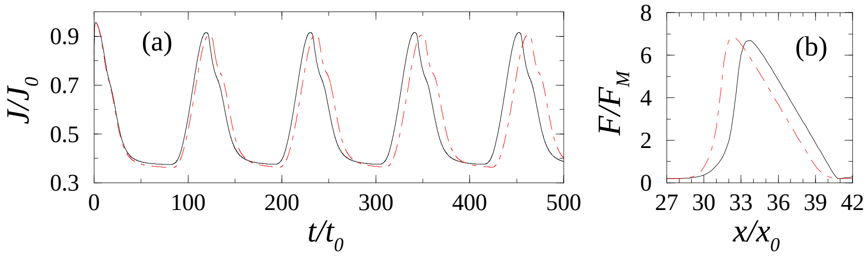

Fig. 1. (a) Current vs. time during self-oscillations, and (b) fully developed dipole wave. Solid line: QDDE, dashed line: GDDE.

3. Wigner description of a two-miniband superlattice

e shall consi er a 2 × 2 amiltonian $H\; x, -i\partial/\partial x$ in hich

$$H\; x, k \quad h_0\; k \; - eW\; x \; \sigma_0 \quad h\; k \cdot \sigma$$

$$\equiv \begin{pmatrix} \alpha \quad \gamma \;\; 1 - \cos kl \;\; - eW\; x \quad g & -i\beta \sin kl \\ i\beta \sin kl & \alpha - \gamma \;\; 1 - \cos kl \;\; - eW\; x \; - g \end{pmatrix}. \quad (3.1)$$

hen $h_0 \quad \alpha\; 1 - \cos kl \quad h_1 \quad 0 \quad h_2 \quad \beta \sin kl \quad h_3 \quad \gamma\; 1 - \cos kl \quad g$ an

$$\sigma_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (3.2)$$

he amiltonian 3.1 correspon s to the simplest 2 × 2 ane mo el in hich the a ratic an linear terms $kl\; ^2/2$ an kl are replace b $1 - \cos kl$ an $\sin kl$ respecti el . For a S ith t o miniban s $2g$ is the miniban gap an $\alpha \quad _1 \quad _2\; /4$ an $\gamma \quad _1 - \quad _2\; /4$ pro i e $_1$ an $_2$ are the miniban i ths. n the case o a lateral S $g \quad \gamma \quad 0$ an $h_2\sigma_2$ correspon s to the precession term in the ashba spin orbit interaction.[7] he other term the inters bban co pling epen s on the moment m in the y irection an e ha e not incl e it here. Small mo i cations o 3.1 represent a single miniban S ith il te magnetic imp rities in the presence o a magnetic el B: $g \quad \gamma \quad h_2 \quad 0$ an $h_1 \quad \beta\; B$.[6] As in the case o a single miniban S $W\; x$ is the electric potential.

he energ miniban s $E^{\pm}$ k are the eigen al es o the ree amiltonian H_0 k h$_0$ k σ_0 h k $\cdot \sigma$ an are gi en b

$$E^{\pm} k \quad h_0 k \pm |h k|. \tag{3.3}$$

he correspon ing spectral pro ections are $P^{\pm}$ k $\sigma_0 \pm \nu$ k $\cdot \sigma$ /2 ith ν h/|h k | so that e can rite H_0 k E^{+} k P^{+} k E^{-} k P^{-} k .

e shall no rite the e ations or the igner matrix rit ten in terms o the a li matrices σ_i:

$$f \; x,k,t \quad \sum_{i=1}^{3} f^{i} \; x,k,t \; \sigma_i \quad f^{0} \; x,k,t \; \sigma_0 \quad f \; x,k,t \; \cdot \sigma. \tag{3.4}$$

he igner components are real an can be relate to the coe cients o the ermitian igner matrix b f_{11} f^0 f^3 f_{12} $f^1 - if^2$ f_{21} f^1 if^2 f_{22} $f^0 - f^3$. ereina ter e shall se the e i alent notations

$$f \quad \begin{pmatrix} f^0 \\ f \end{pmatrix} \quad \begin{pmatrix} f^0 \\ f^1 \\ f^2 \\ f^3 \end{pmatrix}. \tag{3.5}$$

he pop lations o the miniban s ith energies $E^{\pm}$ are the moments:

$$n^{\pm} \; x,t \quad \frac{l}{2\pi} \int_{-\pi/l}^{\pi/l} f^0 \; x,k,t \; \pm \nu \cdot f \; x,k,t \; dk, \tag{3.}$$

an the total electron ensit is n^+ n^-.

e shall restrict o rsel es to the ashba case g γ h_3 0 rom no on. hen ν 0, 1, 0 an $n^{\pm}$ are the ensities o electrons ha ing spin $\pm$. A ter some algebra e can obtain the ollo ing e ations or the igner components

$$\frac{\partial f^0}{\partial t} \quad \frac{\alpha}{} \sin kl \quad {}^{-}f^0 \quad \frac{\beta \cos kl}{} \quad {}^{-}f^2 - \quad f^0 \quad Q^0 \; f \; , \tag{3.}$$

$$\frac{\partial f}{\partial t} \quad \frac{\alpha \sin kl}{} \quad {}^{-}f \quad \frac{\beta}{} \nu \quad {}^{-}f^0 \cos kl \quad {}^{+} \; \nu \times f \; \sin kl$$

$$- \quad f \quad Q \; f \; , \tag{3.8}$$

$$\varepsilon \frac{\partial^2 W}{\partial x^2} \quad \frac{e}{l} \; n^+ \quad n^- - N_D \; , \tag{3.9}$$

$$f^{i} \; x,k,t \quad \sum_{j=-\infty} \frac{ejl}{i} F \; x,t \; {}_{j} e^{ijkl} f^{i}_{j} \; x,t \; . \tag{3.10}$$

r collision mo el contains t o terms: a term hich tries to sen $f^0 \pm f^2$ to its local e ilibri m approximate b olt mann statistics at T 300 an a scattering term hich tries to e ali e n^+ an n^-:[6]

$$Q^0 f \quad -\frac{f^0 - \quad ^0}{\tau}, \quad Q f \quad -\frac{f - }{\tau} - \frac{f}{\tau_{sc}}, \qquad 3.11$$

$$_0 \quad \frac{\varphi^+ \quad \varphi^-}{2}, \qquad \frac{\varphi^+ - \varphi^-}{2} \nu, \qquad 3.12$$

$$\varphi^\pm \; k \; n^\pm \quad \frac{m^* k_B T}{\pi^2 \hbar^2} \exp \frac{\mu^\pm - E^\pm \; k}{k_B T}, \qquad 3.13$$

$$\frac{l}{2\pi} \int_{-\pi/l}^{\pi/l} \varphi^\pm \; k \; n^\pm \; dk \quad n^\pm. \qquad 3.14$$

n 3.13 $\mu^\pm$ $\mu^\pm$ $n^\pm$ sol e 3.14 . r collision mo el satis es charge contin it . n act 3. to 3.9 iel :

$$\frac{\partial n^\pm}{\partial t} \quad \frac{l}{2\pi\hbar} - \int_{-\pi/l}^{\pi/l} \alpha \sin kl \;\; f^0 \pm f^2 \quad \beta \cos kl \;\; f^2 \pm f^0 \;\; dk$$

$$\frac{l}{2\pi} - \int_{-\pi/l}^{\pi/l} Q^0 f \; \pm Q^2 f \;\; dk \quad \frac{n^+ - n^-}{\tau_{sc}}, \qquad 3.15$$

here e ha e emplo e $\int f^{0,2} dk$ 0. hen e obtain:

$$\frac{\partial}{\partial t} \; n^+ \quad n^- \quad - \frac{l}{\pi\hbar} \int_{-\pi/l}^{\pi/l} \alpha \sin kl \, f^0 \quad \beta \cos kl \, f^2 \;\; dk \quad 0. \; 3.1$$

Since $^-u \; x$ $l \, \partial u \; x \; _1 / \partial x$ 3.1 pro i es charge contin it . From 3.9 an 3.1 e get Ampere s la J t is the total c rrent ensit :

$$\varepsilon \frac{\partial F}{\partial t} \quad \frac{el}{\pi\hbar} \int_{-\pi/l}^{\pi/l} \alpha \sin kl \, f^0 \quad \beta \cos kl \, f^2 \;\; dk \quad _1 \quad J \; t \; . \qquad 3.1$$

4. uantum drift-diffusion equations

n the simpler case o a lateral S ith the precession term o ashba spin orbit interaction b t no inters bban co pling e can obtain explicit rate e ations or $n^\pm$ b means o the metho . he general case 3.1 ill be treate else here. First o all e sho l eci e the or er o magnit e o the terms in the e ations 3. an 3.8 in the h perbolic limit. ecall that in this limit the collision re enc $1/\tau$ an the loch re enc $eF_M l/\hbar$ are o the same or er abo t 10 or the S o Section 2. he scattering time τ_{sc} is m ch longer than the collision time τ an e shall consi er τ/τ_{sc} O λ 1. From 3. an 3.8 e can rite the scale e ations as ollo s:

$$Lf \equiv f - \tau \quad f \quad - \lambda \quad \tau \frac{\partial f}{\partial t} \quad f \quad , \qquad 4.1$$

$$f \quad \frac{\tau}{\tau_{sc}} \quad \begin{matrix} 0 \\ f \end{matrix} \quad \frac{\alpha\tau}{} \sin kl \quad ^{-} f \quad \frac{\beta\tau}{} \cos kl \quad - \begin{pmatrix} f^2 \\ 0 \\ f^0 \\ 0 \end{pmatrix} \qquad 4.2$$

$$\frac{\beta\tau}{} \sin kl \quad ^{+} \begin{pmatrix} 0 \\ f^3 \\ 0 \\ -f^1 \end{pmatrix} .$$

o eri e the re ce balance e ations e se the ollo ing ansat :

$$f \quad x, k, t \quad f^{(0)} \quad k \quad n^+, n^-, F \quad \sum_{m=1} f^{(m)} \quad k \quad n^+, n^-, F \quad \lambda^m, \qquad 4.3$$

$$\varepsilon \frac{\partial F}{\partial t} \quad \sum_{m=0} J_m \quad n^+, n^-, F \quad \lambda^m \quad J \quad t \; , \qquad 4.4$$

$$\frac{\partial n^{\pm}}{\partial t} \quad \sum_{m=0} A^{\pm}_m \quad n^+, n^-, F \quad \lambda^m . \qquad 4.5$$

$A^{\pm}_m$ an J_m are relate thro gh the oisson e ation 3.9 so that

$$A^+_m \quad A^-_m \quad -\frac{l}{e} \frac{\partial J_m}{\partial x} . \qquad 4.$$

nserting 4.3 to 4.5 into 4.2 e get

$$Lf^{(0)} \quad , \qquad 4.$$

$$Lf^{(1)} \quad - \tau \left. \frac{\partial f^{(0)}}{\partial t} \right|_0 - \quad f^{(0)}, \qquad 4.8$$

$$Lf^{(2)} \quad - \tau \left. \frac{\partial f^{(1)}}{\partial t} \right|_0 - \quad f^{(1)} - \tau \left. \frac{\partial f^{(0)}}{\partial t} \right|_1 , \qquad 4.9$$

an so on. he s bscripts 0 an 1 in the right han si e o these e ations mean that e replace $\varepsilon\, \partial F/\partial t|_m$ $J\delta_{0m} - J_m$ $\partial n^{\pm}/\partial t|_m$ $A^{\pm}_m$. Moreo er inserting 4.3 into 3. iel s the ollo ing compatibilit an sol abilit con itions:

$$f_0^{(m)0} \quad f_0^{(m)2} \quad 0 \quad Lf^{(m)0}{}_0 \quad Lf^{(m)2}{}_0 \quad 0, \quad m \geq 1. \qquad 4.10$$

o sol e 4. or $f^{(0)} \equiv \phi$ e rst note that

$$-\tau \quad \phi \quad \sum_{j=-\infty} i\vartheta_j \phi_j e^{ijkl}, \quad \vartheta_j \equiv \frac{\tau e j l}{\ } F_j. \qquad 4.11$$

hen 4. an 3.12 iel

$$\phi_j^0 \quad \frac{\varphi_j^+ \quad \varphi_j^-}{2} \frac{1 - i\vartheta_j}{1 \quad \vartheta_j^2}, \quad \phi_j^1 \quad \phi_j^3 \quad 0, \quad \phi_j^2 \quad \frac{\varphi_j^+ - \varphi_j^-}{2} \frac{1 - i\vartheta_j}{1 \quad \vartheta_j^2}, \quad 4.12$$

here e ha e se that the Fo rier coe cients $\varphi_j^\pm$ are real beca se $\varphi^\pm$ are e en nctions o k. Similarl the sol tion o 4.8 is $f^{(1)} \equiv \psi$ ith

$$\psi_j^m \quad r_j^m \frac{1 - i\vartheta_j}{1 \quad \vartheta_j^2} \quad m \quad 0, 2 \ , \quad \psi_j^1 \quad \psi_j^3 \quad 0. \qquad 4.13$$

ere r is the right han si e o 4.8 . he balance e ations can be o n b calc lating $A_m^\pm$ or m 0, 1 rom the sol abilit con itions or 4.8 an 4.9 or b simpl inserting the sol tions 4.12 an 4.13 in 3.15 an 3.1 . n both cases the res lt is:

$$\frac{\partial n^\pm}{\partial t} \quad {}^- D_\pm \ n^+, n^-, F \quad R \ n^+, n^-, F \ , \qquad 4.14$$

$$\varepsilon \frac{\partial F}{\partial t} \quad e D_+ \quad D_- {}_1 \quad J, \qquad 4.15$$

$$D_\pm \quad -\frac{\alpha}{\ } \ \mathrm{m} \ \phi_1^0 \pm \phi_1^2 \quad \psi_1^0 \pm \psi_1^2$$

$$\pm \frac{\beta}{\ } \ \mathrm{e} \ \phi_1^0 \pm \phi_1^2 \quad \psi_1^0 \pm \psi_1^2 \ , \quad R \quad \frac{n^+ - n^-}{2\tau_{sc}}. \qquad 4.1$$

A straight or ar calc lation o 4.1 iel s

$$D_\pm \quad \frac{\alpha\vartheta_1 \pm \beta \ \varphi_1^\pm}{\ 1 \quad \vartheta_1^2} \quad \frac{\tau \ \varphi_1^+ - \varphi_1^- \quad 2\alpha\vartheta_1 \pm \beta \ 1 - \vartheta_1^2}{2 \tau_{sc} \ 1 \quad \vartheta_1^2 \ {}^2} \qquad 4.1$$

$$\frac{2\alpha\vartheta_1 \pm \beta \ 1 - \vartheta_1^2 \ \ \tau}{\ 1 \quad \vartheta_1^2 \ {}^2} \frac{\partial \varphi_1^\pm}{\partial n^\pm} \quad - \quad \frac{\alpha\vartheta_1 \pm \beta}{\ 1 \quad \vartheta_1^2} \varphi_1^\pm \quad \pm \frac{n^+ - n^-}{2\tau_{sc}}$$

$$\frac{\alpha \ \ 3\vartheta_1^2 - 1 \ \pm \beta\vartheta_1 \ 3 - \vartheta_1^2}{\ 1 \quad \vartheta_1^2 \ {}^3} \frac{l\tau^2 \varphi_1^\pm}{\ \varepsilon} \frac{J}{e} - \frac{\alpha \ \ \varphi_1^+ \quad \varphi_1^- \ \vartheta_1}{\ 1 \quad \vartheta_1^2} \ {}_{1 \quad 1}$$

$$- \frac{\beta \ \ \varphi_1^+ - \varphi_1^-}{\ 1 \quad \vartheta_1^2} \ {}_{1 \quad 1} - \frac{\alpha^2 \quad \beta^2 \ \tau}{2 {}^2 \ 1 \quad \vartheta_1^2} \quad {}^- n^\pm$$

$$\frac{\tau}{2 {}^2 \ 1 \quad \vartheta_1^2} \quad \alpha^2 - \beta^2 \ 2\alpha\beta\vartheta_1 \quad - \quad \frac{\varphi_2^\pm}{1 \quad \vartheta_2^2}$$

$$\beta^2 - \alpha^2 \ \vartheta_1 \ 2\alpha\beta \quad - \quad \frac{\vartheta_2 \varphi_2^\pm}{1 \quad \vartheta_2^2} \quad .$$

he ollo ing al es o the parameters are t pical o a aAs Al aAs S : α 10 me β 2.1 me l 5 nm T 300 τ 10^{-13} s τ_{sc} 10^{-12} s N_D 10^{10} cm^{-2}. Fig re 2 sho s the electron elocit v $Jl/\, eN_D$ meas re in nits o v_M $\alpha l/$. $\times$ 10^6 cm s as a nction o el meas re in nits o F_M 13.2 cm at hich the electron elocit reaches its maxim m or a homogeneo s sol tion o . 4.15 ith constant F an n^+ n^- $N_D/2$. e obser e that there is a local maxim m ollo e b a region o negati e slope negati e i er ential elocit hich s ggests a nn t pe instabilit as in Section 2: sel s staine oscillations o the c rrent thro gh the S e to motion o charge ipole a es n er s cientl high c oltage bias.[1,2,4]

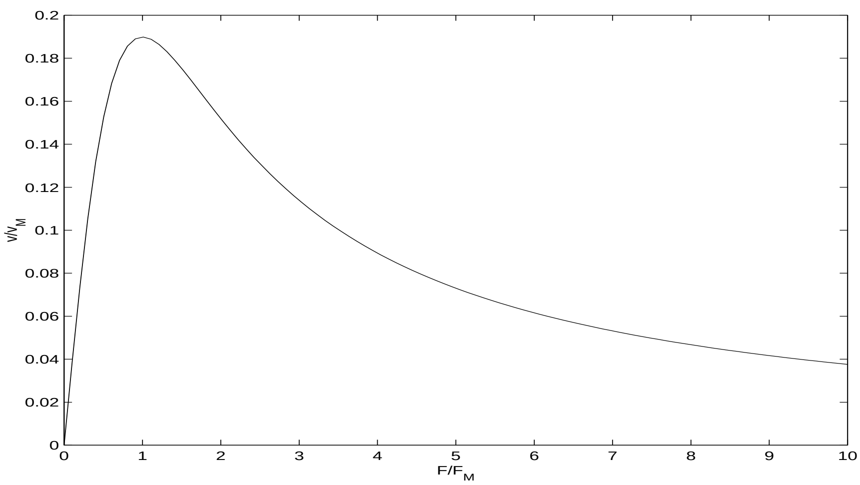

Fig. 2. Electron velocity vs. field for a homogeneous solution of (4.14) - (4.17).

5. Conclusions

For strongl co ple S s ha ing onl one pop late miniban e ha e ritten a igner oisson s stem o e ations an eri e a ant m ri t i sion e ation or the el b sing the hapman ns og pert r bation metho . ith appropriate oltage bias a n merical sol tion o this e ation iel s sel s staine oscillations o the c rrent e to rec cling an motion o charge ipole omains. For S s ha ing t o pop late mini ban s co ple thro gh a ashba spin orbit interaction e ha e intro ce

a perio ic ersion o the ane amiltonian an eri e the correspon ing s stem o e ations. sing the metho e ha e eri e ant m ri t i sion e ations or the miniban pop lations hich con tain generation recombination terms. he spatiall homogeneo s sol tion o these e ations pro i es an electron elocit hich has a region o neg ati e slope as a nction o el . his hints to the possible existence o oscillator instabilities an sel oscillations o the c rrent e to motion o charge ipole a es n er s cientl high c oltage bias.

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