Asymptotic power of goodness of fit tests based on Wasserstein distance

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We present a preliminary study for the power of Wasserstein goodness of fit test. Under $H_0$, $X_1, \ldots, X_n$ are i.i.d. with distribution function $F$, density function $f$ and quantile function $F^{-1}$. The Wasserstein test is based on the statistic:

$$n \int_0^1 (F_n^{-1}(t) - F^{-1}(t))^2 \, dt - a_n,$$

where $F_n^{-1}$ is the empirical quantile function and $a_n$ are some coefficients which depend on $n$ and on $F$. Under tail conditions, this statistic converges to

$$\int_0^1 B^2(t) - t(1-t) \, f^2(F(t)) \, dt,$$

where $B$ is a Brownian bridge (see, e.g., del Barrio et al. [1]). Let $W$ be the Brownian motion such that $B(t) = W(t) - tW(1)$. We prove that almost surely, (1) is equal to

$$\int \int \tilde{K}(s,t)dW(s)dW(t),$$

where $\tilde{K}$ is a kernel associated to $F$. The multiple integral must be understood as in Nualart [2].

Similarly to this expression, we derive a new statistic based on the kernel $\tilde{K}$, which has interesting regularity properties. This statistic can be completely treated under some contiguous alternatives within the frame of Gaussian shifts (see, e.g., Janssen [3]).

References

