Design of continuos regression tests by transforming the accumulated residuals process.

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Transformed empirical processes have been used since [**?**] (see [**?**] and nested references) in the construction of goodness-of-fit tests, consistent for all alternatives, focused on the sequences of contiguous alternatives chosen by the user, and asymptotically distribution free under the null hypothesis of fit as well as under the alternatives of focusing. After transforming, these limiting laws are the same, regardless there are estimated parameters or not.

In this work we consider sequences of linear models with continuous regressors $x_h, h = 0, 1, \ldots, p-1$, that, eventually changing variables to obtain a normalized form, is written as

$$Y_{n,j} = \sum_{h=0}^{p=1} \beta_h x_h(j/(n+1)) + \sigma Z_{n,j}, Z_{n,j} \text{ i.i.d.}(j = 1, \ldots, n), EZ_{n,1} = 0, EZ_{n,1}^2 = 1.$$

With suitable consistent estimators $\hat{\beta}_{n,h}(h = 0, \ldots, p-1), \hat{\sigma}$ of the parameters, the estimated residuals are $\hat{e}_{n,j} = \hat{\sigma}_n^{-1}(Y_{n,j} - \sum_{h=0}^{p=1} \hat{\beta}_h x_h(j/(n+1)))$. We introduce the process of accumulated estimated residuals (AER) $\hat{r}_n(t) = \sum_{j=1}^{[nt]} \hat{e}_{n,j}$ that under infill asymptotics ($n \to \infty$), converges in law to a standard Wiener process on $[0, 1]$.

The same kind of transformation applied to the empirical process in the articles referred above is applied in the present work to the AER process, leading to a *transformed accumulated residual process* that is also asymptotically Gaussian, from which Cramér-von Mises tests statistics of Watson type ([**?**]) are constructed, thus providing tests with are also consistent under all alternatives, can be focused to the alternatives of interest of the user, and have distribution free asymptotic laws.

As a particular application, a simple test for polynomial regression exploiting properties of the Legendre Polynomials is constructed and its behaviour is described empirically.

[1] Cabaña, A., Transformations of the empirical measure and Kolmogorov-Smirnov tests, *Ann. Statist.* **24** (1996) 2020-2035.

[2] Cabaña, A., Cabaña, E.M., Goodness-of-fit to the exponential distribution, focused on Weibull alternatives, *Communications in Statistics, Simulation and Computation* **34** (2005), 711-724.

[3] Watson, G. S., Goodness - of - fit tests on a circle, *Biometrika* **48** (1961) 109-114.